SOLVING FULLY FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM UNDER FUZZY CIRCUMSTANCES

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Abstract: Several methods currently exist for solving fuzzy linear fractional programming problems under nonnegative fuzzy variables. However, due to the limitation of these methods, they cannot be applied for solving fully fuzzy linear fractional programming (FFLFP) problems where all the variables and parameters are fuzzy numbers. So, this paper is planning to fill in this gap and in order to obtain the fuzzy optimal solution we propose a new efficient method for FFLFP problems utilized in daily life circumstances. Here, FFLFP problem transformed into an equivalent Multi- Objective Linear Fractional Programming (MOLFP) problem. Then MOLFP converted into an equivalent multi objective linear programming problem by using Mathematical programming approach. This proposed method is based on crisp linear fractional programming and has a simple structure. To show the efficiency of our proposed method some numerical examples have been illustrated.

Keywords: Linear programming problem; Triangular fuzzy numbers; Fuzzy mathematical programming.

1. **Introduction:** Nowadays, the problem of linear fractional programming has significant application in different real life areas such as production planning, financial sector, health care and all engineering fields. However, in real world applications, certainty, reliability and precision of data is often illusory. The optimal solution of an LP only depends on a limited number of constraints; therefore much ofthe collected information has little impact on the solution. It is useful to consider the knowledge of experts about the parameters as fuzzy data.

The fuzzy linear fractional programming problems in which all the parameters and variables are represented by fuzzy numbers are known as fully fuzzy linear fractional programming (FFLFP) problems. In the actual cases the parameters may be uncertain or a vague estimation about the variables is known as those are found in general by some experiment. To overcome the uncertainty and vagueness, one may use the fuzzy numbers in the place of the crisp numbers. Thus the crisp system of linear fractional programming problem becomes a fuzzy system of FLFP problem or FFLFP problem. In the FFLFP problem all the parameters and variables are considered to be fuzzy numbers. Nowadays, the problem of FLFP problems has significant application in different real life areas such as production planning, financial sector, health care and all engineering fields. For this reason, this is an important area of research in the recent years. In this paper, we consider the FFLFP problem. In recent years, many methods currently exist for solving FFLFP problem under nonnegative fuzzy variables.

The concept of fuzzy set and fuzzy numbers was first introduced [8]. In [11], the authors proposed a method to solve multi-objective linear fractional programming (MOLFP) problem under a fuzzy satisfied. A general concept of Pareto optimal solution and use two types of fuzzy goals (called fuzzy equal and fuzzy min) was introduced in [12]. In [8] the concept of linguistic variation was introduced. The FFLP problem by establishing all the coefficients and variables of a linear program as being fuzzy quantities was introduced in [7]. Recently Pop and Minasian[2], proposed a method for solving fully falsified linear fractional programming problems where all the parameters and variables are triangular fuzzy numbers. In [3-4], they considered the same problem of [2] for solving fully fuzzy linear fractional programming problem. In this paper, we modified the methods of [2-3]. First we transform the FFLFP problem into a FFLP problem with the help of Charnes-Cooper method [1]. Thenusing a new technique, the FFLP problem will be converted into a multi-objective linear programming (MOLP) problem. We also prove that this solution can be considered as an exact solution of FFLFP problem. Finally, we show that advantages of the proposed method over the existing method [2, 3]. This paper is organized as follows: some basic definitions and notations are present in section 2. In section 3, we discuss the LFP problem. In section 4, we present our proposed method. A real life example is provided to validate the proposed method in section 5.

2. **Preliminaries:** We have presented some basics concept of fuzzy triangular number, which was very useful in this paper.

Definition 2.1: Let X denotes a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}} : X \to [0,1]$; which assigned a real number $\mu_{\widetilde{A}}(X)$ in the interval [0, 1], to each element $x \in X$, where the values of $\mu_{\widetilde{A}}(X)$ at x shows the grade of membership of x in $\widetilde{A}$. A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(X)$ and is often written $\widetilde{A} = (x, \mu_{\widetilde{A}}(x)) : x \in X$ is called a fuzzy set.

Definition 2.2: A fuzzy number $\widetilde{A} = (b,c,a)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(X) = \begin{cases} \dfrac{(x-b)}{(c-b)}, & b \le x \le c, \\ \dfrac{(x-a)}{(c-a)}, & c \le x \le a, \\ 0, & else. \end{cases}$$

Definition 2.3: Two triangular fuzzy number $\widetilde{A} = (b,c,a)$ and $\widetilde{B} = (e,f,d)$ are said to be equal if and only if $b = e, c = f, a = d$.

Definition 2.4: A ranking is a function $R : F(R) \to R$ where *F(R)* is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{A} = (b,c,a)$ is a triangular fuzzy number then $\Re(\widetilde{A}) = \dfrac{b + \quad -a}{4}$. **Definition 2.5:** Let $\widetilde{A} = (b, \quad)$, $\widetilde{B} = (e,f,d)$ be two triangular fuzzy numbers then:

(i) $\widetilde{A} + \widetilde{B} = (b,c,a) + (e,f,d) = (b+e, c+f, a+d)$,

(ii) $\widetilde{A} - \widetilde{B} = (b,c,a) - (e,f,d) = (b-d, c-f, a-e)$,

(iii) If $\widetilde{A} = (b,c,a)$ be any triangular fuzzy number and $\widetilde{B} = (e,f,d)$ be a non-negative triangular fuzzy number then,

$$\widetilde{A} \otimes \widetilde{B} = \widetilde{A}\widetilde{B} = \begin{cases} (be, cf, ad) & if\ b \ge 0, \\ (bd, cf, ad) & if\ b < 0, a \ge 0, \\ (bd, cf, cd) & if\ c < 0, \end{cases}$$

Definition 2.6: Let $\widetilde{A} = (b,c,a)$, $\widetilde{B} = (e,f,d)$ be two triangular fuzzy numbers. We say that $\widetilde{A}$ is relatively less than $\widetilde{B}$, if and only if:

(i) $b < e$ or

(ii) $b = e$ and $(b-c) > (e-f)$ or

(iii) $b = e$, $(b-c) = (e-f)$ and $(a+b) = (d+e)$.

Note: It is clear from the definition 2.7 that $b = e$, $(b-c) = (e-f)$ and $(a+b) = (d+e)$ if and only if $\widetilde{A} = \widetilde{B}$.

3. Linear Fractional programming

The general form of LFP may be written as:

$$\text{Max } \frac{c^t z + q}{d^t z + r} = \frac{F(z)}{G(z)}$$

subject to:

$$Az \leq = \geq b,$$
$$z \geq 0.$$

where z, c^t, $d^t \in R^n$ and $A \in R^{m\times n}, \alpha, \beta \in R$ (1)

For some values of z, G(z) may be equal to zero. To avoid such cases, one requires that either $\{z \geq 0,\ Az \leq = \geq b, \Rightarrow G(z) > 0\}$ or $\{z \geq 0, Az \leq = \geq b, \Rightarrow G(z) < 0\}$

For satisfaction, we assume that (1) satisfies the condition that:

$$\{z \geq 0,\ Az \leq = \geq b, \Rightarrow G(z) > 0\} \quad (2)$$

Theorem 3.1 [11]**:** Assume that no point (x,0) with $x \geq 0$ is feasible for the following linear programming (LP) problem:

Max $c^t x + qt$

subject to

$$d^t x + rt = 1,$$
$$Ax - bt = 0, \quad (3)$$
$$t \geq 0,\ x \geq 0$$

Now assume that the equation (2) then the LFP (1) us equivalent into linear programming problem (3).

4. Fully fuzzy linear fractional programming problem: Linear fractional programming problem is evidently an uncertain optimization problem due to its variations in the maximum daily requirements. So the amount of each product of ingredient will be uncertain. Hence, we will model the fully fuzzy linear fractional programming problem where all the variables and all the parameters are triangular fuzzy numbers to avoid uncertain.

Let us consider a general format of fully fuzzy linear fractional programming problem as follows:

$$\text{Max } \tilde{Z} = \frac{\tilde{c}^t \tilde{x} + \tilde{q}}{\tilde{d}^t \tilde{x} + \tilde{r}}$$
$$\text{s.t. } \tilde{A}\tilde{x} \leq \tilde{b} \quad (9)$$
$$\tilde{x} \geq 0.$$

Consider the Eq.(9) and let $\tilde{x}^* = (x^*, y^*, z^*)$ be an optimal solution of this FFLFP. Furthermore, let all the parameters $\tilde{x}$, $\tilde{c}, \tilde{q}, \tilde{d}, \tilde{r}, \tilde{b}$ and $\tilde{z}$ are represented by triangular fuzzy numbers (x, y, z), (p, q, r), $(\alpha_1, \alpha_2, \alpha_3), (u, v, w), (\beta_1, \beta_2, \beta_3), (b_1, b_2, b_3)$ and (z_1, z_2, z_3) respectively. Then we can rewrite the mentioned FFLFP as follows:

$$\text{Max } (z_1, z_2, z_3) = \frac{(p,q,r)^t \otimes (x,y,z) + (\alpha_1, \alpha_2, \alpha_3)}{(u,v,w)^t \otimes (x,y,z) + (\beta_1, \beta_2, \beta_3)}$$
$$\text{subject to } (b,c,a) \otimes (x,y,z) \leq (b_1, b_2, b_3)$$
$$(x,y,z) \geq 0$$

5. Numerical Example: In Jamshedpur City, India, A Wooden company is the producer of two kinds of products A and B with profit around (5, 1, 3) and around (4, 1, 6) dollar per unit, respectively. However the cost for each one unit of the above products is around (4, 6, 5) and around (6, 3, 9) dollars respectively. It is assume that a fixed cost of around (1, 2, 6) dollar is added to the cost function due to expected duration through the process of production. Suppose the raw material needed for manufacturing product A and B is about (3, 2, 1) units per pound and about (6, 4, 1) units per pound respectively, the supply for this raw material is restricted to about (13, 5, 2) pounds. Man-hours per unit for the product A is about (4, 1, 2) hour and product B is about (6, 5, 4) hour per unit for manufacturing but total Man-hour available is about (6, 3, 9) hour daily. Determine how many products A and B should be manufactured in order to maximize the total profit.

This real life problem can be formulated to the following FLFP problem:

$$\text{Max } \frac{(5,1,3)(x_1, y_1, z_1) + (4,1,6)(x_2, y_2, z_2)}{(4,6,5)(x_1, y_1, z_1) + (6,3,9)(x_2, y_2, z_2) + (1,2,6)}$$
$$\text{s.t. } (3,2,1)(x_1, y_1, z_1) + (6,4,1)(x_2, y_2, z_2) \leq (13,5,2) \quad (10)$$

$$(4,1,2)(x_1,y_1,z_1)+(6,5,4)(x_2,y_2,z_2)\le(6,3,9)$$
$$(x_1,y_1,z_1),\quad (x_2,y_2,z_2)\ge 0.$$

The problem (10) is converted into the MOLFP problem as follows

$$\text{Max}\left\{\frac{5x_1+4x_2}{4x_1+6x_2+1},\frac{y_1+y_2}{6y_1+3y_2+2},\frac{y_1+y_2}{6y_1+3y_2+2}\right\}$$

subject to $3x_1+6x_2\le 13,$

$$\begin{aligned}&2y_1+4y_2\le 5,\\&z_1+z_2\le 2,\\&4x_1+6x_2\le 6,\\&y_1+5y_2\le 3,\\&2z_1+4z_2\le 9,\\&x_1,x_2,y_1,y_2,z_1,z_2\ge 0.\end{aligned}\qquad(11)$$

The problem (11) is transformed into an equivalent multi objective linear programming problem as follows:

$$\begin{aligned}&\text{Max } Z_1=5y_1+4y_2\\&\text{Max } Z_2=z_1+z_2\\&\text{Max } Z_3=3x_1+6x_2\end{aligned}\qquad(12)$$

subject to $4y_1+6y_2+t\le 1,$

$$\begin{aligned}&6z_1+3z_2+2t\le 1,\\&5x_1+9x_2+6t\le 1,\\&3y_1+6y_2-13t\le 0,\\&2z_1+4z_2-5t\le 0,\\&x_1+x_2-2t\le 0,\\&4y_1+6y_2-6t\le 0,\\&z_1+5z_2-3t\le 0,\\&2x_1+4x_2-9t\le 0,\\&x_1,x_2,y_1,y_2,z_1,z_2\ge 0.\end{aligned}$$

The problem (12) can be written as follows:

$$\begin{aligned}&\text{Max } Z_1=5y_1+4y_2\\&\text{Max } Z_2=5y_1+4y_2-z_1-z_2\\&\text{Max } Z_3=5y_1+4y_2+3z_1+6z_2\end{aligned}\qquad(13)$$

subject to $4y_1+6y_2+t\le 1,$

$$\begin{aligned}&4y_1+6y_2+t-6z_1-3z_2-2t\le 0,\\&4y_1+6y_2+t+5x_1+9x_2+6t\le 2,\\&3y_1+6y_2-13t\le 0,\\&3y_1+6y_2-13t-2z_1-4z_2+5t\le 0,\\&3y_1+6y_2-13t+x_1+x_2-2t\le 0,\\&4y_1+6y_2-6t\le 0,\\&4y_1+6y_2-6t-z_1-5z_2+3t\le 0,\\&4y_1+6y_2-6t+2x_1+4x_2-9t\le 0,\end{aligned}$$

Solving the problem (13) we get $y_1=0.214$, $y_2=0$, $x_1=0$, $x_2=0.016$, $z_1=0.067$, $z_2=0.102$, $t=0.142$

Hence the solution of the problem (10) is $Z_1=1.07$, $Z_2=0.169$, $Z_3=0.09$.

Example: 2. In TATA Hospital Jamshedpur, India has two nutritional experiments (Vitamin A and Calcium) with two products Milk (glass) and Salad (500mg) with profit around 6 dollars and around 2 dollars per unit respectively. However, the cost for each one unit of the above product is around 1 and around 1 dollars respectively. Consider that a fixed cost of around 2 dollars as added to the cost function. Determine the *maximum* profit of these two products.

Table 1.Information of Example 2

Nutrient	Milk (glass)	Salad (500mg)	Min nutrient required
Vitamin A	1	1	7
Calcium	2	3	17

Solution: In this case, let x_1 and x_2 to be the amount of units of Vitamin A and Calcium to be produced. Then the above problem can be formulated as:

Max $\tilde{z} = \dfrac{\tilde{6}\tilde{x}_1 + \tilde{2}\tilde{x}_2}{\tilde{x}_1 + \tilde{x}_2 + \tilde{2}}$

Subject to $\tilde{x}_1 + \tilde{x}_2 \le \tilde{7}$

$\tilde{2}\tilde{x}_1 + \tilde{3}\tilde{x}_2 \le 1\tilde{7}$

$\tilde{x}_1, \tilde{x}_2 \ge 0.$

Let us we take $\tilde{x}_1 = (y_1, z_1, x_1)$, $\tilde{x}_2 = (y_2, z_2, x_2)$ $\tilde{z} = (z_1, z_2, z_3)$.Nowweconsider the coefficients $\tilde{7} = (3,7,11)$, $1\tilde{7} = (7,17,27)$, $\tilde{6} = (4,6,8)$, $\tilde{2} = (1,2,3)$, $\tilde{3} = (2,3,4)$ and $\tilde{1} = (0,1,2)$.The problem can be written as follows:

Max $(z_1, z_2, z_3) = \dfrac{(4,6,8) \otimes (y_1, z_1, x_1) + (1,2,3) \otimes (y_2, z_2, x_2)}{(0,1,2) \otimes (y_1, z_1, x_1) + (0,1,2) \otimes (y_2, z_2, x_2) + (1,2,3)}$

Subject to $(0,1,2) \otimes (y_1, z_1, x_1) + (0,1,2) \otimes (y_2, z_2, x_2) \le (3,7,11),$

$(1,2,3) \otimes (y_1, z_1, x_1) + (2,3,4) \otimes (y_2, z_2, x_2) \le (7,17,27),$

$y_1, y_2, z_1, z_2, x_1, x_2 \ge 0.$

Then, weobtain the fuzzy optimal solutionas:

$\tilde{x}_1 = (y_1, z_1, x_1) = (5.5,7,7),$ and $\tilde{x}_2 = (y_2, z_2, x_2) = (0,0,0).$

And the optimal value of the problem as:

$\tilde{Z}_1 = (Z_1, Z_2, Z_3) = (3.14, 4.6, 28).$

Conclusions: In this paper, a new solving procedure has been suggested to solve the FFLFP problem. Using Charnes-Cooper transformation method, we transformed the fully fuzzy linear fractional programming problem into fully fuzzy linear programming problem. After that,the fully fuzzy linear programming problem is converted into its equivalent MOLP problem. It is our belief that the proposed method for solution of FFLFP problem in real life problem as well as simple problem may be of considerable interest for mathematician working in this field.

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