VISCOUS DARK ENERGY AND PHANTOM EVOLUTION DUE TO THE APPLICATION OF FRIEDMANN'S EQUATIONS

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Abstract: In Physical Cosmology and Astronomy, dark energy is regarded as the hypothetical form of energy which is extremely responsible for the exponential expansion of the universe. The_standard cosmological model indicates that the total mass-energy of the universe contains 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy. In standard cosmology, with the evolution of the universe, the matter density and thermodynamic pressure gradually decreases. Also in the course of evolution, the matter in the universe obeys (or violates) some restrictions or energy conditions. If the matter distribution obeys strong energy condition (SEC), the universe is in a decelerating phase while violation of SEC indicates an accelerated expansion of the universe. In the period of accelerated expansion, the matter may be either of quintessence nature or phantom nature depending on the fulfilment of the weak energy condition (WEC) or violation of it. As recent observational evidences demand that the universe is undergoing through an accelerated expansion, so matter should be either of quintessence or phantom in nature. Finally, we investigate the possibility of occurrence of any singularity in phantom era. Here in this paper, we have investigated the dissipative processes in the universe by introducing the Friedmann Equations to study whether the bulk viscosity induces a big rip singularity on the at Friedmann-Robertson-Walker (FRW) cosmologies or not.

Keywords: Dark energy, Viscosity, Big rip, Singularity, Cosmology, Thermodynamics.

Introduction: The astrophysical observations indicate that the universe media is not a Perfect fluid [1] and the viscosity is concerned in the evolution of the universe [2, 3, 4]. On the other hand, in the standard cosmological model, if the Equation of State (EOS) parameter ω is less than -1, the universe shows the future finite singularity called Big Rip [5, 6]. Several ideas are proposed to prevent the big rip singularity as though it un-physical, like by introducing quantum effects terms in the action [7], or by including universe viscosity media for the Universe evolution [8]. The universe at present should have a phase of deceleration in the context of standard cosmology. Modification of the geometry indicates introduction of some modified gravity theory (f(R) gravity, Brane scenario etc) while change in the matter part indicates inclusion of some unknown kind of matters having large negative pressure so that SEC (ρ + 3p > 0) is violated. Such an unknown matter is known as dark energy (DE). In literature, there are various DE model to match with observational data. The simplest model representing differential equation is the Cosmological Constant which was introduced by Einstein himself. The critical boundary where universe make a transition from quintessence era to the phantom era is known as phantom divide line or phantom crossing. It is extraordinary that recent observations have confirmed that the Universe is undergoing a phase of accelerated expansion. Evidence of this cosmological expansion, coming from measurements of supernovae of type Ia (SNe Ia) [9, 10] and independently from the cosmic microwave background radiation [11, 12], shows that the Universe additionally consists of some sort of negative pressure dark energy. The Wilkinson Microwave Anisotropy Probe (WMAP), designed to measure the CMB anisotropy with great precision and accuracy, has recently confirmed that the Universe is composed of approximately 70 percent of dark energy [13]. A simple way to parameterize the dark energy is by an equation of state of the form $\varpi = \frac{p}{\rho}$, where p is the spatially homogeneous pressure and ρ, the energy density of the dark energy [14]. A value of $\varpi < -\frac{1}{3}$ is required for cosmic expansion, and corresponds to a cosmological constant [15]. The particular case of $\varpi = -\frac{2}{3}$ is extensively analyzed in [16]. However, a note on the choice of the imposition $\varpi > -1$ is in order. Note that the dark energy density is positive, ρ > 0. Matter with the property $\varpi < -1$ has been denoted phantom energy. Apart from the null energy condition violation phantom energy possesses other strange properties, namely, phantom energy probably mediates a long-range repulsive force [17], phantom thermodynamics leads to a negative entropy (or negative temperature) [18, 19], and the energy density increases to infinity in a finite time [20, 21], at which point the size of the Universe blows up in a finite time. This is known as the Big Rip. To an observer on Earth this corresponds to observing the galaxies being stripped apart, the Earth being itself ripped from its gravitational attraction to the Sun, before being eventually ripped apart, followed by the dissociation of molecules and atoms, and finally of nuclei and nucleons [22]. However, it has been shown that in certain models the presence of phantom energy does not lead to the above-mentioned [23, 24, 25, 26, 27, and 28]. It was also shown that quantum corrections may prevent [29], or at least delay, the Big Rip by taking into account the back reaction of conformal quantum fields near the singularity [30, 31, 32], and the universe may end up in a de-Sitter phase before the scale factor blows up. (It has also been shown that quantum effects, without a negative kinetic term, could also lead to a super-accelerated phase of inflation, with a weak energy condition violating, on average and not just in fluctuations, dark energy equation of state ω< -1 on cosmological scales [33, 34]). In an interesting paper it was shown that the masses of all black holes tend to zero as the phantom energy universe approaches the Big Rip [35]. In this paper, we will be interested in constructing solutions using the equation of state

$p = \omega\rho$,

with $\omega < -1$, that describes phantom energy in cosmology.

The existence of an exotic cosmic fluid with negative pressure which constitutes about the 70 percent of the total energy of the universe, has been perhaps the most surprising discovery made in cosmology. This dark energy is supported by the astrophysical data obtained from Wilkinson Microwave Anisotropy Probe (WAMP) (MAP) and high red-shift surveys of super novae. The dark energy is considered to be a fluid categorized by a negative pressure and usually represented by the equation of state $\omega = \frac{p}{\rho}$ where ω lies very close to -1, majority of the possibility is that it lies below the value -1. Now dark energy with $\omega < -1$ the phantom component of the universe leads to uncommon cosmological scenarios as it was pointed out by Cadwell in [36]. Initially, there was a violation of the dominant energy condition (DEC). The energy density grows up to infinity in a finite time, which leads to a big rip, characterized by a scale factor blowing up in this finite time. These sudden future singularities are, nevertheless, not necessarily produced by a fluids violating DEC. Barrow[2] showed with explicit examples which develop a big rip singularity at a finite time even if the matter fields satisfy the strong-energy conditions $\rho > 0$ and $p + 3\rho > 0$.

The role of dissipative processes in the evolution of early universe has also been extensively studied. In the case of isotropic and homogeneous cosmologies, any dissipation process in FRW cosmology is scalar and therefore may be modelled as a bulk viscosity within a thermo dynamical approach. There is a well-known result of the FRW cosmological solutions, corresponding to universes filled with perfect fluid and bulk viscous stresses is the possibility of violating DEC[37].

Formulation of the Problem: The bulk viscosity introduces dissipation by only redefining the effective pressure, Peff , according to

$$P_{eff} = p + \Pi = p + 3\xi H \quad (1)$$

where $\Pi = -3\xi H$

and Π=bulk viscous pressure, ξ = co-efficient of bulk viscosity, H= Hubble Parameter.

The equation of energy balance is

$$\dot{\rho} + 3H(p + \rho + \Pi) = 0 \quad (2)$$

Eckert Theory:

The FRW metric for a homogeneous and isotropic at universe is given by

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)\right) \quad (3)$$

where a(t) is the scale factor and t represents the cosmic time. In the following, we use the units $8\pi G$ = 1. In the first order thermodynamic theory of Eckart, the field equations in the presence of bulk viscous stresses are

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3} \quad (4)$$

and

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P_{eff}) \quad (5)$$

with $P_{eff} = p + \Pi$ (6)

where

$$\Pi = -3\xi H \quad (7)$$

We have the conservation equation as

$$\dot{\rho} + 3H(p + \rho + \Pi) = 0 \quad (8)$$

Assuming that the dark component obeys the state equation.

Let us first explain the Friedmann's Equations.

Friedmann's equations: There are two independent Friedmann's equations for modelling a homogeneous, Isotropic universe. The first equation of Friedmann is as:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\Pi G\rho + \Lambda c^2}{3}$$

which is derived from the 'oo' component of Einstein's field equations.

Again, the second equation is

$$\frac{\ddot{a}}{a} = -\frac{4\Pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (9)$$

which is derived from the first equation together with the trace of Einstein's field equations.

$H \equiv \frac{\dot{a}}{a}$ is the Hubble Parameter. G, Λ and c are universal constants.

G= Newton's gravitational constant.

Λ = Cosmological constant.

c= Speed of light in vacuum.

Using the first equation, the second equation can be re-expressed as

$$\dot{\rho} = -3H\left(\rho + \frac{p}{c^2}\right) \quad (10)$$

which eliminates Λ and expresses the conservation of mass- energy,

$$T^{\alpha\beta}_{;\beta} = 0$$

Now from equation (10), we have,

$$6H\dot{H} = -3H\left(3H^2 + \frac{p}{c^2}\right)$$
$\Rightarrow p = -3c^2.H^2 - 2c^2.\dot{H}$
where 0 ≤c ≤3 ×10^8 m/sec (in vacuum).
Writing, $-3c^2$= A and $-2c^2$= B where c is the velocity of light. Although c is a function of time, but here for the problem considered in this paper, we shall take specified value for c, so that c can be treated as a constant. Therefore, we have, $p = AH^2 + B\dot{H}$ (11)
Substituting (11) in equation (8) and using equation (4) and (7), we get,
$$(2 + B)\dot{H} + (3 + A)H^2 = 3\xi H$$
$$\Rightarrow \dot{H} + \left(\frac{3 + A}{2 + B}\right)H^2 = \left(\frac{3}{2 + B}\right)H\xi$$
$$\Rightarrow \dot{H} + EH^2 = DH\xi \quad (12)$$
Where $E = \frac{3+A}{2+B}$ and $D = \frac{3}{2+B}$
Thus the equation (12) becomes,
$$\frac{dH}{dt} - DH\xi = -EH^2 \quad (13)$$
$$\Rightarrow H(t) = \frac{exp[D\int \xi(t)dt]}{\int E[exp[D\int \xi(t)dt]dt] + F} = \frac{exp[D\int \xi(t)dt]}{F + E\int[exp[D\int \xi(t)dt]dt]} \quad (14)$$
where F is the constant of integration.
Again the equation (4) which is
$\frac{\dot{a}}{a} = H$ we can find the expression for the scalar factor a(t) which is as follows:
$$\log a - \log K = \int\left[\frac{D\int \xi(t)dt}{F+E\int[exp[D\int \xi(t)dt]dt]}\right] \quad (15)$$
Where log K is the new constant of integration.
Thus,
$$a(t) = K[F + E\int[exp[D\int \xi(t)dt]dt]]^{\frac{2}{3}} \quad (16)$$
where K is a new constant of integration.
where
$$E = \frac{3 + A}{2 + B} = \frac{3 - 3c^2}{2 - 2c^2} = \frac{3}{2}$$
since c is the velocity of light which is a function of time i.e., t. Thus, for a given ξ(t), we have the expression for a(t) as in equation (16), we can have the expression for p(t) and ρ(t) as follows:
Since $\rho = 3H^2$
Therefore
$$\rho(t) = 3H^2(t) = 3\left[\frac{exp[D\int \xi(t)dt]}{F+E\int[exp[D\int \xi(t)dt]dt]}\right]^2 \quad (17)$$
Again, $\rho(t) = AH^2(t) + B\dot{H}(t)$
$$\Rightarrow \rho(t) = -3c^2\left(\frac{\dot{a}}{a}\right)^2 + B\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) \quad (18)$$
where the values of the scale factor a(t) and its derivative $\dot{a}(t)$are obtained from the equation (16). Now from equation (12), for the case E = 0, we have,
$$\dot{H} = D\xi H$$
$$\Rightarrow \xi = \frac{1}{D}.\frac{\dot{H}}{H} \quad (19)$$
Now substituting this expression into equation (8), we have,
$$\dot{\rho} + 3H\left[(3 + A)H^2 + \left(B - \frac{3}{D}\right)\dot{H}\right] = 0$$
Here, we have taken
$$E = 0$$
i.e.,
$$A = -3$$
Also
$$B = -2c^2$$
and
$$D = \frac{3}{2 + B} = \frac{3}{2 - 2c^2}$$
Therefore, we have from,
$$\dot{\rho} + 3H\left[(3 - 3)H^2 + \left(B - \frac{3}{D}\right)\dot{H}\right] = 0$$
as
$$\dot{\rho} = 6H\dot{H} \quad (20)$$
$$\Rightarrow \int d\rho = 3H^2 + \text{constant} \quad (21)$$
Comparing this expression with (4), we have that the above constant in equation (21), is zero. Again from equation (16) we have,
$$a(t) = K\left[F + \int \frac{3}{2}\left[exp\left[\frac{3}{2-0}\int \xi(t)dt\right]dt\right]\right]^{\frac{2}{3}} \quad (22)$$
Thus we shall have the expression for the scale factor a(t), defined by the field equations as shown in equation (22). So, for a given value of a(t), we can write H and obtain the expressions for the energy density ρ from equation(4), which is obtained in equation (17) and the bulk viscosity from equation (19).
Again if
$$\xi = 0, \frac{1}{D}\frac{\dot{H}}{H} = 0$$
We have,
H =constant = H_0 (say) (23)
Again, from equation (4), we have,
$\log a = H_0\int dt = H_0 t$ [by equation (23)]
Thus the well known de-Sitter scale factor a(t) is obtained as : $a(t)$=$e^{H_0 t}$ and for c = 0, the Friedmann equation (11) gives pressure p = 0
As there are three independent equations for the four unknown functions a(t), ρ(t), ξ(t) and p(t), we can have the solutions of the field equations in terms of ξ(t) or a(t),
Now, we shall examine if there is any possibility of the existence of cosmological models with viscous matters causing to the formation of a big rip singularity.
The Case For ($E \neq 0$)Or ($c \neq 0$): Firstly, let us consider the case for $c \neq 0$.
If the viscous fluid satisfies DEC, then the condition 0 ≤c ≤3 ×10^8 must be satisfied.
Although the value of E is calculated to be
$$E = \frac{3 + A}{2 + B} = \frac{3 - 3c^2}{2 - 2c^2} = \frac{3}{2}$$
The condition 0 ≤ E ≤1:5 must be satisfied i.e., E ∈[0; 1:5]. Thus, for E > 0, we have a Phantom Cosmology.
From Thermodynamics, if ξ > 0 and if E < 0, equation (16) implies that we

can have no singularity at a finite value of the cosmic time. Here we have considered some examples to see this more clearly. From equation (19), the well known standard case for a perfect fluid i.e., ξ= o takes the form as: $a(t) = K\left[F + \frac{3}{2}t\right]^{\frac{2}{3}}$ The scale factor may be written as $a(t) = KF\left[1 + \frac{3}{2}.\frac{1}{F}t\right]^{\frac{2}{3}} = a_0\left[1 + \frac{3}{2}H_0 t\right]^{\frac{2}{3}}$

Writing, $a_0 = KF$ as another constant and $\frac{1}{F} = H_0$ also the energy density is given by equation (4). Thus we have, $a = a_0\left(1 + \frac{3}{2}H_0 t\right)^{\frac{2}{3}} = a_0(1 + Jt)^{\frac{2}{3}}$

where $J = \frac{3}{2}H_0$, a constant. Therefore,

$$\dot{a} = a_0 H_0 (1 + Jt)^{-\frac{1}{3}}$$

Therefore,

$$\rho = \frac{H_0}{\left(1+\frac{3}{2}H_0 t\right)}$$

But

$$\rho = 3H^2 = 3\left(\frac{\dot{a}}{a}\right)^2 = \frac{3{H_0}^2}{\left(1+\frac{3}{2}H_0 t\right)^2}\frac{\rho_0}{\left(1+\frac{3}{2}H_0 t\right)^2} \quad (24)$$

where $\rho_0 = 3{H_0}^2$, in order to have, $H(t_0 = 0) = H_0 > 0$

If E < o, the cosmic time $t_{br} = -\frac{2}{3}H_0 > t_0 = 0$, we do not have any big rip singularity at a finite value of cosmic time t_{br}

In the special case of $\xi(t) = \xi_0$ =constant.

We get,

$$a(t) = K\left[F + \frac{3}{2(D\xi_0)}e^{D\xi_0 t}\right]^{\frac{2}{3}} \quad (25)$$

We can rewrite this equation into the form

$$a(t) = a_0\left[1 + \frac{3}{2}\frac{H_0}{\xi_0}.\frac{3}{2}(1-c^2)\left(e^{\left(\frac{3(1-c^2)\xi_0 t}{2}-1\right)}\right)\right]$$

(26)

from which we obtain the energy density

$$\rho(t) = \rho_0\frac{e^{3\xi_0 t}}{1+\frac{3}{2}\frac{2H_0}{3\xi_0}\left((1-c^2)\left(e^{\left(\frac{3(1-c^2)\xi_0 t}{2}-1\right)}\right)^2\right)} \quad (27)$$

Where $\rho_0 = 3{H_0}^2$

As earlier, for E < o

i.e., $E = \frac{3+A}{2+B} < 0 \Rightarrow -2c^2 - 1 > -3c^2 \Rightarrow c > \pm 1$

Since $A = -3c^2$ and $B = -2c^2$

i.e., for E < o or $c > \pm 1$ and (c is not less than zero), we have no big rip singularity at a finite value of cosmic time.

$$t_{br} = \frac{2}{3\xi_0}I_n\left(1 - \frac{H_0}{\xi_0}\right) > t_0 = 0$$

Now we take the condition $\xi(t) = \xi(\rho(t))$ to fix the unknown problem by using any additional condition on the system of the field equations Now from equation (12), we have, $\dot{H} + \frac{3}{2}H^2 = DH.\alpha\sqrt{3H}$

Since $\xi = \alpha\rho^{\frac{1}{2}} \Rightarrow \frac{1}{H} = -\frac{3}{2}\left(\frac{\sqrt{3}\alpha}{1-c^2} - 1\right)(t - t_0) + \frac{1}{H_0^*}$

where H_0^* is the constant of integration and thus we have $-\frac{1}{H_0^*} = \frac{1}{H_0}$

$$\Rightarrow \frac{1}{H} = \frac{1}{H_0} - \frac{3}{2}\left(\frac{\sqrt{3}\alpha}{1-c^2} - 1\right)(t - t_0) \quad (28)$$

Where $H_0 = H(t - t_0)$ and t_0 correspond to the time where dark component begins to become dominant. Also

$$a(t) = a_0\left(1 - \frac{t-t_0}{t_{br}}\right)^{\frac{2}{3\left(\frac{\sqrt{3}\alpha}{1-c^2}\right)-1}} \quad (29)$$

Where $a_0 = a(t - t_0)$

Now to have the occurrence of a big rip in the future cosmic time, then we has the following constraint on the parameterα. Thus $1 - c^2 > \sqrt{3}\alpha$ (30)

leading the scale factor to blow up to infinity at a finite time$t_{br} > t_0$, whose expression is

$$t_{br} = \frac{2}{3\left(\frac{\sqrt{3}\alpha}{1-c^2}-1\right)}H_0^{-1} \quad (31)$$

In terms of time tbr, the Hubble Parameter is given by $H(t) = H_0\left(1 - \frac{t-t_0}{t_{br}}\right)^{-1}$ (32)

From equation (4) and the parameterized equations (31) and (32), for the scale factor and the Hubble Parameter respectively, we shall obtain the expression for the increasing density of the dark component in terms of the scale factor

$$\rho(a) = 3H_0^2\left(\frac{a}{a_0}\right)^{3\left(\frac{\sqrt{3}\alpha}{1-c^2}-1\right)} \quad (33)$$

Now substituting this in equation (15), we can replace it. The bulk viscosity is given by

$$\xi(t) = \left(\frac{\sqrt{3}\alpha}{1-c^2}\right)H_0\left(1 - \frac{t-t_0}{t_{br}}\right)^{-1} \quad (34)$$

The Case For (E = o): Now let us notice that the structure of equation (12) changes, if E = o and ξ is an arbitrary function of density, since the quadratic term in H appears. Nevertheless, if $\xi \sim \rho^{\frac{1}{2}}$ the structure of equation (12) is the same for any value of E in the range o ≤ E ≤ 2, except in the case when $\frac{\sqrt{3}\alpha}{1-c^2} = 1$ where equation (12) becomes $\dot{H} = 0$ i.e., H=constant.

Findings: Here we have showed that the power law solution, found by Barrow [6] within the framework of the non-causal thermodynamics, for dissipative universe with $\xi = \alpha\rho^{\frac{1}{2}}$ yields cosmologies which present big rip singularity when the constraint given in equation (30) holds. If we consider that the dark component is quintessence, i.e., $0 \le \sqrt{3}\alpha \le \frac{2}{3}$ with a sufficiently large bulk viscosity will make this quintessence behaves like a phantom energy. However, it is not clear for us that how it can be interpreted with a radiation fluid e.g., with a large bulk viscosity leading to high negative pressures and increasing densities. At the boundary between the quintessence sector and the phantom sector, i.e, E = o or c = 0 there also exist cosmologies with a big rip singularity.

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