

Combination of Adaptive Filter Algorithms for Noise Cancellation

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Abstract: In today's digital communication and mobile multimedia world, adaptive signal processing is one of the most important areas of real time DSP implementation. The ability of adaptive filter to operate satisfactorily in an unknown environment and track time variations of input statistics makes the adaptive filter a powerful device for real time signal processing and control applications. The main objective of this project is to cancel the noise by using the Least-Mean-Square algorithm, is one of the most widely used algorithm for adaptive signal processing because of its simplicity and robustness. But, its performance in terms of convergence rate and tracking capability depends on the Eigen value spread of the input signal correlation matrix. By using an approach of combining transversal filtering and linearly constrained optimization, a new structure for the affine combination is proposed. Furthermore, an optimal affine combiner is found using two approaches like stochastic gradient approach and error power based scheme is proposed. The interpretations of the affine combination as a linearly constrained processing are then considered in adaptive filtering, and a power normalized and time-varying step-size LMS algorithm is suggested for updating the parameters of the proposed scheme. Finally, simulation results obtained with the algorithm are presented and compared with the standard LMS and recursive least squares (RLS) algorithms. The ability of adaptive filter to operate satisfactorily in an unknown environment and track time variations of input statistics makes the adaptive filter a powerful device for real time signal processing and control applications.

1. INTRODUCTION

Digital Signal processing has become one of the very important fields in the areas of Communication, Speech Processing, Instrumentation and control systems, Image processing, Industrial automation, Robotics, computer vision etc. In today's digital communication and mobile multimedia world, adaptive signal processing is one of the most important areas of real time DSP implementation. Adaptive filters learn the statistics of their operating environment and continually adjust their parameters accordingly. The Least-Mean-Square algorithm is one of the most widely used algorithms for adaptive signal processing because of its simplicity and robustness. But, its performance in terms of convergence rate and tracking capability depends on the Eigen value spread of the input signal correlation matrix. An example of a wideband signal whose Fourier spectrum overlaps a narrowband interference signal.

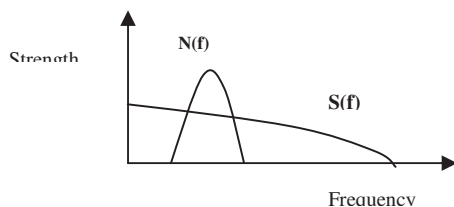


Fig. 1 A Strong narrowband interference $N(f)$ in a wideband signal $S(f)$

This situation can occur frequently when there are various modulation technologies operating in the same range of frequencies. In fact, in mobile radio systems co-channel interference is often the limiting factor rather than thermal or other noise sources. It may also be the result of intentional signal jamming, a scenario that regularly arises in military operations when competing sides intentionally broadcast signals to disrupt their enemies' communications. Furthermore, if the statistics of the noise are not known a priori, or change over time, the coefficients of the filter cannot be specified in advance. In these situations, adaptive algorithms are needed in order to continuously update the filter coefficients.

2. ADAPTIVE FILTERING

The goal of any filter is to extract useful information from noisy data. Whereas a normal fixed filter is designed in advance with knowledge of the statistics of both the signal and the unwanted noise, the adaptive filter continuously adjusts to a changing environment through the use of recursive algorithms. This is useful when either the statistics of the signals are not known beforehand or change with time. The operation of a linear adaptive filtering algorithm involves two basic processes:

- A filtering process designed to produce an output in response to a sequence of input data and

- An adaptive process, the purpose of which is to provide a mechanism for the adaptive control of an adjustable set of parameters used in the filtering process.

These two process work interactively with each other. Naturally, the choice of a structure for the filtering process has a profound effect on the operation of the algorithm as a whole. The discrete adaptive filter accepts an input $u(n)$ and produces an output $y(n)$ by a convolution with the filter's weights, $w(k)$. A desired reference signal, $d(n)$, is compared to the output to obtain an estimation error $e(n)$. For a FIR filter with N coefficients the output of the adaptive filter can be expressed as

$$y(n) = W^T(n)u(n)$$

Where $W = [w(0), w(1), \dots, w(M-1)]^T$ is a vector containing the coefficients of the FIR filter, and $U(n) = [U(n), U(n-1), \dots, U(n-M+1)]^T$ is a vector containing the input samples. The error signal, which has been mentioned in the previous paragraph, can be expressed as

$$e(n) = d(n) - y(n)$$

This error signal is used to incrementally adjust the filter's weights for the next time instant. Several algorithms exist for the weight adjustment, such as the Least-Mean-Square (LMS) and the Recursive Least-Squares (RLS) algorithms. The choice of training algorithm is dependent upon needed convergence time and the computational complexity available, as statistics of the operating environment.

An adaptive algorithm tries to minimize an objective function, usually denoted as J_w . The choice of this function has a profound impact on the properties of the whole algorithm. It influences the rate at which the iterative adjustment process converges to its steady state. This state represents a point at which the objective function achieves its minimal value. Moreover, the objective function determines the robustness of the system and its stability. By carefully selecting the objective function we can condition its shape to have a continuous, convex character with a single "easy-to-track" minimum.

Among the most popular objective functions that are used in adaptive systems are The mean-square error (MSE):

$$J_w = E[e^2(n)]$$

The least-square error (LS):

$$J_w = \frac{1}{N} \sum_{i=1}^N e^2(i)$$

The weighted least-squares error (WLS):

$$J_w = \sum_{i=1}^N \lambda^{(N-i)} e^2(i)$$

the signal $e(n)$ usually denotes an estimation error. The index w in the definition of the functions above signifies their dependence on the tap-weight vector w . Since adaptive methods are iterative, the values of the tap-weights are continuously updated based on the current value of the objective function. It is therefore one of the most useful quantities to describe the state of the filtering process. In this project, Mean-square error is used as Object function because of its advantages over the other two, such as it leads to tractable mathematics. In particular, the choice of the mean-square-error criterion results in second-order dependence for the object function on the unknown coefficients in the impulse response of the filter. Moreover, the object function has a distinct minimum that uniquely defines the optimum statistical design of the filter.

Properties of adaptive algorithms

There is no unique solution to the linear adaptive filtering problem. Rather, we have a kit of tools represented by a variety of recursive algorithms, each of which offers desirable features of its own. However in a wide sense the performance of an adaptive filtering algorithm is evaluated based on one or more of the following factors

Convergence rate It is the primary parameter at which we look when comparing different adaptive algorithms together. It determines the number of iterations required to get to the vicinity of a steady-state solution. It is desirable to achieve the highest rate possible. Since in many applications the system has to meet stringent deadlines, the convergence must be fast enough to meet the deadlines and not to affect the performance. This is also the case of speech signal processing, since speech is considered stationary only in short frames not longer than 30 ms.

Computational Requirement

The parameters of interest include the number of operations required to complete one iteration of the algorithm and the amount of memory needed to store the required data and also the program. These quantities influence the price of the computer needed to implement the adaptive filter.

Misadjustment

This quantity describes steady-state behavior of the algorithm. This is a quantitative measure of the amount by which the ensemble averaged final value of the mean-squared error exceeds the minimum mean-squared error produced by the optimal wiener filter.

Robustness This may be viewed as a combined requirement of maximum immunity against internal errors, such as

quantization and round-off errors and insensitivity to external errors. Sometimes, however, it is better for the algorithm to be sensitive to certain changes of the environment, such as non stationary of speech signals and noise processes. The trade-off between sufficient sensitivity and relative robustness is often a difficult task to solve.

Numerical Properties When an algorithm is implemented numerically, inaccuracies are produced due to quantization error. An adaptive filtering algorithm is said to be numerically robust when it is insensitive to variation in the word length used in its digital implementation.

3. AN AFFINE COMBINATION OF TWO ADAPTIVE FILTERS

3.1 Introduction

The design of many adaptive filters requires a tradeoff between convergence speed and steady-state mean-square error (MSE). A faster (slower) convergence speed yields a larger (smaller) steady-state mean-deviation (MSD) and MSE. This property is usually independent of the type of adaptive algorithm, i.e., least mean-square (LMS), normalized least mean-square (NLMS), recursive least squares (RLS), or affine projection (AP). This design tradeoff is usually controlled by some design parameter of the weight update, such as the step size in LMS or AP, the step size or the regularization parameter in NLMS or the forgetting factor in RLS. Variable step-size modifications of the basic adaptive algorithms offer a possible solution to this design problem. A Combination algorithm is proposed, which uses a convex combination of two fixed step-size adaptive filters as shown in Fig. 2, where adaptive filter $W_1(n)$ uses a larger step size than adaptive filter $W_2(n)$.

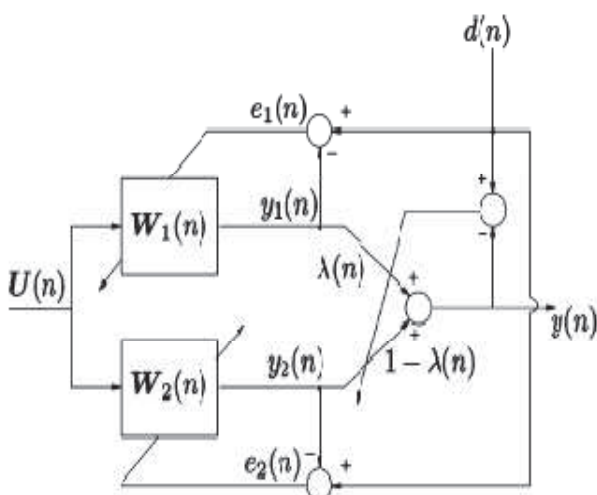


Fig. 2. Affine Combination of Two-LMS Filters

The key to this scheme is the selection of the scalar mixing parameter $\lambda(n)$ for combining the two filter outputs. The mixing parameter is defined as a sigmoid function the quadratic error of the overall filter, whose free parameter is adaptively optimized using a stochastic gradient search which minimizes the achievable performance. The achievable performance is studied for an affine combination of two LMS adaptive filters using the structure with stationary signals. Here, the combination parameter $\lambda(n)$ is not restricted to the range $(0,1)$. Each adaptive filter is estimating the unknown channel impulse response using the same input data. Thus, $W_1(n)$ and $W_2(n)$ are statistically dependent estimates of the unknown channel. There exists a single combining parameter sequence $\lambda(n)$ which minimizes the MSD.

The parameter $\lambda(n)$ does not necessarily lie within $(0,1)$ for all n . Thus, the output $y(n)$ is an affine combination of the individual outputs $y_1(n)$ and $y_2(n)$. The convex combination is a particular case. The adaptive scheme is first studied from the view point of an optimal affine combiner. The value of $\lambda(n)$ that minimizes the MSE for each n conditioned on the filter parameter at iteration n is determined as a function of the unknown system response. This leads to an optimal affine sequence $\lambda_0(n)$. The statistical properties of an optimal affine combiner are then studied. It is shown that $\lambda(n)$ can be outside of the interval $(0,1)$ for several iterations. Most importantly, $\lambda_0(n)$ is usually negative in steady-state.

It is of interest to compare the performance of the adaptive filter using a sub optimal but feasible adjustment algorithm for with that of the optimal affine combiner. Although the latter is unrealizable, its performance provides an upper bound on the performance of any realizable affine combiner. Suppose a sub optimal (but realizable) algorithm leads to a performance close to that of the optimal affine combiner. Then, there is sufficient motivation for a more detailed study of the algorithm with respect to analysis and implementation issues. Finally, two realizable schemes for updating $\lambda(n)$ are proposed. The first scheme is based on a stochastic gradient approximation to $\lambda_0(n)$. The second scheme is based on the relative values of averaged estimates of the individual error powers. Both schemes are briefly studied, and their performances are compared to that of the optimal affine combiner. Numerical results support the theoretical finding and show that the analysis closely predicts the probabilistic behavior of the algorithms as observed in Monte Carlo simulations, especially in the neighborhood of the intersection of the MSDs of the individual filters when the hand-off from one filter to the other filter occurs.

3.2 Optimal Affine Combiner

The system under investigation is shown in Fig4.1. Each filter uses the LMS adaptation rule but with different step sizes μ_1 and μ_2

$$W_i(n+1) = W_i(n) + \mu_i e_i(n) U(n), \quad i=1,2$$

Where $e_i(n) = d(n) - W_i^T(n)U(n)$

$$d(n) = e_0(n) + w_0^T U(n)$$

where $W_i(n), i=1, 2$ are the N-dimensional adaptive coefficient vectors, is assumed zero-mean, and statistically independent of any other signal in the system, and the input process is assumed wide-sense stationary. $U_i(n)$ is the input vector. It will be assumed, without loss, that $\mu_1 > \mu_2$, so that will, in general, $W_1(n)$ converges faster than $W_2(n)$. Also, $W_2(n)$ will converge to the lowest individual steady-state weight misadjustment. The weight vectors $W_1(n)$ and $W_2(n)$ are coupled both deterministically and statistically through $U(n)$ and $e_0(n)$.

The outputs of the two filters are combined as

$$Y(n) = \lambda(n)y_1(n) + [1-\lambda(n)] y_2(n)$$

Where $Y_i(n) = W_i^T(n)U(n) \quad i=1,2$ and

overall system error is $e(n) = d(n) - y(n)$

Equation can be re-written as

$$\begin{aligned} Y(n) &= \lambda(n) W_1^T(n) U(n) + [1-\lambda(n)] W_2^T(n) U(n) \\ &= \{ \lambda(n) [w_{12}(n) - w_2(n)] + w_2(n) \}^T U(n) \\ &= \{ \lambda(n) w_{12}(n) + w_2(n) \}^T U(n) \end{aligned}$$

where $W_{12}(n) = W_1(n) - W_2(n)$

$y(n)$ can be interpreted as a combination of $W_2(n)$ and a weighted version of the difference filter $W_{12}(n)$. It also shows that the combined adaptive filter has an equivalent weight vector given by

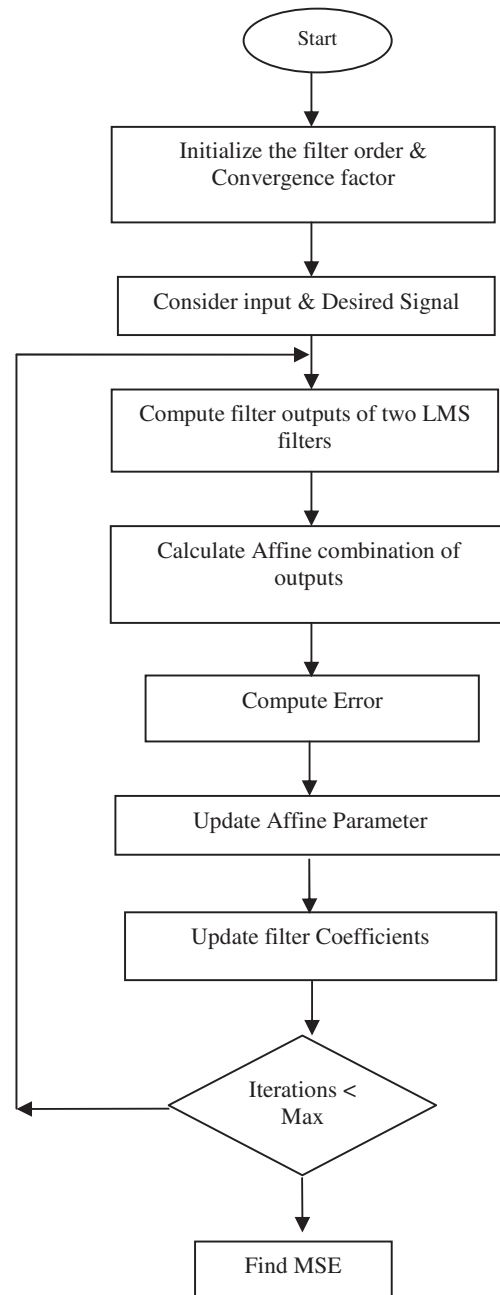
$$w_{eq} = \lambda(n) w_{12}(n) + w_2(n)$$

A rule to find λ , which minimizes MSE

$e(n) = e_0(n) + [w_{02}(n) - \lambda(n)w_{12}(n)]^T U(n)$ which is the expression for the optimum affine combiner, as a function of unknown system response.

3.3 Iterative Algorithms to Adjust Affine Combiner:

The previous derivation of the optimal linear combiner was based upon prior knowledge of the unknown system response. Clearly, this is not the case in reality.



Flow chart of Affine Combination Algorithm

Performance close to the optimal suggests that further analytical study of a new algorithm could be worth the effort. This is especially important for the adaptive combiner structure. A detailed algorithm is based upon a stochastic gradient search for the optimal. The other is based on the ratio of the average error powers from each individual adaptive filter.

4. PERFORMANCE ANALYSIS

In this chapter the simulations results of the developed affine algorithm is presented. For simulations a sinusoidal signal of frequency 500HZ is used as desired input. The input to the filter is a noisy signal consisting of multiple sine frequencies and Gaussian random noise. As discussed flow chart for the simulation, the input noisy signal and the desired signal and the filter parameters are same for all the four simulation procedures and they are characterized as follows-

Input signal parameters:

Amplitude: 1, Frequency: 500Hz, 1500Hz,

Sampling Frequency: 10000Hz, Initial Phase: 0

Noise Parameters:

Amplitude: 0.15, Type: Gaussian, Mean: 0, Variance: 1, Initial Seed: 10

Filter Parameters

Filter Type: FIR, Order: 32, Structure: Direct form-I

Window: Rectangular, Convergence Factor: time varying

Desired signal parameters:

Sinusoidal signal of 500Hz frequency with amplitude 1.

Table: Comparing MSE of LMS, RLS, DCT-LMS and Affine LMS algorithms

Iterations	standard	RLS	DCT	Affine LMS
50	0.0484	0.0272	0.0485	0.0465
100	0.0349	0.0146	0.0316	0.0314
150	0.0276	0.0104	0.0191	0.0236
200	0.0229	0.0062	0.0119	0.0186
250	0.0197	0.006	0.0097	0.0145
300	0.0183	0.0055	0.0092	0.0118
350	0.0148	0.0049	0.0091	0.009
400	0.0138	0.0045	0.009	0.0077
450	0.0126	0.0039	0.0089	0.0057
500	0.0124	0.0038	0.0089	0.0057

Performance comparison of adaptive algorithms:

We can observe MSE decreases, when filter adapts for more number of iterations. As the number of iterations increase,

filter approaches wiener filter. As iterations increase, acquisition time increases. So we should comprise between number of iterations and Mean square error.

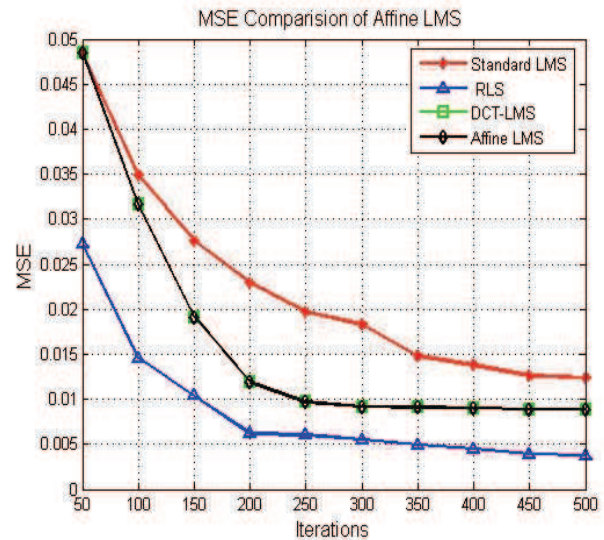


Fig 3.. 6.19 MSE comparisons of LMS, RLS, DCT-LMS and Affine LMS algorithm for denoising the noisy ECG signal

The good performance of the Affine algorithm can be observed in terms of convergence rate when compared with the standard LMS algorithm. As far as computational burden is concerned, its simplicity is noticed when compared with the RLS algorithm and DCT transform. More over its performance is close to that of RLS and DCT-LMS algorithms. Such an aspect, together with convergence improvement and less complexity, is definitive in justifying its application.

5. CONCLUSION

From the simulation results obtained it is proven that the developed algorithm for improving the convergence rate of the adaptive filters in real time is excellent compared to the standard LMS algorithm. And its performance is very close that of RLS and DCT-LMS algorithms, whose implementation is not possible in real time due to their computational complexity. The developed affine combination algorithm is used for system identification and denoising of the acoustic signal and in ECG signals using Mat lab and the results are compared with that of the standard LMS, DCT-LMS and RLS algorithms. It is observed that the performance of the RLS and DCT-LMS algorithms are better than the standard LMS and the proposed affine LMS, but as already stated their implementation is difficult and the computational complexity is more when compared to the standard LMS and Multi-Split LMS algorithm. So what LMS is the most widely used algorithm in the adaptive filters.

It is observed that the proposed affine combination algorithm performs better than the standard LMS algorithm in terms of MSE and also the convergence rate. That is for particular MSE of 0.0061 the standard LMS takes 650 iterations where as affine LMS takes only 250 iterations. Moreover, it is also observed that the proposed algorithm's performance is close to the performance of RLS algorithm. This project implements a new structure of affine combination of transversal filtering. The same procedure can also be repeated with time-varying step size algorithms and split adaptive transversal filtering. The input vector is split as low frequency part and high frequency part, each part is separately applied adaptive filtering algorithm, which leads to sub band adaptive algorithm.

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