NEW SEPARATION AXIOMS IN EXTENDED BITOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce and study some new separation axioms using the$(1, 2)^+\alpha$-open sets in extended bitopological space .We also investigate some of their basic properties and establish the relationship between them .

Keywords :Extended bitopological spaces,Ultra+-T_i(i = 0, 1, 2),Ultra+-$T_{1/2}$ and Ultra+-D_i(i = 0, 1, 2)spaces .

Introduction: In 1963, Kelly [2] initiated the study of the bitopological space which is to be a set X equipped with two topologies $\tau_1$and $\tau_2$on X. Maheswari et al.[7] is defined new separation axioms called semi-Ti(i = 0, 1, 2). Another set of new separation axioms αT_i (i = 0, 1), were characterized by Mahi et al.[8]. Recently, LellisThivagar [5] introduced new bitopological notions of $\tau_1\tau_2$-open sets and $\tau_1\tau_2$-closed sets. After that LellisThivagar et al [5] initiated the concept of Ultra space by using(1, 2)α-open sets in bitopological spaces and also proved [3] that each (1, 2)α-open sets is (1, 2)-semi open and (1, 2) -pre open but the converse of each is not true . In 1964, Levin[6] initiated the simple extensions of topologies. The purpose of this paper is to introduce new separation axioms and to discuss its various aspects by using $(1, 2)^+\alpha$-open sets.

Preliminaries:

Definition 2.1.[2] A non-empty set X together with two topologies $\tau_1$and $\tau_2$is called a bitopological spaces and is denoted by (X, τ_1, τ_2).

Definition 2.2.[3] A subset S of X is called

(i) $\tau_1\tau_2$-open if $S\in \tau_1\cup\tau_2$

(ii) $\tau_1\tau_2$-closed if $S\in (\tau_1\cup\tau_2)^c$.

The family of all $\tau_1\tau_2$-open (resp. $\tau_1\tau_2$-closed) sets is denoted by $\tau_1\tau_2O(X)$ (resp. $\tau_1\tau_2C(X)$).

Definition 2.3.[4] Let(X, τ_1, τ_2) be a bitopological space and $\tau_1\tau_2O(X\}\subset (\tau_1\tau_2)^+$. Then $(\tau_1\tau_2)^+$ will be termed a simple extension of $\tau_1\tau_2O(X)$ if and only if there exists an $A \notin \tau_1\tau_2O(X)$ such that $(\tau_1\tau_2)^+ = (\tau_1\tau_2)^+(A) = \{G_1\cup (G_2\cap$

A): G_1, $G_2 \in \tau_1\tau_2 O(X)$}. We call (X, $(\tau_1\tau_2)^+(A)$) an extended bitopological space of (X, τ_1,τ_2) w.r.t A.

Throughout this paper (X, $(\tau_1\tau_2)^+(A)$),(Y, $(\tau_1\tau_2)^+(B)$) [or simply X, Y] denote the extended bitopological space on which no separation axioms are assumed unless explicitly stated.

Definition 2.4.[4] Let (X, $(\tau_1\tau_2)^+(A)$) be an extended bitopological space and $S \subseteq X$. Then $(\tau_1\tau_2)^+$ closure of S is defined as $(\tau_1\tau_2)^+cl(S) = \cap\{F : S \subseteq F$ and F is $(\tau_1\tau_2)^+$ closed} and $(\tau_1\tau_2)^+$interior of S is defined as $(\tau_1\tau_2)^+int(S)= \cup\{G : G \subseteq S$ and G is $(\tau_1\tau_2)^+$ open}.

Theorem 2.5.[4] Let (X,τ_1,τ_2) be a bitopological space which is T_o, T_1or T_2 and $A \notin \tau_1\tau_2 O(X)$. Then (X, $(\tau_1\tau_2)^+(A)$) is T_o, T_1 or T_2.

Definition 2.6. [4] Let (X,$(\tau_1\tau_2)^+(A)$) be an extended bitopological space. A subset S of X is called

(i) $(l,2)^+\alpha$-open if $S \subset \tau_1^+ int((\tau_1\tau_2)^+cl(\tau_1^+ int(S)))$

(ii) $(l,2)^+$semi-openif $S \subset (\tau_1\tau)^+cl(\tau_1^+ int(S))$ and

(iii) $(1,2)^+$pre-open if $S \subset \tau_1^+ int((\tau_1\tau_2)^+cl(S))$.

The Complements of the sets mentioned above from (i) to (iii) are called their respective closed sets.

The collection of all $(1,2)^+\alpha$-open, $(1,2)^+$semi-open and $(1,2)^+$pre-open sets of X are denoted by $(l,2)^+\alpha O(X,A)$,$(1,2)^+SO(X,A)$,$(1,2)^+PO(X,A)$respectively.

Remark 2.7.[4] Let (X,τ_1,τ_2) be a bitopo-logical space, if $(1,2)\alpha O(X)$ is form a topology, then $(1,2)^+\alpha O(X,A)$ is also form a topology. We call the topology $(1,2)^+\alpha O(X,A)$ as a $(1,2)^+\alpha$-topology or an Ultra$^+$ space. Here $(1,2)^+\alpha cl(S)$[resp.$(1,2)^+scl(S)$ and $(1,2)^+pcl(S)$] is defined as the intersection of all (1,2)+α-closed [resp. $(1,2)^+$ semiclosed and $(1,2)^+$pre-closed] sets containing A.

Theorem 2.8.[4] Let X be an extended bitopological space.

$S \subseteq X$ is a $(l,2)^+\alpha$-open if and only if S is a $(l,2)^+$semi-open and a $(1, 2)^+$pre-open .

Definition 2.9.[4] Let X be an extended bitopological space. A subset S of X is called a

- $(1,2)^+\alpha$g-closedset if $(l,2)^+\alpha cl(S) \subseteq U$ whenever$S \subseteq U$and $U \in (1,2)^+\alpha O(X,A)$,
- $(1,2)^+$sg-closed set if $(l,2)^+scl(S) \subseteq U$whenever$S \subseteq U$ and $U \in (1,2)^+SO(X,A)$,
- $(1,2)^+$pg-closed set if $(l,2)^+pcl(S) \subseteq U$whenever$S \subset U$ and $U \in (1,2)^+PO(X, A)$.

The complement of above closed sets are called their respective open sets.

Theorem 2.10.[4] In an extended bitopological space X,a set S of X is called a $(1,2)^+\alpha$g-closed set iff $[(1, 2)^+\alpha cl(S)-S]$ contains no nonempty $(1, 2)^+\alpha$-closed set.

Theorem 2.11.[4] A $(1, 2)^+\alpha$g-closed set is $(1,2)^+\alpha$-closed iff $[1,2)^+\alpha cl(S)-S]$ is $(1,2)^+\alpha$-closed.

Theorem 2.12.[4]If S and T are $(1,2)^+\alpha$g-closed, thenS $\cup$T is also $(1,2)^+\alpha$g-closed.

Lemma 2.13.[4] For an extended bitopological space X, every singleton set $\{x\}$ is either $(1,2)^+\alpha$-closed or $\{x\}^c$is $(1, 2)^+\alpha$g-closed.

Extended Ultra $-T_i(I = 0, 1, 2)$ **Spaces**

In this section, we introduce and investigate the concept of Ultra$^+$-T_i(i = 0, 1, 2) spaces in an extended bitopological space using $(1, 2)^+\alpha$-open sets.

Definition 3.1. An extended bitopological space X is called an Ultra$^+$-T_0-space iff for every disjoint point x, y $\in$X ,there exists a $(1, 2)^+\alpha$-open set containing x but not y or a $(1, 2)^+\alpha$-open set containing y but notx.

Definition 3.2. An extended bitopological space X is called an Ultra$^+$-T_1-space iff for every disjoint point x, y $\in$X there exists a $(1,2)^+\alpha$-open set containing x but not y and a $(1, 2)^+\alpha$-open set containing y but not x.

Definition 3.3. An extended bitopological space X is called an Ultra$^+$-T_2-space iff for every disjoint point x, y $\in$X there exists two disjoint $(1,2)^+\alpha$-open setsG and H such that $x \in G$ and $y\in H$.

Example 3.4. Let X = {a, b, c}, τ_1= {ϕ,X}, τ_2={ϕ,X} . Then $\tau_1\tau_2O(X)$= {ϕ,X}. Let A ={a} $\notin \tau_1\tau_2O(X)$. Then $(\tau_1\tau_2)^+(A)$ = {ϕ,X, {a}}.$\tau_1^+(A)$ = {ϕ,X, {a}}. Then $(1, 2)^+\alpha O(X,A)$={ϕ,X, {a}, {a, b}, {a, c}} . Here X is an Ultra$^+$-T_0-space.

Theorem 3.5. A space, where $\tau_1^+$is T_0, is an Ultra$^+$-T_0-space.

Proof: The proof is obvious from the definition.

Remark 3.6. The converse of the above theorem is need not be true . In the above example 3.4, we have $\tau_1^+(A)$ = {ϕ,X, {a}}.

Then $(1, 2)^+\alpha O(X,A)$={ϕ,X, {a}, {a, b}, {a, c}} . Here X is an Ultra$^+$-T_0-space.But $\tau_1^+(A)$ is not a T_0-space.

Theorem 3.7. Every Ultra$^+$-T_1-space is an Ultra$^+$-T_0-space.

Proof: The proof is obvious from the definition.

Remark 3.8. The converse of the above theorem is need not be true . Let X = {a, b, c},τ_1 = {ϕ,X, {a}}, τ_2= {ϕ,X, {b, c}}.

Then $\tau_1\tau_2O(X)$={ϕ,X, {a}, {b, c}}. LetA = {b} $\notin \tau_1\tau_2O(X)$. Then $(\tau_1\tau_2)^+(A)$ ={ϕ,X, {a}, {b}, {a, b}, {b, c}}. $\tau_1^+(A)$ ={ϕ,X, {a}, {b}, {a, b}}. Then $(1, 2)^+\alpha O(X,A)$={ϕ,X, {a}, {b}, {a, b}} . Here X is an Ultra$^+$-T_0-space but not an Ultra$^+$-T_1-space.

Theorem 3.9. An extended bitopological space(X, $(\tau_1\tau_2)^+$) is an Ultra$^+$-T_0-space iff the $(1,2)^+\alpha$-closure of distinct points are distinct.

Proof: Let $x \neq y$ implies $(1, 2)^+\alpha cl(\{x\}) \neq (1, 2)^+\alpha cl(\{y\})$. Then there exists atleast one $z\in(1,2)^+\alpha cl(\{x\})$ but $z \notin (1, 2)^+\alpha cl(\{y\})$. Letx $\in (1, 2)^+\alpha cl(\{y\})$. Then $(1, 2)^+\alpha cl(\{x\}) \subseteq (1, 2)^+\alpha cl(\{y\})$, which is a contradiction that $z \notin (1, 2)^+\alpha cl(\{y\})$. Hence $x \in [(1, 2)^+\alpha cl\{y\}]^c$. Conversely, let X be an Ultra$^+$–T_0-space. Take x, y $\in$Xand $x \neq y$. Then there exists a $(1, 2)^+\alpha$-open set G such that $x \in G$ and $y\notin G$, which implies $y \in X - G = F$(say). Now $(1,2)^+\alpha cl(\{y\}) = \cap\{F/\{y\} \subseteq F$ and F is a $(1,2)^+\alpha$-closed set$\}$. This implies thaty$\in (1, 2)^+\alpha cl(\{y\})$ and $x \notin ((1, 2)^+\alpha cl(\{y\})$.

Theorem 3.10. An extended bitopological space(X, $(\tau_1\tau_2)$+) is an Ultra$^+$-T_1-space iff every singleton subset {x} of X is a $(1, 2)^+\alpha$-closed set.

Proof: Let x, y $\in$X and $x \neq y$. If {x} and {y} are the $(1, 2)^+\alpha$-closed sets of x and y respectively such that $\{x\} \neq \{y\}$, then $\{x\}^c$and $\{y\}^c$are $(1, 2)^+\alpha$-open sets such that $y \in \{x\}^c$and $x \notin \{x\}^c$, also $x \in \{y\}^c$and $y \notin \{y\}^c$. Then X is an Ultra$^+$-T_1-space.

Conversely, suppose X is an Ultra$^+$-T_1- space. Then for x, y $\in$X and $x \neq y$, there are two $(1, 2)^+\alpha$-open sets G and H such that $x \in H$, $y \notin H$ and $y \in G$ andx $\notin G$. So $G \subseteq \{x\}^c$, and also $\cup\{G/y \neq x\} \subseteq \{x\}^c$and $\{x\}^c \subseteq \{G/y \neq x\}$. Hence $\{x\}^c = \cup\{G/y \neq x\}$,which is a $(1, 2)^+\alpha$-open set. Then {x} is a $(1, 2)^+$-α-closed set.

Theorem 3.11. An extended bitopological space is an Ultra$^+$-T_1-space iff every subset of X is a$(1, 2)^+\alpha$-closed set.

Proof: Assume that every singleton subset {x} of Xis a $(1, 2)^+\alpha$-closed set. Since a finite subset of X is the union of finite number of singleton sets. Hence(1, 2)+α-closed. Conversely, every singleton set {x} is a finite subset of X.

Remark 3.12. Every finite Ultra$^+$-T_1-space is a discrete space.

Definition 3.13. Let X be an extended bitopological space and x$\in$X. Then a subset $N_x$of x is called a$(1,2)^+\alpha$ -nbd (neighbourhood)of x, if there exists a$(1, 2)^+\alpha$-open set G such that $x \in G \subseteq N_x$.

Theorem 3.14. If X is an Ultra$^+$-T_1-space, then the intersection of $(1, 2)^+\alpha$-nbds of an arbitrary point of X is a singleton set.

Proof: Let X be an Ultra$^+$-T_1-space. Also let $x \in X$ and $N_x$be the $(1,2)^+\alpha$-nbd of x. If y be a point of X and $y \neq x$, then there exists a $(1,2)^+\alpha$-open set containing x but not y. Since y is arbitrary, N_x has no point other than x. Conversely, the intersection of $(1,2)^+\alpha$-nbd of x is the singleton set {x}, which does not contain any other point, say, y. Hence X is Ultra$^+$-T_1-space.

Definition 3.15.A space X is called an Ultra$^+$-$T_{1/2}$ -space iff every $(1,2)^+\alpha$g-closed subset of Xis a $(1,2)^+\alpha$-closed set.

Theorem 3.16. An extended bitopological space X is an Ultra$^+$-$T_{1/2}$- space iff either {x} is $(1,2)^+\alpha$-open or $(1,2)^+\alpha$-closed.

Proof: Let X be an Ultra$^+$-$T_{1/2}$ -space. If {x} is not a $(1,2)^+\alpha$-closed set, then by lemma 2.13 ,A =X–{x} is $(1,2)^+\alpha$g-closed. Also by assumption, A is$(1,2)^+\alpha$-closed, which implies that {x} is $(1,2)^+\alpha$ open. Conversely, let A $\subseteq$X be a $(1, 2)^+\alpha$g-closed set with $x \in (1,2)^+\alpha cl(A)$. If {x} is $(1,2)^+\alpha$-open, then$\{x\} \cap A \neq \phi$. Otherwise {x} is $(1, 2)^+\alpha$-closed and so $\phi \neq (1,2)^+\alpha cl(x) \cap A$. In both the cases, $x \in A$ and A is $(1, 2)^+\alpha$-closed.

Theorem 3.17. Every Ultra$^+$-T_1-space is an Ultra$^+$-$T_{1/2}$ -space.

Proof: The proof is obvious from the definition.

Remark 3.18. The converse of the above theorem is need not be true .Let X = {a, b, c}, τ_1= {ϕ,X}, andτ_2= {ϕ,X, {a, c}}. Then $\tau_1\tau_2$O(X)={ϕ,X, {a, c}}.Let A = {a} $\notin \tau_1\tau_2$O(X). Then $(\tau_1\tau_2)^+(A)$ ={ϕ,X, {a}, {a, c}}. $\tau_1^+(A)$ = {ϕ,X, {a}}. Then
$(1, 2)^+\alpha O(X,A)$= {ϕ,X, {a}, {a, b}, {a, c}} and$(1, 2)^+\alpha GCL(X,A)$= {ϕ,X, {b}, {c},{b, c}} in which X is an Ultra$^+$-$T_{1/2}$ -space but not an Ultra$^+$-T_1- space.

Theorem 3.19. Every Ultra$^+$-$T_{1/2}$ -space is an Ultra$^+$-T_0-space.

Proof: Let X be an Ultra$^+$-$T_{1/2}$-space but not an Ultra$^+$-T_0space. Then there exists x, y $\in$X, x $\neq$y such that $(1, 2)^+\alpha cl(\{x\}) = (1, 2)^+\alpha cl(\{y\})$. Let A=$(1, 2)^+\alpha cl(\{x\}) \cap \{x\}^c$. To claim A is $(1, 2)^+\alpha$ g closed:

Let A $\subseteq$G, where G is $(1, 2)^+\alpha$-open. To show that $(1, 2)^+\alpha cl(A) \subseteq G$, it is enough to show that (1, 2)+αcl({x}) $\subseteq$G. That is, it is enough to show that x $\in$G. If not let x $\notin$ G, then $x \in G^c$. So$(1, 2)^+\alpha cl(\{x\}) \subseteq G^c$. By assumption, there exits ay$\in (1, 2)^+\alpha cl(\{x\}) \subseteq G^c$. So $y \in (1, 2)^+\alpha cl(\{x\})\cap\{x\}^c = A \subseteq G$. Hence we get $y \in G \cap G^c$,
which is a contradiction. Therefore x $\in$ G and hence A is $(1, 2)^+\alpha$g-closed.

To claim A is not $(1, 2)^+\alpha$-closed: Let x $\in$U and U is $(1, 2)^+\alpha$-open. By assumption, $x \in (1, 2)^+\alpha cl(\{x\}) = (1, 2)^+\alpha cl(\{y\})$.So $\{y\} \cap U \neq \phi$ and so $\{y\} \subseteq (1, 2)^+\alpha cl(\{y\}) \cap U$,which implies that $\{y\} \subseteq (1, 2)^+\alpha cl(\{x\})\cap U$. Then$\{y\}\cap\{x\}^c \subseteq [(1, 2)^+\alpha cl(\{x\})$
$\cap U] \cap \{x\}^c= A\cap U$. Hence we have $x \in (1, 2)^+\alpha cl(A)$ but $x \notin A$. HenceA is not $(1, 2)^+\alpha$-closed and X is not an Ultra$^+$-$T_{1/2}$-space, which is a contradiction.

Remark 3.20. The converse of the above theorem need not be true. Let X

= {a, b, c, d},τ_1 = {ϕ,X, {a, b}} ,τ_2= {ϕ,X, {b}, {a, b, c}}. Then $\tau_1\tau_2$O(X)={ϕ,X, {b}, {a, b}, {a, b, c}}. LetA = {a} ∉ $\tau_1\tau_2$O(X). Then $(\tau_1\tau_2)^+$(A) ={ϕ,X, {a}, {b}, {a, b}, {a, b, c}} and τ_1^+(A) ={ϕ,X, {a}, {a, b}}. Then $(1, 2)^+\alpha$O(X,A)={ϕ,X, {a}, {a, b}, {a, b, c}, {a, b, d}} and $(1, 2)^+\alpha$GCL(X,A)={ϕ,X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}}. The space X is an Ultra$^+$-T_0space but not an Ultra$^+$-$T_{1/2}$-space.

Definition 3.21.A subset X is called an Ultra$^+$-Door space iff each subset of X is either $(1, 2)^+\alpha$-open or $(1, 2)^+\alpha$-closed .

Remark III.22. Every Ultra$^+$-Door space is an Ultra$^+$-$T_{1/2}$ -space.

Example 3.23. The following example shows that the converse of Remark 3.22, need not always be true. Let X = {a, b, c, d},τ_1= {ϕ,X, {a}, {a, b}, {a, b, c}, {a, b, d}}, τ_2={ϕ,X, {a}, {a, b}}. Then$\tau_1\tau_2$O(X)={ϕ,X, {a}, {a, b}, {a, b, c}, {a, b, d}}.Let A = {b} ∉ $\tau_1\tau_2$O(X). Then $(\tau_1\tau_2)$+(A) ={ϕ,X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} andτ_1^+(A) = {ϕ,X, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}.Then $(1, 2)^+\alpha$O(X,A)= {ϕ,X, {a}, {b}, {a, b},{a, b, c}, {a, b, d}} . Here X is an Ultra$^+$-$T_{1/2}$ –space but not an Ultra$^+$-Door space. Since the subset {a,d} is neither $(1, 2)^+\alpha$-open nor $(1, 2)^+\alpha$-closed.We can also note that X is also not an Ultra$^+$-T_1-space.

Definition 3.24.A space X is called an Ultra$^+$-T_D-space iff the $(1, 2)^+\alpha$-derived set of each singleton set is $(1, 2)^+\alpha$-closed. Let us denote the set of all derived set of {x} as $(1, 2)^+\alpha$-D(x) and define it by $(1, 2)^+\alpha$-D({x}) = $(1, 2)^+\alpha$cl({x})-{x}.

Theorem 3.25. Every Ultra$^+$-$T_{1/2}$ -space is an Ultra$^+$-T_D-space.

Proof: Let X be an Ultra$^+$-$T_{1/2}$-space. Then by theorem 3.16, each singleton set {x} of X is either$(1, 2)^+\alpha$-open or $(1, 2)^+\alpha$-closed . If {x} is $(1, 2)^+\alpha$-open, then $(1, 2)^+\alpha$-D({x}) = $(1, 2)^+\alpha$cl({x})-{x} is $(1, 2)^+\alpha$-closed. If {x} is $(1, 2)^+\alpha$-closed, then$(1, 2)^+\alpha$-D({x}) = ϕ.

Remark 3.26. The converse of the above theorem need not be true .

Let X = {a, b, c, d},τ_1= {ϕ,X, {a, b}}, τ_2= {ϕ,X, {b}, {a, b, c}}.Then $\tau_1\tau_2$O(X)={ϕ,X, {b}, {a, b}, {a, b, c}}. LetA = {a} ∉ $\tau_1\tau_2$O(X). Then $(\tau_1\tau_2)^+$(A) ={ϕ,X, {a}, {b}, {a, b}, {a, b, c}} and τ_1^+(A) ={ϕ,X, {a}, {a, b}}. Then $(1, 2)^+\alpha$O(X,A)={ϕ,X, {a}, {a, b}, {a, b, c}, {a, b, d}} and (1, 2)+α GCL(X,A)={ϕ,X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}}. The spaceX is an Ultra$^+$-T_D space but not an Ultra$^+$-$T_{1/2}$-space.

Theorem 3.27. Every Ultra$^+$-T_2-space is an Ultra$^+$-T_1-space.

Proof: The proof is obvious from the definition.

Definition 3.28.A function f :X → Y is called a $(1, 2)^+\alpha$g-irresolute if the inverse image of every$(1, 2)^+\alpha$g-closed set in Y is a $(1, 2)^+\alpha$g-closed set in X.

Definition 3.29.A map f :X → Y is called an Ultra$^+$-bi closed (resp. Ultra$^+$-bi open) if

(i) for any $(1, 2)^+\alpha$-closed (resp. $(1, 2)^+\alpha$- open)subset V of Y, the inverse image $f^{-1}(V)$ is a$(1, 2)^+\alpha$-closed (resp. $(1, 2)^+\alpha$-open) in X and

(ii) the image V of a $(1, 2)^+\alpha$-closed (resp. $(1, 2)^+\alpha$-open) subset of X is a $(1, 2)^+\alpha$-closed set (resp.$(1, 2)^+\alpha$-open set) of Y.

Theorem 3.30. If X is an Ultra$^+$-$T_{1/2}$-space andf : X → Y is a surjective , $(1, 2)^+\alpha$g-irresolute and Ultra$^+$-bi closed map , then Y is Ultra$^+$-$T_{1/2}$.

Proof: Let X be an Ultra$^+$-$T_{1/2}$-space ,$B\subseteq Y$ be a$(1, 2)^+\alpha$g-closed set. Then by assumption, $f^{-1}(B)$ isa $(1, 2)^+\alpha$g-closed set in X. Since X is Ultra$^+$-$T_{1/2}$-space ,$f^{-1}(B)$ is a $(1, 2)^+\alpha$-closed set in X. Then $f(f^{-1}(B)) = B$ is $(1, 2)^+\alpha$-closed set in Y, since f : X → Y is a surjective.

Theorem 3.31. Let X and Y be two extended bitopological spaces and f :X → Y be a surjective, Ultra$^+$-bi open map such that for each $y \in Y$, $f^{-1}(\{y\})$ isa finite set. Then Y is Ultra$^+$-$T_{1/2}$-space if X is an Ultra$^+$-$T_{1/2}$-space.

Proof: Let us assume that $y \in Y$,$f^{-1}(\{y\}) =\{x_1, x_2, ..., x_n\}$ and f is Ultra$^+$-bi open map .Since X is Ultra$^+$-$T_{1/2}$-space , for some i ,$\{x_i\} \in (1, 2)^+\alpha O(X)$ and so $\{y\} = \{f(\{x_i\})\} \in (1, 2)^+\alpha O(Y)$. Otherwise $\{x_i\}^c \in (1, 2)^+\alpha O(X)$ andso $\{y\}^c= \{f(\{x_i\}^c)\} \in (1, 2)^+\alpha O(Y)$, which implies$\{y\}^c$is $(1, 2)^+\alpha$-open. Then {y} is $(1, 2)^+\alpha$-closed .

Hence Y is an Ultra$^+$-$T_{1/2}$-space

4.Extended Ultra -D_i(i = 0, 1, 2) **Spaces**

In this section, we introduce the notion of difference sets and corresponding separation axioms in accordance to $(1, 2)^+\alpha$-open sets.

Definition 4.1.A subset A of X is called a $(1, 2)^+\alpha$-difference set (in short $(1, 2)^+\alpha$D-set) if there exists two $(1, 2)^+\alpha$-open sets $U_1$and U_2such that A $=U_1\backslash U_2$and $U_1\neq X$.

The family of all $(1, 2)^+\alpha$D-sets is denoted by$(1, 2)^+\alpha$-D(X).

Definition 4.2. An extended bitopological space X is called an

(i) Ultra$^+$-D_0-space if for x, $y \in X$ and $x \neq y$, there exists a $(1, 2)^+\alpha$D-set containing x but not y, or containing y but not x.

(ii) Ultra$^+$-D_1-space if for x, $y \in X$ and $x \neq y$, there exists two $(1, 2)^+\alpha$D-sets G and H such that$x \in G$, $x \notin H$ and $y \in H$, $y \notin G$

(iii) Ultra+-D2 -space if for x, $y \in X$ and $x \neq y$, there exists two disjoint $(1, 2)^+\alpha$D-sets G and H such that $x \in G$ and $y \in H$.

Theorem 4.3. Every $(1, 2)^+\alpha$-open set is a $(1, 2)^+\alpha$D-set.

Proof: A is a $(1, 2)^+\alpha$-open set. We can write $A =U_1\backslash U_2$, where $U_1$and $U_2$are

$(1, 2)^{+}\alpha$-open sets and $U_2 = \phi$.

Theorem 4.4. An extended bitopological space X is Ultra^{+}-T_o-space iff X is an Ultra^{+}-D_o-space.

Proof: By theorem 4.3, we can say every Ultra^{+}-T_o-space is an Ultra^{+}-D_o-space. Conversely, let X be Ultra^{+}-D_o-space. Then for every $x, y \in X$, $x \neq y$, there exists a $(1, 2)^{+}\alpha D$-set S such that $x \in S$ and $y \notin S$, where $S = U_1 \setminus U_2$ and $U_1, U_2 \in (1, 2)^{+}\alpha O(X)$. As $x \in S$ implies $x \in U_1$ and $x \notin U_2$, $y \notin S$ implies $y \notin U_1$ or $y \in U_1$ and $y \in U_2$. Now we have the following two cases:

Case I : $x \in U_1$ and $y \notin U_1$ implies X is Ultra^{+}-T_o-space.

Case ii: $x \notin U_2$ and $y \in U_2$ implies X is Ultra^{+}-T_o-space.

Remark 4.5. In an extended bitopological space X, if X is

(i) Ultra^{+}-D_i, then it is Ultra^{+}-D_{i-1} $(i = 1, 2)$

(ii) Ultra^{+}-T_i, then it is Ultra^{+}-D_i $(i = 0, 1, 2)$.

Theorem 4.6. If X is Ultra^{+}-D_1, then it is Ultra^{+}-T_o but the converse need not always be true.

Proof: Ultra^{+}-$D_1 \Rightarrow$ Ultra^{+}-D_o and Ultra^{+}-$D_o \Rightarrow$ Ultra^{+}-T_o (by theorem.4.4).

Example 4.7. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{b, c\}, a, b, c\}\}$. Then $\tau_1\tau_2 O(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{c\} \notin \tau_1\tau_2 O(X)$. Then $(\tau_1\tau_2)^{+}(A) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, $(1, 2)+\alpha O(X) = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\{a, b, c\}\}$ and $(1, 2)^{+}\alpha D(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ .Here X is an Ultra^{+}-T_o but not an Ultra^{+}-D_1.

Remark 4.8. The following example illustrate that Ultra^{+}-D_1 does not imply Ultra^{+}-T_1.

Example 4.9. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$. Then $\tau_1\tau_2 O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{b\} \notin \tau_1\tau_2 O(X)$. Then $(\tau_1\tau_2)^{+}(A) =$
$\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and τ_1^{+}
$(A) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then $(1, 2)^{+}\alpha O(X,A) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $(1, 2)^{+}\alpha D(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$ .Here X is an Ultra^{+}-D_1-space but not an Ultra^{+}-T_1-space.

Theorem 4.10. An extended bitopological space X is Ultra^{+}-D_1 iff it is Ultra^{+}-D_2.

Proof: Ultra^{+}-$D_2 \Rightarrow$ Ultra^{+}-D_1 is obvious.

Suppose X is Ultra^{+}-D_1. Then for each $x, y \in X, x \neq y$, we have $(1, 2)^{+}\alpha D$-sets S_1 and S_2 such that $x \in S_1$ and $x \notin S_2$ and $y \in S_2$, $y \notin S_1$. Let $S_1 = U_1 \setminus U_2$ and $S_2 = U_3 \setminus$

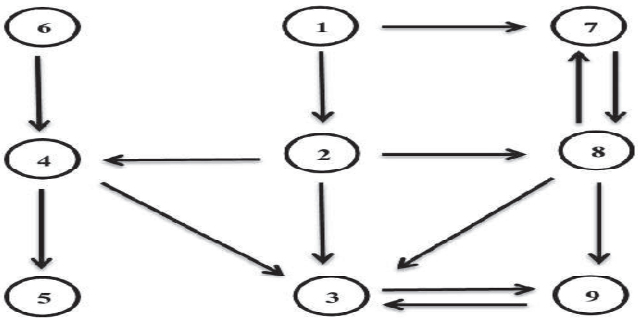
U_4, where U_1,U_2,$U_3$and$U_4 \in (1, 2)^+\alpha O(X,A)$. $x \in S_1$ implies $x \in U_1$andx $\notin U_2$, x $\notin S_2$ implies $x \notin U_3$or $x \in U_3$and $x \in U_4$and $y \in S_2$implies $y \in U_3$and $y \notin U_4$, y $\notin S_1$impliesy $\notin U_1$or $y \in U_1$and $y \in U_2$.

Case i : Let $x \notin S_2$and $y \notin S_1$this implies $x \notin U_3$and $y \notin U_1$. If $x \in S_1$, then x $\in U_1 \setminus U_2$, which implies $x \in U_1 \setminus U_3 \cup U_2$and if $y \in S_2$, then $y \in U_3 \setminus U_4$,which implies $y \in U_3 \setminus U_4 \cup U_1$ and also$(U_1 \setminus U_3 \cup U_2)$ $(U_3 \setminus U_4 \cup U_1) = \phi$.

Case ii : $y \in U_1$ and $y \in U_2 \Rightarrow x \in U_1 \setminus U_2$, $y \in U_2$and $(U_1 \setminus U_2) \cap U_2 = \phi$.

Case iii : $x \in U_3$and $x \in U_4 \Rightarrow y \in U_3 \setminus U_4$, $x \in U_4$and $(U_3 \setminus U_4) \cap U_4 = \phi$.

From the theorems and examples we have the following diagram. We depict by arrow the implications between the separation axioms.



(1) Ultra$^+$-T_2
(2) Ultra$^+$-T_1
(3) Ultra$^+$-T_0
(4)Ultra$^+$-$T_{1/2}$
(5) Ultra$^+$-T_D
(6) Ultra$^+$-Door
(7) Ultra$^+$-D_2
(8) Ultra$^+$-D_1
(9) Ultra$^+$-D_0

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