MODELING AND ESTIMATION OF RUBIDIUM ATOMIC CLOCK NOISES

DEEPAK MISHRA, ALAK BANIK, TVS RAM, VAIBHAV VATS

Abstract: Noise estimation of atomic clock is one of the important research areas in the field of atomic clock development and applications. Various literature reports and good amount of research material are available in the field of noise estimation pertaining to atomic clock. However, in this paper, an effort is made to present and adopt an alternate approach of least-squares normalized-error (LSNE) regression algorithm for noise estimation.

This algorithm weights the error from the curve fit by the reciprocal of the value of the curve fit. This fitting method is applied to the Allan variance of frequency stability analysis to estimate at five typical noise coefficients from the given time domain phase dataset. The proposed technique presented here is less complex, requires less computation time, and does not have convergence issues compared to iterative logarithmic methods that are typically used for fitting Allan variance data.

A MATLAB based application with graphical user interface (GUI) is developed to generate the Allan variance from time domain data and calculate the curve fit by estimating different type of noise profile .The software estimate the deterministic noises behavior from the corresponding clock measured phase and frequency data and also compares its stability with best possible realizes Rubidium clock stability.

Keywords: LSNE, Rb, GUI, FP,WF, RWF.

Introduction: Today the Atomic clocks are back bone of navigation systems. The performances of these atomic clocks are limited by inherent noises present in the corresponding clocks. These limitations can be assessed with the help of modeling of noise process present in the clock. The most important parameter of an atomic clock is stability, which is mostly affected by different types of noises, inherently present in clock. The Allan Variance technique [1] has long been applied to the characterization of different noise in atomic clock. In this paper, an attempt is made to present least-squares normalized-error (LSNE) regression method for interpreting different types of noise present in clock data. Although least squares normalized-error regression method is already used for estimation of different noises in various subsystems [2][3][4]&[5],for atomic clock noise estimation it was never applied. In this paper we have presented a new approach using least squares normalized-error regression method for noise estimation of atomic clock.

Modeling of atomic clock noise: in the field of satellite navigation, various simulation tools are developed for the design and validation of atomic clock stability, however the effect of clock errors due to noise are not taken into account accurately. therefore to predict the clock behavior accurately, precise clock errors modeling are required. the clock errors consist of clock noise errors and systematic clock errors. the clock noise errors consist of five types of error sources such as wf (white noise on frequency), wp (white noise on phase), ff (flicker oise on frequency), fp (flicker noise on phase) and rwf (random walk noise on the frequency) whereas, the systematic noise errors are mainly due to bias, drift and drift rate acceleration. in this paper, we have considered rubidium atomic clock for modeling purpose. in this rubidium atomic clock, wf, ff and rwf clock noises are the dominating noise sources throughout the frequency [8]. in this paper, clock noises affecting rubidium atomic clock are modeled using matlab and simulink. these noise models are used to predict the clock stability.

It is well documented that the instability of most frequency sources can be modeled by a combination of power-law noises [8] having a spectral density of their fractional frequency fluctuations of the form $S_y(f) = kf^{\alpha}$, where f is the Fourier or sideband frequency in hertz, and α is the power law exponent. Table I [9] shows typical value of the power law exponent for clock noises. Figure 1 shows the theoretical (or usual behavior) characteristics of power spectral density of the clock noises.

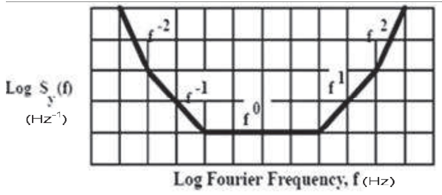

Figure 1: Power Spectral Density of Clock noise error sources [8]

TABLE I: Value of the power law exponent for clock noises

Noise Type	α (Power Law Exponent)
White PM(WP)	2
Flicker PM(FP)	1
White FM(WF)	0
Flicker FM(FF)	-1
Random Walk FM(RWF)	-2

Time domain frequency stability measure (Allan Deviation) is related to the spectral density of the fractional frequency fluctuations by the relationship given below [10]:

$$\sigma^2(M,T,\tau) = \int_0^\infty S_Y(f).|H(f)|^2.df \qquad (1)$$

Where, σ^2(M,T,t) is the M - sample Allan Variance for time t and sampling period T. S_y(f) is the power spectral density of fractional frequency fluctuations. $|H(f)|^2$ is the transfer function of Allan (two sample) time domain stability, which is given in equation 2.

$$|H(f)|^2 = 2[\sin^4(\pi tf)/(\pi tf)^2] \qquad (2)$$

Theoretical Allan deviation plot for the Atomic Clock noises is given in the figure 2.

Figure 2: Allan Deviation (σ_y(t)) of the Clock Noise Error Sources [6]

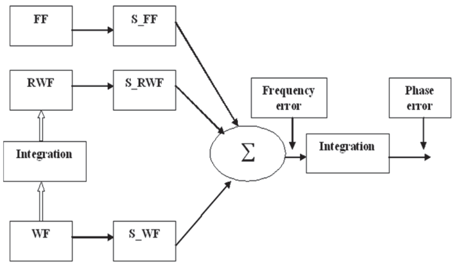

figure:3 Rb clock noice clock error source

Simulation of different noises:

In Rb clocks dominating noises [8] are simulated as follows.

- WF noise is generated using the random number generator in SIMULINK.
- RWF noise is generated by integrating the WF noise.
- FF noise can be generated by filtering the WF noise [11]. FF having PSD S(f)= f^{-1} is difficult to generate using white Gaussian random variables. Several previous studies such as the IDFT (Inverse Discrete Fourier Transform) method [12], FIR (Finite Impulse Response) and IIR (Infinite Impulse Response) filtering method have been proposed for modeling and generating the sequence of f^{-1}[13]. In this paper, FF is generated by passing a white noise through Autoregressive filter [11]. Impulse response of Autoregressive filter is given as follows

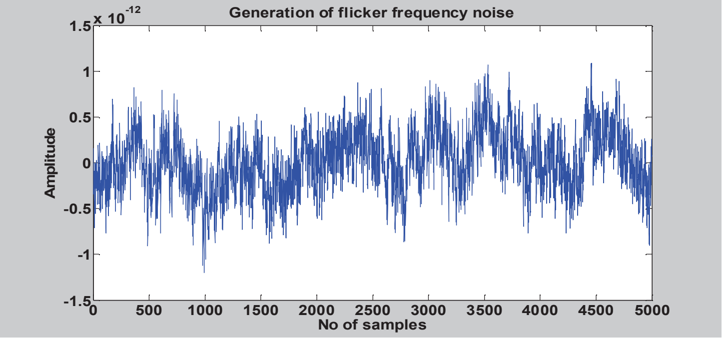

figure: 4 Generating of flicker frequency noice.

$$h_o = 1$$

$$h_k = (K\text{-}1+(\alpha/2))\, h_{k-1} / k\text{-}1. \quad (3)$$

Where, K > 0 and $\alpha = -1$ (Power law exponent for FF).

Table II: Equations to calculate scaling factor

Scaling Factor	**Equations**
S_WF	$h_o/2t$
S_FF	$h_{-1}2\ln(2)$
S_RWF	$h_{-2}(2\pi 2t/3)$

Basic block diagram of the Rb clock noise error source is given in figure 3. Figure 4 shows the generated different clock noise error generic to Rb clock. The clock noise error sources are multiplied by the scaling factor, and then the scaled clock noise error sources are being uncorrelated, they are summed and root squared to generate the total clock frequency error. Integration of frequency error generates clock phase error. S_WF, S_FF and S_RWF are the scaling factors for WF, FF and RWF, respectively. The scale factor can be computed using equations given in table II

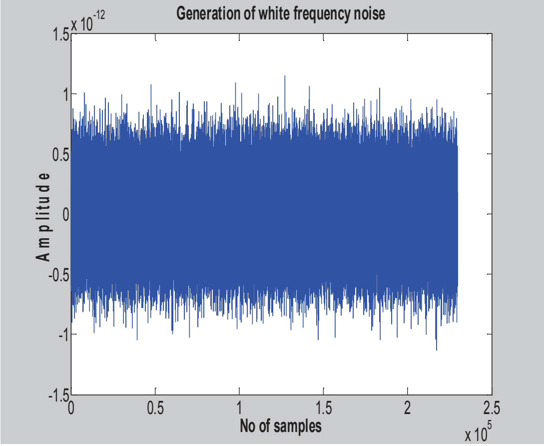

Figure 4: (a) Generation of white frequency noise

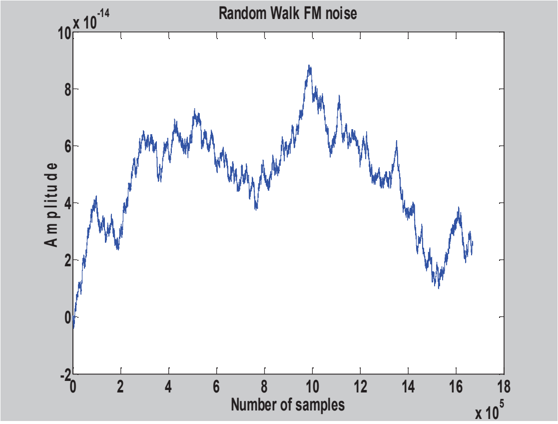

Figure 4: (b) Generation walk frequency noise

Clock error modeling algorithms: a number of algorithms have been proposed and used during the past 20 years for noise estimation. each one possesses some advantages and limitations compared to the well-established ones. the major algorithm proposed by vernotte *et al* [17] ,is basically based on probability distribution of different noises, however this approach required large simulation time and may suffer from convergence of result in case of more than one present in clock. the approach proposed by grantham *et al* [18] is also based upon least square method; this approach optimizes the complete measurement. for this reason this approach will not converge, where more than one noises is present together.

therefore this approach is not suitable for estimation of noises. the approach adopted by stable32 software is basically normalized weighted fitting method. more specifically, the weight applied to each point is scaled inversely by w, the span of its confidence interval (error bars) divided by the nominal sigma. this is theoretically desirable, and also provides better fits, especially for stability data comprising more than one power law noise type. because the line slope is fixed for a particular user-selected noise type, the noise or aging lines are simply fitted to the weighted average of expressions that represent them on log-log coordinates.

$$\sigma\hat{}(1) = \frac{\sum_{i=1}^{N} \frac{\sigma(nominal)i}{w_i.\tau^{\mu/2}}}{\sum_{i=1}^{N} 1/w_i}$$

Where

N= # stability data points

$w_i = \frac{\sigma(max)i - \sigma(min)i}{\sigma(nominal)i}$ = *weighing function*

$\mu = variance\ slope: \sigma_y^2(\tau) \propto \tau^{\mu}$, -2≤ μ ≤2 *for unmodified variances*

$\sigma_y^2(\tau) \propto \tau^{\mu'}$, -3≤ μ' ≤2 *for modified variances*

The drawback of this approach is that it requires very long measurement of data set for proper estimation of noises.

Proposed algorithm: The regression algorithm proposed in this paper begins with the Allan Variance calculation from time domain data as indicated by Tehrani [14]. The algorithm assumes each noise has different signature. This algorithm estimates the signature of different random noises present in each decade of log-log plot. The slope of different noise in the Allan variance against time (tau) is mentioned table III [8]:

These noises are uncorrelated. Hence the equation describing the total Allan Variance is [15].

$$\sigma_{Tot}^2 = \sigma_{WPM}^2 + \sigma_{FPM}^2 + \sigma_{WFM}^2 + \sigma_{FFM}^2 + \sigma_{RWFM}^2 \quad (4)$$

Where σ_{Tot}^2 is the total Allan Variance, σ_{WPM}^2 is the variance due to white PM noise, σ_{FPM}^2 is variance due to flicker PM NOISE, σ_{WFM}^2 is the variance due to white FM noise , σ_{FFM}^2 is due to flicker FM noise and σ_{RWFM}^2 is due to random walk FM noise. However in case of rubidium clock the total Allan Variance is only affected by white FM, flicker FM and random walk FM noise [8].

Relationship between power spectral density (PSD) and Allan Deviation is given in the table 4. Values of Allan Variance in table IV are as per equation 1. The high-frequency cut-off f_h is defined as the upper limit in the spectral bandwidth of the system.

The proposed algorithm begins by normalization of data generated by clock measurement. Since this generated data span several Kbytes, it was desirable to normalize the errors to the localized magnitude of the data. Sargent and Wyman [15] accomplished this by calculating the logarithm of the data. In this paper we have used an alternate method of normalization by dividing the data by the value of the curve fit at the associated measurement time

.

TABLE IV: Relationship between PSD and Allan variance of Atomic Clock noise

Noise Type	PSD	Allan Variance (σ_y^2(⍰))
White PM	h_2f^2	$3h_2f_h/4\pi^2$⍰^{2}
Flicker PM	h_1f^1	$h_1(1.038+2\ln(2\pi f_h t)/\ 4\pi^2$⍰$^2)$
White FM	h_0f^0	$h_0/2$⍰
Flicker FM	$h_{-1}f^{-1}$	$h_{-1}2\ln(2)$
Random Walk FM	$h_{-2}f^{-2}$	$h_{-2}(2\pi^2$⍰$/3)$

The error equation is given by

$$\varepsilon = \frac{\sigma_{Tot}^2 - \hat{\sigma}_{Tot}^2}{\hat{\sigma}_{Tot}^2} \qquad (5)$$

Where ε is defined as the "normalized error" and $\hat{\sigma}_{Tot}^2$ is the estimate of the total Allan Variance for a given measurement time.However, the estimate of the total Allan Variance from the curve fit is not available initially. Therefore, a modified definition of the normalized error is created using the actual data at that value of measurement time or

$$\varepsilon = \frac{\sigma_{Tot}^2 - \hat{\sigma}_{Tot}^2}{\sigma_{Tot}^2} \qquad (6)$$

This modified definition of the normalized error allows for a simplified derivation of the least-squares normalized error (LSNE) regression.

The derivation of the least-squares normalized error curve fit follows the least-squares algorithm very closely. The error equation shown in (6) can be generalized as

$$\varepsilon = \frac{y - \ddot{y}}{y} \quad (7)$$

Where the measured variance in (6) has been replaced by y and the value of the curve fit has been replaced by $\ddot{y}$. The equation for $\ddot{y}$ can be generalized as [16]

$$\ddot{y} = C_m\tau^m + C_n\tau^n + C_p\tau^p + C_g\,g \quad (8)$$

It is desired to minimize the sum of the squares of the normalized error from the curve fit by solving for the coefficients that minimize this error. The sum of the squares of the normalized error is defined as

$$\sum_{\tau_{min}}^{\tau_{max}} \epsilon^2 = \sum_{\tau_{min}}^{\tau_{max}} [\ \frac{y-\ddot{y}}{y}]^2 \quad (9)$$

The proposed algorithm divides the complete measurement time in decade of log-log plot. For each decade sum of the normalized error is minimized

.The predicated slope of each decade is map to one of noise characteristic, whose behavior close to predicated slope. Figure 5 shows the estimated curve fit and estimated noise present using proposed method. The flow chart of proposed approach is mention in figure 6.

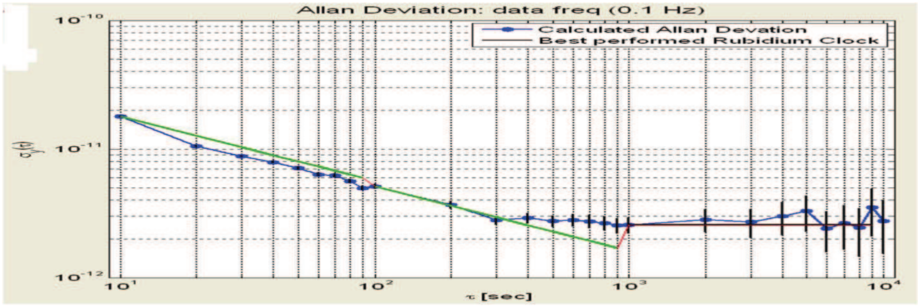

Figure 5: Estimated curve fit of different noises in Allan variance plot

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Allan: Minimum ADEV value: 2.41811e-12 at tau = 6000 seconds
White FM noise present between tau 10.000000 to 100.000000 with slope 5.666947e-12 tau^-1/2
White FM noise present between tau 100.000000 to 1000.000000 with slope 5.109194e-13 tau^-1/2
Flicker FM noise present between tau 1000.000000 to 10000.000000 with value  2.561431e-12
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Advantages Of Proposed Algorithm:

The Proposed Algorithm Has Following Advantages Compared To Earlier Proposed Least-Squares Normalized Error (Lsne) Regression Algorithms:

The proposed algorithm divides the Allan deviation log–log plot decade wise and the complete estimation is a summation of decade wise estimation. The advantage of this approach is that normalization process happen per decade with local maxima; as a result the estimated result is more accurate compared to other approach [18]. Fig 6 and 7 shows the estimated noises of rubidium frequency data in stable32 software and in our proposed approach respectively. In Stable32 software it estimated. White FM noise presented throughout the tau range with slope of 1.8048060e-11 tau$^{-1/2}$.

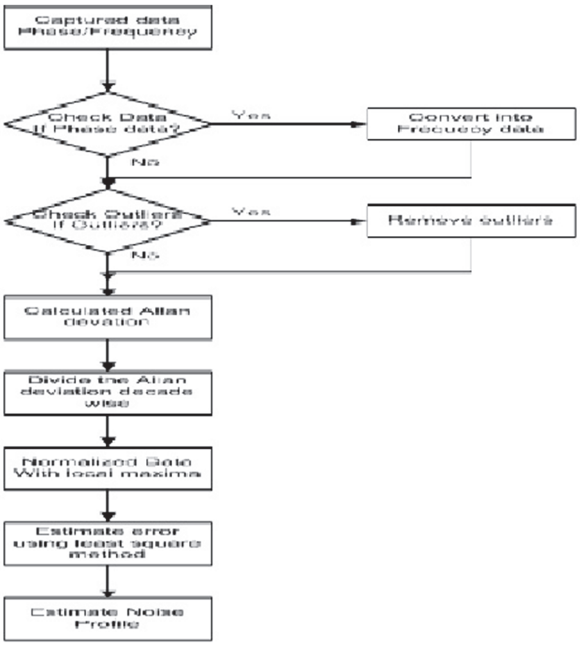
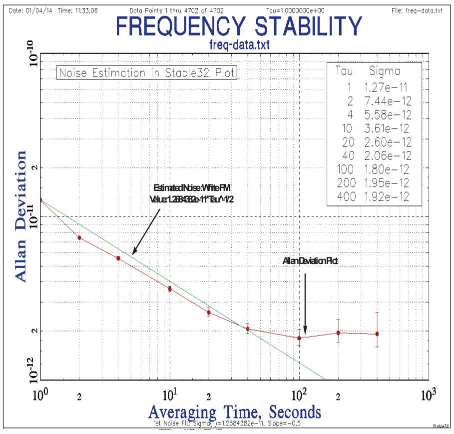


Figure 6: Flow chart of proposed algorithm

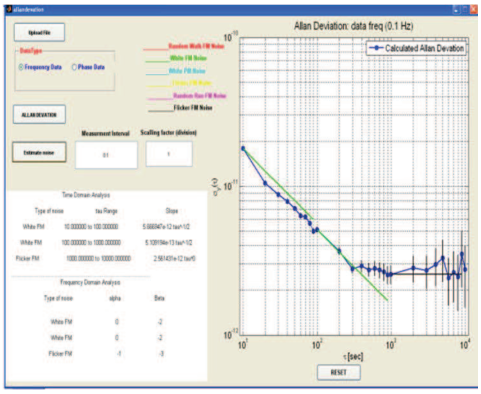
While in the proposed algorithm software it estimated white FM noise and flicker FM noise at different tau range with different slope equation as mention in figure 7. Result clearly shows that estimation of proposed algorithm is much correct and closer to actual values.

White PM and flicker PM noise are difficult to analyze in time domain log-log plot of Allan deviation. The proposed algorithm analyzes these noises in frequency domain log-log plot of power spectral density using same regression algorithm by applying per decade.



Tau	Sigma
1	1.27e−11
2	7.44e−12
4	5.58e−12
10	3.61e−12
20	2.60e−12
40	2.06e−12
100	1.80e−12
200	1.95e−12
400	1.92e−12

figure 6: estimation of noise in allan deviation plot using stable32 software



Measurment Interval: 0.1

Scalling factor (division): 1

Time Domain Analysis

Type of noise	tau Range	Slope
White FM	10.000000 to 100.000000	5.666947e-12 tau^-1/2
White FM	100.000000 to 1000.000000	5.109194e-13 tau^-1/2
Flicker FM	1000.000000 to 10000.000000	2.561431e-12 tau^0

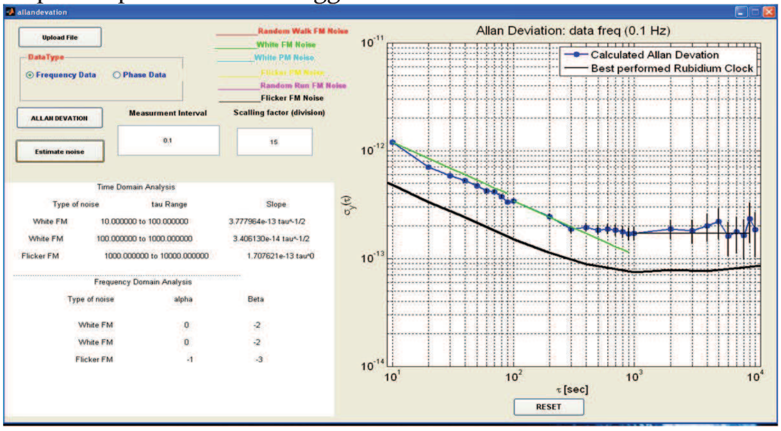
Frequency Domain Analysis

Type of noise	alpha	Beta
White FM	0	-2
White FM	0	-2
Flicker FM	-1	-3

Figure 7: Estimation of noise in Allan deviation plot using proposed least-squares normalized error (LSNE) regression algorithms

The proposed estimation also provides an option of scaling and different measurement interval option of raw data for analyzing a different type of noises.

Software Implementation: A GUI based (software) implementation of the LSNE fit method is developed for Microsoft Windows using MATLAB 13.0a. The application was used to generate the LSNE fit data presented in the plot section. The different color for different noise is showed in the plot. The application uses data files stored in column-based text format. A data scale factor may also be specified. Allan Variance data are generated using a user specified maximum number of points per decade as suggested by Sargent and Wyman [15]; this shortens computation time and weights data more equally across all measurement times. An auto fit function is provided to compute automatically estimated noise behavior based on minimum least squares normalized error. The GUI compares the stability plot of captured data with theoretically best estimated Rubidium clock performance [19] for ease of reference to user. The GUI also displays type of noise and basic equation or behavioral characteristic of noise on per decade. Fig 8 shows the screenshot of developed GUI.



Measurment Interval: 0.1

Scalling factor (division): 15

Time Domain Analysis

Type of noise	tau Range	Slope
White FM	10.000000 to 100.000000	3.777964e-13 tau^-1/2
White FM	100.000000 to 1000.000000	3.406130e-14 tau^-1/2
Flicker FM	1000.000000 to 10000.000000	1.707621e-13 tau^0

Frequency Domain Analysis

Type of noise	alpha	Beta
White FM	0	-2
White FM	0	-2
Flicker FM	-1	-3

Figure 8: Snapshot of a GUI software for different noise estimation using least-squares normalized-error (LSNE) regression method

Conclusions: This paper describes LSNE based clock error model. The clock noise model is generated from various measured phase data of rubidium atomic clock and compared in parts with Stable-32 as there is no such software available to compare the result. The result shows that proposed clock error model is more accurate and efficient compared to Stable-32 noise error model. The fit technique is less complex and does not have convergence issues compared to iterative logarithmic methods that are typically used for fitting Allan variance data.

A user friendly GUI is developed, which analyzes the clock noise error model and shows its profile and value decade wise in the stability plot of atomic clock.

Future Scope: In this paper, we have estimated the deterministic noises of atomic clock .However the stability of atomic clock is also affected by non-deterministic noises, which are present in clock due to temperature, solar radiation, magnetic effect etc. So for complete estimation of noises in the atomic clock, analyses of non-stationary noises are also required.

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References:

1. Allan, D. W. (1966), "Statistics of Atomic Frequency Standards", Proceedings of IEEE, vol. 54, no. 2, pp. 221-230, Feb, 1966.
2. King, A. D., "Characterization of Gyro In-Run Drift, "*Symposium Gyro Technology 1984*, Oct. 1984, pp. 10.0-10.56.
3. Ng, Lawrence C., and Pines, Darryll J.,"Characterization of Ring Laser Gyro Performance Using the Allan Variance Method,", *Journal of Guidance Control, and Dynamics*, Vol. 20, No. 1, January-February 1997, pp. 211-214.
4. Park B K, Kim J H and Kim K C 1991 Performance evaluation of an X- band RF switching radiometer *IEEE Trans. Instrum. Meas.*40 483–5.
5. Goodberlet M A and Mead J B 2006 Two-load radiometer precision and accuracy *IEEE Trans. Geosci. Remote Sensing* 44 58–66.
6. L. Galleani and P. Tavella, "The Characterization of Clock Behavior with the Dynamic Allan Variance", *Proc. 2003 Joint FCS/EFTF Meeting*, pp. 239-244, 2003.
7. L. Galleani and P. Tavella, "Tracking Nonstationarities in Clock Noises Using the Dynamic Allan Variance", *Proc. 2005 Joint FCS/PTTI Meeting*.
8. Atomic Clock Error Modeling for GNSS Software Platform by M. Y. Shin, C. Park and S. J. Lee.
9. J. Rutman, "Characterization of phase and frequency instabilities in precision frequency sources: Fifteen years of progress," *Proc. IEEE*, vol. 66, no. 9, pp. 1048–1075, Sep. 1978.
10. Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision Clocks and Oscillators by DAVID W. ALLAN.
11. Discrete Simulation of Colored Noise and Stochastic Processes and 1/f^alfa Power Law Noise Generation by N. JEREMY USDIN.
12. Jack J. Lennon, "Red-shifts and red herrings in geographical ecology," ECOGRAPHY 23, pp. 101-113, Copenhagen, Denmark, 2000.
13. Efficient Modeling of $1/f^{\alpha}$ Noise Using Multirate Process by Jongsun Park, Member, IEEE, Khurram Muhammad, Member, IEEE, and Kaushik Roy, Fellow, IEEE.
14. Tehrani, M. M., "Ring Laser Gyro Data Analysis with Cluster Sampling Technique," *Proceedings of the SPIE*, Vol. 412, 1983, pp. 207-222.
15. Sargent, D., and Wyman, B. O., "Extraction of Stability Statistics from Integrated Rate Data,", *Proc. A1AA Guidance and Control Conference*, Aug. 11-13, 1980.
16. Ng, Lawrence C., and Pines, Darryll J., "Characterization of Ring Laser Gyro Performance Using the Allan Variance Method,", *Journal of Guidance Control, and Dynamics*, Vol. 20, No. 1, January-February 1997, pp. 211-214.
17. Francois Vernotte and Eric Lantz', "Statistical Biases and Very-Long-Term Time Stability Analysis", IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 59, no. 3, March 2012
18. Grantham, Brian E., Bailey, Mark A., "A Least-Squares Normalized Error Regression Algorithm with Application to the Allan Variance Noise Analysis Method," *Proceedings of IEEE/ION PLANS 2006*, San Diego, CA, April 2006, pp. 750-756.
19. Tom van Baak (www.leapsecond.com)

Scientist/Engineer-SE, Scientist/Engineer-H, Group Director-ODCG,
Scientist/Engineer-G, Head-OSPD, Scientist/Engineer-SE,
Building no:22,24, Space Application Centre,ISRO,Satellite,Ahmedabad-15,
deepakmishra@sac.isro.gov.in, alak@sac.isro.gov.in
tvsram@sac.isro.gov.in, vaibhav_vats@sac.isro.gov.in