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Acceptance Sampling Based on Life Tests: Exponentiated Pareto Model

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Abstract: Exponentiated Pareto model is considered as a life-testing model. The problem of acceptance sampling when the life test is truncated at a pre-assigned time is discussed with known shape parameters. For various acceptance numbers, confidence levels and values of the ratio of the fixed experimental time to the specified mean life, the minimum sample size necessary to assure a specified mean life time worked out. The operating characteristic functions of the sampling plans are obtained. Producer's risk is also discussed. A table for the ratio of true mean life to a specified mean life that ensures acceptance with a pre-assigned probability is provided. The results are given by an example.

Keywords: Exponentiated Pareto model, Operating Characteristic function, sampling plans, Truncated life tests.

1. INTRODUCTION

The cumulative distribution function G(x) and probability density function g(x) of Type II Pareto distribution are respectively given by

$$G(x;\alpha,\theta)=1-\left(1+\frac{x}{\sigma}\right)^{-\alpha}, \quad x>0,\alpha>0,\sigma>0,\theta>0$$

$$g(x;\alpha,\theta)=\frac{\alpha}{\sigma}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}, \quad x>0,\alpha>0,\sigma>0,\theta>0$$

where σ is the scale parameter and α is shape parameter.

R.C. Gupta, R.D. Gupta and P.L. Gupta [16] (1998) introduced the exponentiated Pareto distribution as life time model, with cumulative distribution function

$$F(x;\alpha,\theta)=\left[1-\left(1+\frac{x}{\sigma}\right)^{-\alpha}\right]^{\theta}, \quad x>0,\alpha>0,\sigma>0,\theta>0 \qquad \ldots(1)$$

The probability density function with two shape parameters α and θ is given by

$$f(x;\alpha,\theta)=\frac{\alpha\theta}{\sigma}\left[1-\left(1+\frac{x}{\sigma}\right)^{-\alpha}\right]^{\theta-1}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}$$
$$x>0,\alpha>0,\sigma>0,\theta>0 \quad \ldots(2)$$

when θ=1, the above model corresponds to Pareto distribution of the second kind.

Acceptance sampling with new life test objectives is given by M. Sobel, J. A. Tischendrof [14] (1959). Acceptance sampling plans based on truncated life tests are studied by Several authors: S.S. Gupta, P. A. Groll [5] (1961) developed accepting sampling plans based on life tests for Gamma distribution; H.P. Goode and J. H. K. Kao [4] (1961) for Weibull; R.R.L. Kantam, K. Rosaiah and G. Srinivisarao [7] (2001) for log logistic distribution; K. Rosaiah, R.R.L. Kantam, and Ch. Santosh Kumar [11],[12] for exponentiated log-logistic distribution. K. Rossaia and R.R.L.Kantam [9] developed accepting sampling plans based on Inverse Rayleigh distribution, (2005) and for half logistic distribution [6] (1998). K. Rosaiah, R. R. L. Kantam and R. Subba Rao [10] (2009) given Pareto distribution in acceptance sampling based on truncated life tests. G. Kulldorff and K. Vannman [8] (1973) considered the estimation of location and scale parameters of Pareto distribution by linear functions of order statistics and K. Vannman [17] (1976) discussed the estimation of parameters based on selected order statistics from Pareto distribution. A.A. Abdel- Ghaly, A.F. Attia and H.M. Aly [1] (1998) have discussed estimation of parameters of Pareto distribution using accelerated life testing. The Exponentiated Pareto distribution in acceptance sampling plans based on life tests has not paid much attention so far.

In the present paper it is assumed that the probability distribution of a life time random variable is Exponentiated

Pareto distribution with known shape parameters. The problem considered is that of finding the minimum sample size necessary to assure a certain average life when the life test is terminated at a pre-assigned time t and when the observed number of failures does not exceed a given acceptance number. The decision procedure is to accept a lot only if the specified average life can be established with a pre-assigned high probability p^*, which provides the protection to the consumer. The decision to accept the lot can take place only at the end of time t and only if the number of failures does not exceed the given acceptance number c. The life test experiment gets terminated at the time at which $(c+1)^{th}$ failure is observed or at the end of time t whichever is earlier. In the first case the decision is to reject the lot.

In section 2, we have obtained the minimum sample sizes necessary for various acceptance numbers - c, for various confidence levels - p^* and various ratios of the test time-t to the specified average life σ_0 using cumulative Binomial probabilities and cumulative Poisson probabilities for Exponentiated Pareto distribution with known shape parameters (α = 2, θ = 2,3,4). Section 3 deals with the operating characteristic and producer's risk of the sampling plans. The results are observed for α = 2, θ = 2, 3, 4 and presented in this paper only for α = 2, θ = 2 due to the space constraints. The use of the numerical tables is described through an illustration presented in Section 4. The results are explained by an example in Section 5.

2. RELIABILITY TEST PLAN

A common practice in life testing is to terminate a life test by a predetermined time t and observe the number of failures (assuming that a failure is well-defined). One of the objectives of these experiments is to set a lower confidence limit on the average life. It is then desired to establish a specified average life with a given probability of at least p^*. The decision to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time t does not exceed a given number c – called the acceptance number. The test may get terminated before the time t is reached when the number of failures exceeds c – the decision then being to reject the specified average life. For such a truncated life test and the associated decision rule, we are interested in obtaining the smallest sample size necessary to achieve the objective.

A sampling plan consists of

i) the number of units n on test,

ii) an acceptance number c such that if c or fewer failures occur during the test time t, the lot is accepted and

iii) a ratio t/σ_0 where σ_0 is the specified average life.

We fix the consumer's risk the probability of accepting a bad lot (the one for which the true average life is below the specified life σ_0) not to exceed $1-p^*$, so that p^* is a minimum confidence level with which a lot of true average life below σ_0 is rejected by the sampling plan. For a fixed p^* our sampling plan is characterized by (n, c, t/σ_0). Here we considered a lot of infinitely large size. Mathematically, given a number p^* (0 < p^* < 1), a value σ_0 of σ and an acceptance number c, we want the find the smallest positive integer n such that

$$\sum_{i=0}^{c} \left(n_{c_i} \right) p^i \left(1 - p\right)^{n-i} \leq 1 - p^* \quad ...(3)$$

where $p = F(t;\sigma_0)$ given by Equation (1). Since Equation (1) depends only on the ratio t/σ, the experiment needs to specify only this ratio. If the number of observed failures before t is less than or equal to c, from (3) we obtain

$$F(t;\sigma) \leq F(t;\sigma_0) \iff \sigma \geq \sigma_0 .$$

That is, the true average life is more than the specified average and the lot is accepted as a good lot. The minimum values of n satisfying the inequality (3) have been obtain for p* = 0.75, 0.90, 0.95, 0.99 and t/σ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712. The results are presented in Table 1 for α = 2 and θ = 2.

If $p = F(t;\sigma)$ is small and n is large (as is true in some cases of our present work), the binomial probability is approximated by Poisson probability with parameter $\lambda = np$ so that the equation (3) can be written as

$$\sum_{i=0}^{c} \left(\frac{e^{-\lambda} \lambda^i}{i!} \right) \leq 1 - p^* \qquad ...(4)$$

where $\lambda = n\,F(t;\sigma)$. The minimum values of n satisfying the inequality (4) have also been obtained for the same combination of p^*, t/σ_0 as those used in inequality (3) and are given in Table 2 for α = 2 and θ = 2.

3. OPERATING CHARACTERISTIC FUNCTION OF SAMPLING PLAN

The operating characteristic of the sampling plan (n, c, t/σ_0) gives the probability of accepting the lot. It can be seen that operating characteristic is an increasing function of σ. For given p^*, t/σ_0, the choice of c and n will be made on the basis of operating characteristics. Values of operating characteristics as a function of σ/σ_0 for a few sampling plans are given in Table 3.

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For a given value of the producer's risk say 0.05, one may be interested in knowing what value of σ/σ_0 will ensure producer's risk less than or equal to 0.05, if a sampling plan under discussion is adopted. It should be noted that the probability p may be obtained as a function of σ/σ_0, as $p = F(t/\sigma) = F[(t/\sigma_0)/(\sigma_0/\sigma)]$. The value σ/σ_0 is the smallest positive number for which the following inequality holds;

$$\sum_{i=0}^{c}\left(n_{c_i}\right)p^i(1-p)^{n-i} \geq 0.95 \qquad ...(5)$$

For a given sampling plan (n, c, t/σ_0) at specified confidence level p^* (i.e., consumer's risk ($1-p^*$), we have computed the minimum values of σ/σ_0 satisfying the inequality (5) and are given in Table 4.

4. TABLES DESCRIPTION

Suppose that the lifetime distribution is Exponentiated Pareto distribution with $\alpha = 2$, $\theta = 2$ and that the experimenter is interested in establishing that the true unknown average life is at least 1000 hours with confidence $p^* = 0.75$. It is desired to stop the experiment at $t = 628$ hours. Then, for an acceptance number $c = 2$, the required n in Table 1 is 10. If, during 628 hours, no more than 2 failures out of 10 are observed, then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours. If the Poisson approximation to Binomial probability is used, the value of n = 11 is obtained from Table 2 for the same situation.

In general, all the values of n tabulated by us are found to be less than the corresponding values of n tabulated in R. R. L. Kantam, K. Rosaiah and G. Srinivisarao. [7] (2001) for log-logistic model.

For the sampling plan (n =10, c = 2, t/σ_0 = 0.628) and confidence level p^* = 0.75 under Exponentiated Pareto distribution with α = 2 and θ = 2 the values of the operating characteristic function from Table 3 are as follows:

σ/σ0	**2**	**4**	**6**
L(p)	0.7456	0.9776	0.9965
σ/σ0	8	10	12
L(p)	0.9992	0.9997	0.9999

The above values show that if the true mean life time is twice the required mean life time ($\sigma/\sigma_0 = 2$) the producer's risk is approximately 0.2544. The producer's risk is 0.0035 when the true mean life is 6 times or more the specified mean life ($\sigma/\sigma_0 \geq 6$)

From Table 4, we can get the values of the ratio σ/σ_0 for various choices of c, t/σ_0 in order that the producer's may not exceed 0.05. For example if $p^* = 0.75$, $t/\sigma_0 = 0.628$, $c = 2$, table 4 gives a reading of 3.29. This means the product can have an average life of 3.29 times the required average life time in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95. The actual average life time necessary to accept 95 percent of the lot is provided in Table 4.

5. NUMERICAL EXAMPLE

Consider the following ordered failure times of the release of software given in terms of hours from starting of the execution of the software up to the time at which a failure of the software is occurred (A. Wood [18]). This data can be regarded as an ordered sample of size n = 9 with observations: 254, 788, 1054, 1393, 2216, 2880, 3593, 4281, and 5180.

Let the required average life time be 1000 hours and the testing time be t = 628 hours, this leads to rate of $t/\sigma_0 = 0.628$ with a corresponding sample size n = 9 and an acceptance number c = 1, which are obtained from Table 1 for p^* = 0.90. Therefore, the sampling plan for the above sample data is (n = 9, c = 1, t/σ_0 = 0.628). Based on the observations, we have to decide whether to accept the product or reject it. We accept the product only if the number of failures before 628 hours is less than or equal to 1. However, the confidence level is assured by the sampling plan only if the given life times follow Exponentiated Pareto distribution. In order to confirm that the given sample is generated by lifetimes following at least approximately the Exponentiated Pareto distribution, we have compared the sample quantiles and the corresponding population quantiles and found a satisfactory agreement. Thus, the adoption of the decision rule of the sampling plan seems to be justified. In the sample of 9 units, there is a 1 failure at 254 hours before t = 628 hours. Therefore we accept the product.

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Table 1. Minimum sample size for the specified ratio t/σ_0, confidence level p*, acceptance number c, $\alpha = 2$, $\theta = 2$ using the binomial approximation.

							t/σ_0		
P*	C	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	2	1	1	1	1
	1	7	5	4	3	3	2	2	2
	2	**10**	7	5	5	4	4	3	3
	3	13	9	7	6	5	5	5	4
	4	15	11	9	8	7	6	6	6
	5	18	13	11	9	8	7	7	7
	6	21	15	12	11	9	8	8	8
	7	24	17	14	12	10	10	9	9
	8	27	19	16	14	12	11	10	10
	9	30	21	17	15	13	12	11	11
	10	32	23	19	17	14	13	13	12
0.9	0	5	3	3	2	2	2	1	1
	1	**9**	6	5	4	3	3	3	3
	2	12	8	7	6	5	4	4	4
	3	16	11	9	8	6	6	5	5
	4	19	13	10	9	8	7	6	6
	5	22	15	12	11	9	8	8	7
	6	25	17	14	12	10	9	9	8
	7	28	20	16	14	12	10	10	9
	8	31	22	18	15	13	12	11	11
	9	34	24	19	17	14	13	12	12
	10	37	26	21	19	15	14	13	13
0.95	0	7	4	3	3	2	2	2	2
	1	11	7	6	5	4	3	3	3
	2	14	10	8	7	5	5	4	4
	3	18	12	10	8	7	6	6	5

	4	21	15	12	10	8	7	7	6
	5	25	17	13	12	10	9	8	8
	6	28	19	15	13	11	10	9	9
	7	31	21	17	15	12	11	10	10
	8	34	24	19	17	14	12	12	11
	9	37	26	21	18	15	14	13	12
	10	40	28	23	20	16	15	14	13
0.99	0	10	6	5	4	3	3	2	2
	1	15	10	7	6	5	4	4	4
	2	19	12	10	8	6	6	5	5
	3	23	15	12	10	8	7	6	6
	4	26	18	14	12	9	8	8	7
	5	30	20	16	14	11	10	9	8
	6	33	23	18	15	12	11	10	10
	7	37	25	20	17	14	12	11	11
	8	40	27	22	19	15	14	13	12
	9	44	30	24	20	17	15	14	13
	10	47	32	26	22	18	16	15	14

Table 2: Minimum sample size for the specified ratio t/σ_0, confidence level p^*, acceptance number c, $\alpha = 2$, $\theta = 2$ using the Poisson approximation.

							t/σ_0		
P*	C	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	3	3	2	2	2	2	2
	1	7	5	5	4	4	4	3	3
	2	**11**	8	7	6	5	5	5	5
	3	14	10	8	8	7	6	6	6
	4	17	12	10	9	8	8	7	7
	5	20	14	12	11	9	9	9	8
	6	23	16	14	12	11	10	10	10
	7	25	18	15	14	12	11	11	11
	8	28	21	17	15	14	13	12	12
	9	31	23	19	17	15	14	13	13
	10	34	25	21	19	16	15	15	14
0.9	0	6	5	4	4	3	3	3	3
	1	11	8	7	6	5	5	5	5
	2	14	10	9	8	7	7	6	6
	3	18	13	11	10	9	8	8	8
	4	21	15	13	12	10	10	9	9
	5	24	18	15	13	12	11	11	10
	6	28	20	17	15	13	12	12	12
	7	31	22	19	17	15	14	13	13
	8	34	25	21	19	16	15	15	14
	9	37	27	22	20	18	17	16	16
	10	40	29	24	22	19	18	17	17
0.95	0	8	6	5	5	4	4	4	4
	1	13	9	8	7	6	6	6	6
	2	17	12	10	9	8	8	7	7
	3	20	15	13	11	10	9	9	9
	4	24	17	15	13	12	11	10	10
	5	28	20	17	15	13	12	12	12
	6	31	22	19	17	15	14	13	13

	7	34	25	21	19	16	15	15	14
	8	38	27	23	21	18	17	16	16
	9	41	30	25	22	19	18	18	17
	10	44	32	27	24	21	20	19	19
0.99	0	12	9	8	7	6	6	6	5
	1	18	13	11	10	8	8	8	8
	2	22	16	14	12	11	10	10	9
	3	26	19	16	14	13	12	11	11
	4	30	22	18	17	14	14	13	13
	5	34	25	21	19	16	15	15	14
	6	38	27	23	21	18	17	16	16
	7	42	30	25	23	20	19	18	18
	8	45	33	27	25	21	20	19	19
	9	49	35	30	27	23	22	21	20
	10	52	38	32	28	25	23	22	22

Table 3: Values of the operating characteristic function of the sampling plan (n, c, t/σ_0) for given confidence level p^* with $\alpha = 2$, $\theta = 2$.

P*	n	C	t/σ_0	$\sigma/\sigma_0 = 2$	4	6	8	10	12
0.75	10	2	0.628	0.7456	0.9776	0.9965	0.9992	0.9997	0.9999
	7	2	0.942	0.6712	0.9593	0.9925	0.9980	0.9994	0.9997
	5	2	1.257	0.7030	0.9580	0.9914	0.9976	0.9992	0.9997
	5	2	1.571	0.5542	0.9119	0.9787	0.9935	0.9976	0.9990
	4	2	2.356	0.4850	0.8590	0.9574	0.9849	0.9939	0.9972
	4	2	3.141	0.3128	0.7298	0.8971	0.9574	0.9807	0.9905
	3	2	3.927	0.5158	0.8285	0.9346	0.9723	0.9871	0.9935
	3	2	4.712	0.4276	0.7584	0.8955	0.9515	0.9758	0.9871
0.9	12	2	0.628	0.6402	0.9627	0.9939	0.9985	0.9995	0.9998
	8	2	0.942	0.5790	0.9405	0.9885	0.9970	0.9990	0.9996
	7	2	1.257	0.4462	0.8887	0.9742	0.9924	0.9973	0.9989
	6	2	1.571	0.3995	0.8540	0.9618	0.9879	0.9955	0.9981
	5	2	2.356	0.2788	0.7415	0.9120	0.9668	0.9860	0.9935
	4	2	3.141	0.3128	0.7298	0.8971	0.9574	0.9807	0.9905
	4	2	3.927	0.2039	0.5999	0.8175	0.9143	0.9574	0.9775
	4	2	4.712	0.1362	0.4850	0.7297	0.8590	0.9241	0.9574
0.95	14	2	0.628	0.5372	0.9439	0.9904	0.9976	0.9993	0.9997
	10	2	0.942	0.4118	0.8935	0.9776	0.9939	0.9979	0.9992
	8	2	1.257	0.3409	0.8446	0.9618	0.9885	0.9958	0.9983
	7	2	1.571	0.2768	0.7875	0.9399	0.9802	0.9924	0.9967
	5	2	2.356	0.2788	0.7415	0.9120	0.9668	0.9860	0.9935
	5	2	3.141	0.1374	0.5544	0.8038	0.9120	0.9581	0.9787
	4	2	3.927	0.2039	0.5999	0.8175	0.9143	0.9574	0.9775
	4	2	4.712	0.1362	0.4850	0.7297	0.8590	0.9241	0.9574
0.99	19	2	0.628	0.3205	0.8819	0.9773	0.9942	0.9981	0.9993
	12	2	0.942	0.2793	0.8364	0.9627	0.9894	0.9963	0.9985
	10	2	1.257	0.1874	0.7451	0.9296	0.9776	0.9917	0.9965
	8	2	1.571	0.1859	0.7162	0.9135	0.9704	0.9885	0.9950
	6	2	2.356	0.1492	0.6171	0.8541	0.9416	0.9745	0.9879
	6	2	3.141	0.0556	0.3997	0.6988	0.8541	0.9271	0.9618
	5	2	3.927	0.0699	0.3966	0.6777	0.8335	0.9120	0.9516
	5	2	4.712	0.0373	0.2788	0.5543	0.7415	0.8506	0.9120

Table 4. Minimum ratio of true σ and required σ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = 2$, $\theta = 2$.

P*	C	t/σ_0 =0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	8.70	10.41	13.89	17.36	17.47	23.29	29.11	34.93
0.75	1	4.48	5.37	6.11	6.10	9.14	8.33	10.41	12.49
0.75	2	**3.29**	3.79	3.81	4.76	5.72	7.62	6.60	7.92
0.75	3	2.77	3.12	3.32	3.54	4.27	5.69	7.11	5.97
0.75	4	2.36	2.74	3.03	3.34	4.28	4.61	5.76	6.91
0.75	5	2.20	2.50	2.83	2.83	3.63	3.92	4.90	5.87
0.75	6	2.08	2.33	2.47	2.79	3.17	3.43	4.29	5.15
0.75	7	2.00	2.20	2.40	2.49	2.84	3.78	3.85	4.61
0.75	8	1.93	2.11	2.34	2.49	2.99	3.43	3.50	4.20
0.75	9	1.87	2.03	2.14	2.28	2.75	3.15	3.22	3.86
0.75	10	1.78	1.96	2.12	2.30	2.54	2.92	3.65	3.59
0.90	0	11.49	13.05	17.41	17.36	26.03	34.70	29.11	34.93
0.90	1	5.25	6.07	7.16	7.63	9.14	12.19	15.24	18.28
0.90	2	3.72	4.20	5.06	5.58	7.14	7.62	9.53	11.44
0.90	3	3.22	3.66	4.16	4.69	5.30	7.07	7.11	8.53
0.90	4	2.82	3.16	3.35	3.78	5.01	5.71	5.76	6.91
0.90	5	2.57	2.84	3.09	3.54	4.24	4.84	6.05	5.87
0.90	6	2.39	2.61	2.90	3.09	3.70	4.23	5.29	5.15
0.90	7	2.26	2.56	2.77	3.00	3.72	3.78	4.72	4.61
0.90	8	2.16	2.42	2.66	2.71	3.37	3.99	4.29	5.14
0.90	9	2.08	2.31	2.43	2.68	3.09	3.66	3.94	4.72
0.90	10	2.01	2.22	2.38	2.65	2.86	3.39	3.65	4.38
0.95	0	13.76	15.27	17.41	21.75	26.03	34.70	43.39	52.06
0.95	1	5.93	6.72	8.10	8.95	11.44	12.19	15.24	18.28
0.95	2	4.12	4.93	5.60	6.32	7.14	9.51	9.53	11.44
0.95	3	3.49	3.92	4.53	4.69	6.21	7.07	8.83	8.53
0.95	4	3.03	3.54	3.94	4.19	5.01	5.71	7.14	6.91
0.95	5	2.82	3.15	3.33	3.86	4.79	5.65	6.05	7.25
0.95	6	2.60	2.88	3.11	3.36	4.18	4.93	5.29	6.34
0.95	7	2.44	2.67	2.94	3.23	3.72	4.40	4.72	5.67
0.95	8	2.32	2.62	2.81	3.13	3.73	3.99	4.98	5.14
0.95	9	2.22	2.48	2.70	2.86	3.42	4.12	4.57	4.72
0.95	10	2.14	2.37	2.62	2.81	3.16	3.82	4.24	4.38
0.99	0	16.62	19.00	22.99	25.46	32.62	43.49	43.39	52.06
0.99	1	7.12	8.39	8.96	10.12	13.42	15.25	19.07	22.88
0.99	2	5.00	5.58	6.57	7.00	8.37	11.15	11.89	14.27
0.99	3	4.10	4.61	5.22	5.66	7.03	8.28	8.83	10.60
0.99	4	3.51	4.06	4.48	4.93	5.67	6.67	8.34	8.56
0.99	5	3.20	3.58	4.00	4.45	5.30	6.39	7.06	7.25
0.99	6	2.93	3.36	3.66	3.88	4.62	5.57	6.16	7.39
0.99	7	2.78	3.10	3.42	3.67	4.49	4.96	5.50	6.59
0.99	8	2.61	2.89	3.23	3.51	4.06	4.97	5.62	5.98
0.99	9	2.53	2.81	3.08	3.21	4.01	4.55	5.15	5.49
0.99	10	2.42	2.67	2.95	3.12	3.71	4.21	4.77	5.08

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