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ψ^* Closed Sets in Topological Spaces

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Abstract: In this paper, we introduce a new class of sets called ψ^* closed sets and ψ* open sets in topological spaces and study some of their properties.

Keywords: ψ^* closed set , ψ^* open set.

1. INTRODUCTION

In 1970, Levine[6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham[5] introduced the concept of the closure operator cl* and studied some of their properties. S.P.Arya [5], P.Bhattacharyya and B.K.Lahiri [3], J.Dontchev [4], H.Maki, R.Devi and K.Balachandran [9], [10], P.Sundaram and A.Pushpalatha [12], A.S.Mashhour M.E.Abd El-Monsef and S.N. El-Deeb[11], D.Andrijevic [1], and S.N.Maheswari and P.C.Jain [9] introduced and investigated generalized semi closed sets , Semi generalized closed sets , generalized semi preclosed sets, α-generalized closed sets , generalized α-closed sets, strongly generalized closed sets, pre closed sets, semi preclosed sets, α-closed sets respectively. In this paper , we obtain a new generalization of closed sets in the weaker topological space (X, τ^*)

Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise pxpliciyly stated.For a subset of A of a topological space X, int(A), cl(A), cl*(A), scl(A), spcl(A), αcl(A) and A^c denote the interior, closure, closure*, semi closure, semi preclosure, α closure and complement of A respectively.

2. PRELIMINARIES

We recall the following definitions:

Definition 2.1 [6]: Let A subset A of a topological space (X, τ) is called a generalized closed set (g-closed) if cl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X.

Definition 2.2 [3]: Let A subset A of a topological space (X, τ) is called a semi-generalized closed (sg-closed) if scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is semiopen in X.

Definition 2.3 [2]: Let A subset A of a topological space (X, τ) is called a generalized semi closed (gs-closed) if scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X.

Definition 2.4 [8]: Let A subset A of a topological space (X, τ) is called an α open set if α closed set if cl(int(cl(A))) $\subseteq$ A.

Definition 2.5[9]: Let A subset A of a topological space (X, τ) is called a α-generalized closed (αg-closed) if αcl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X.

Definition 2.6 [10]: Let A subset A of a topological space (X, τ) is called a generalized α closed (gα-closed) if αcl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is α open in X.

Definition 2.7 [2]: Let A subset A of a topological space (X, τ) is called a generalized semi preclosed set(gsp -closed) if spcl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X.

Definition 2.8 [12]: Let A subset A of a topological space (X, τ) is called a strongly generalized closed set(strongly g closed) if cl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is g- open in X.

Definition 2.9[11] : Let A subset A of a topological space (X,τ) is called a preclosedset if cl(int(A)) $\subseteq$ A.

Definition2.10[7]: Let A subset A of a topological space(X,τ) is called a semiclosedset if int(cl(A)) $\subseteq$ A.

Definition 2.11[1]: Let A subset A of a topological space (X, τ) is called a semi preclosed set (β closed) if int(cl(int(A))) $\subseteq$ A.

The complements of the above mentioned sets are called their respective open sets.

Definition 2.12: For the subset A of a topological space X, the generalized closure operator cl* [5] is defined by the intersection of all g closed sets containing A.

Definition 2.13: For the subset A of a topological space (i)The semi closure of (briefly scl(A))[7] is defined as the intersection of all semi closed sets containing A

(i)The semi preclosure of A (briefly spcl(A))[1] is defined as the intersection of all semi preclosed sets containing A

(i)The α closure of A (briefly α cl(A))[8] is defined as the intersection of all α closed sets containing A

3. Ψ* CLOSED SETS IN TOPOLOGICAL SPACES

In this section, we introduce the concept of ψ^*closed sets in topological spaces

Definition 3.1 : A subset A of a topological space X is called ψ^* closed set if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is sg open. The complement of ψ^*closed set is called the ψ^* open .

Theorem 3.2 : Every closed set in X is ψ^* closed

Proof : Let A be a closed set. Let $A \subseteq G$ where G is sg open. Since A is closed. Cl(A) = A. but $cl^*(A) \subseteq cl(A)$. Thus we have $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is sg open.

Therefore A is ψ^* closed.

Theorem 3.3 : Every g-closed set in X is a ψ^* closed set but not conversely

Proof : Let A be a g-closed set. Assume that A c G and G is sg open. Then $cl(A) \subseteq G$, since A is g-closed. But $cl^*(A) \subseteq cl(A)$. Therefore $cl^*(A) \subseteq G$. Hence A is ψ^* closed.

The converse of the above theorem need not be true as seen from the following example

Example 3.4 : Consider the topological space X = {a,b,c} with topology τ = { φ, {a}, X}. Then the set is ψ^* closed but not g-closed.

Remark 3.5: The following example shows that is ψ^* closed sets are independent from sp-closed set, sg-closed set, α-closed set, pre closed set, gs-closed set, gsp-closed set, αg-closed, and gα-closed set.

Example 3.6 : Let X = {a,b,c} be a topological space

(i) Consider the topology τ = { φ, {a}, X}. Then the sets {a}, {a,b} and {a,c} are ψ^* closed but not sp-closed.

(ii) Consider the topology τ = { φ, {a,b}, X}. Then the sets {a} and {b} are sp closed but not ψ* closed

(iii Consider the topology τ = { φ, X}. Then the sets {a},{b}, {c}, {a,b}, {b,c} and {a,c} are ψ^* closed but not sg closed

(iv) Consider the topology τ = { φ,{a},{b}, {a,b}, X}. Then the sets {a}, and {b} are sg-closed but not ψ^* closed

(v) Consider the topology τ = { φ, {a,}, X}. Then the sets {a},{b},{c}, {a,b} and {a,c} are ψ^* closed but not α-closed

(vi) Consider the topology τ = { φ, {a},{a,b} X}. Then the set{b} is α-closed but not ψ^* closed.

(vii) Consider the topology τ = C{ φ, {a,}, X}. Then the sets {a}, {a.b} and {a,c} are ψ^* closed but not pre-closed

(viii) Consider the topology τ = { φ, {b},{a,b} X}. Then the set {a} is pre closed but not ψ^* closed.

(ix) Consider the topology τ = { φ, X}. Then the sets {a},{b},{c}, {a,b} and {a,c} are ψ^* closed but not gs-closed

(x) Consider the topology τ = { φ, {b},{a,b} X}. Then the set {a} is gs closed but not ψ^* Closed

(xi) Consider the topology τ = { φ,{a},{b}, {a,b}, X}. Then the sets {b} and {a,b} are gsp closed but not ψ^* closed

(xii) Consider the topology τ = { φ, {a,}, X}. Then the sets {a}, {a,b} and {a,c} are ψ^* closed but not gsp closed

(xiii) Consider the topology τ = C{ φ, {a,}, X}. then the set {a} is ψ^* closed but not αg-Closed

(xiv) Consider the topology τ = { φ, {b},{a,b} X}. Then the sets {a} is αg-closed but not ψ^* Closed

(xv) Consider the topology τ = { φ, {a},{b}, {a,b},{a,c}, X}. Then the set {b} is ψ^* closed but not is gα-closed

(xvi) Consider the topology τ = { φ, {b},{a,b} X}. Then the sets {a} is g α-closed but not ψ^* closed

Theorem 3.7: Union of two ψ^* closed sets in X is a ψ^* closed set in X

Proof : Let A and B be two ψ^* closed sets . Let AUB $\subseteq$ G, where G is sg-open. Since A and b are ψ^* closed sets, $cl^*(A)$ U $cl^*(B)$ $\subseteq$ G. But we know $cl^*(A)$ U $cl^*(B)$ = $cl^*(AUB)$. Therefore $cl^*(AUB)$ $\subseteq$ G. Hence AUB is a ψ^* closed set**.**

Theorem 3.8 : A subset A of X is ψ^* closed if and only if $cl^*(A)$-A contains no non empty sg closed set in X.

Proof : Let A be a ψ^* closed set. Suppose that F is a non empty sg closed subset of $cl^*(A)$-A.

Now F $\subseteq$ $cl^*(A)$-A. Then F $\subseteq$ $cl^*(A) \cap A^c$, since $cl^*(A)$-A = $cl^*(A) \cap A^c$. Therefore F $\subseteq$ $cl^*(A)$ and F $\subseteq$ A^c. Since F^c is sg-open set ans A is a ψ^* closed , $cl^*(A)$ $\subseteq$ F^c. That is F $\subseteq$ $[cl^*(A)]^c$. Hence F $\subseteq$ $cl^*(A) \cap [cl^*(A)]^c = \varphi$. That is F = φ , a contradiction. Thus $cl^*(A)$-A contain no non empty sg closed set in X.

Conversely assume that $cl^*(A)$-A contain no non empty sg closed set in X. Let A $\subseteq$ G, G is sg-open. Suppose that $cl^*(A)$ is not contained in G , then $cl^*(A)$)$\cap G^c$ is a non empty sg- closed set of $cl^*(A)$-A which is a contradiction. Therefore $cl^*(A) \subseteq$ G and hence A is ψ^* closed.

Corollary 3.9: A subset A of X is ψ^* closed if and only if $cl^*(A)$-A contain no non empty closed set in X.

Proof: The proof follows from the theorem (3.8) and the fact that every closed set is sg closed set in X.

Corollary 3.10 : : A subset A of X is ψ^* closed if and only if $cl^*(A)$-A contain no non empty open set in X.

Proof: The proof follows from the theorem (3.8) and the fact that every open set sg-open set in X.

Theorem 3.11: Let A be a ψ^* closed in (X, τ). Then A is g closed if and only if cl*(A) –A is sg open

Proof: Suppose A is g closed in X. Then cl*(A) = A and cl*(A) –A = φ, which is sg open in X. Conversely , suppose cl*(A) – A is sg open in X. Since A is ψ^* closed, by the theorem 3.8, cl*(A)-A contains no non empty ψ^* closed set in X. Then cl*(A)-A = φ. Hence A is g closed.

Theorem 3.12: For x □ X. The set X –{x} is ψ^* closed or sg open.

Proof: Suppose X –{x} isnot sg open. Then X is the only sg open set containing X –{x} . This implies cl*(X –{x}) $\subseteq$ X. Hence X-{x} is a ψ^* closed in X.

Remark 3.17: From the above discussion, we obtain the following implications

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