SOME SPECIAL TYPES OF NEAR IDEMPOTENT SEMIGROUP

T.N.KAVITHA, A. JAYALAKSHMI

Abstract: In this paper we define two special types of near idempotent semigroups namely

- left(right) regular near idempotent semigroup and
- left(right) normal near idempotent semigroup.

We obtain their characterization in terms of certain relations defined on them.

Keywords: Left(right) regular near idempotent semigroup, Left(right) normal near idempotent semigroup.

Introduction: In a previous paper [3], we introduced a near idempotent semigroup and studied its structure. We obtained a near – idempotent semigroup as a generalization of an idempotent semigroup or a band. we have defined a near – idempotent semigroup, as a semigroup S satisfying the identity $xyz = xy^2z$ for all x, y, z in S. and studied the structure of S by defining relations λ, ρ, δ, ξ on S similar to Green's relations. In this paper we introduce some special types of near – idempotent semigroups and characterize them using these relations.

Left(right) regular near – idempotent semigroup

We define below a left (right) regular near idempotent semigroup and study its structure

2.1 Definition: A near idempotent semigroup is said to be left(right) regular near – idempotent semigroup if xyzyw = xyzw (xyzyw = xzyw) for all x, y, z, w in S.

It is easy to prove that a left(right) regular near – idempotent semigroup is a semilattice of near left(right) zero near idempotent semigroups.[7].

2.2 Lemma: Let S be a left(right) regular near – idempotent semigroup. Every δ – class in S is a near left(right) zero near idempotent semigroup.

Proof: Let S be a left regular near – idempotent semigroup. Then

xyzyw = xyzw for all x,y,z,w in S. ....(1)

Let $a \in S$

δ_a is a rectangular near – idempotent subsemigroup of S [3]. Hence

xyzyw = xyw for all x,y,z,w in δ_a ....(2)[7]
(1) and (2) gives
xyzw = xyw for all x, y, z, w in δ_a
Hence δ_a degenerates into a left(right) zero near idempotent semigroup[7].
This leads to
2.3 Theorem: Every left regular near – idempotent semigroup is a semilattice of left zero near idempotent semigroups.
Dually, we can prove that every right regular near idempotent semigroup is a semilattice of right zero near idempotent semigroups.
Let S be a left(right) regular near – idempotent semigroup. Then xyzyw = xyzw for all x, y, z, w $\in$ S. If y δ z, then xyzw = xyzyw = xyw so that y λ z.
From the above discussion it is clear that in the case of a left regular near – idempotent semigroup, $\lambda = \delta$.
In fact the converse is also true.
i.e, if $\lambda = \delta$ on a near idempotent semigroup S, then S is a left regular near – idempotent semigroups.
Let u, v $\in$ S, For all x, y $\in$ S,
$x\, uv\, vu\, uv\, y = x\, uv^2\, u^2\, v\, y$
$= x\, uv\, uv\, y$
$= x\, (uv)^2\, y$
$= x\, uv\, y$
And $x\, vu.uv.vu\, y = xvu\, y$ for all x,y $\in$ S
Thus uv δ vu for all u, v $\in$ S
Since $\delta = \lambda$
uv λ vu for all u, v $\in$ S. Thus
$x\, uv.vu\, y = xuvy$ for all x,u,v,y $\in$ S
i.e, $xuv^2uy = xuvy$ for all x, u, v, y $\in$ S
i.e, $xuvuy = xuvy$ for all x, u, v, y $\in$ S
Hence S is left regular near idempotent semigroup.
Thus we have
2.4 Theorem: A near – idempotent semigroup S is left regular near – idempotent semigroup if and only if $\lambda = \delta$ on S.
Dually,
2.5 Theorem: A near – idempotent semigroup S is right regular near – idempotent semigroup if and only if $\rho = \delta$ on S.
Since $\lambda = \delta$ in the case of a left regular near – idempotent semigroup,

we have $\xi = \lambda \cap \rho = \delta \cap \rho = \rho$.

Conversely, suppose that S is a near – idempotent semigroup in which $\rho = \xi$.
For all $x, y, u, v \in S$
We have
$x\, uvu.uv\, y = x\, uv\, u^2vy = xuvuvy$
$= x(uv)^2y$
$= xuvy$.
and $x\, uv.\, uvu\, y = x(uv)^2uy = xuvuy$.
Thus $uv\ \rho\ uvu$ for all $u, v \in S$.
Since $\rho = \xi$, we have
$x\, uvu\, y = xuvy$.
Hence S is a left regular near – idempotent semigroup.
These discussions lead to

2.6 Theorem: A near – idempotent semigroup S is left regular near – idempotent if and only if $\rho = \xi$ on S.

Dually,

2.7 Theorem: A near – idempotent semigroup S is right regular near idempotent semigroup if and only if $\lambda = \xi$ on S.

3. Left(Right) normal near idempotent semigroup

We now define and characterize a left normal near – idempotent semigroup.

3.1 Definition: A near idempotent semigroup S is called a left normal near – idempotent semigroup if $xuvwy = xuwvy$ for all x, y, u, v, w in S.

3.2 Result: A left normal near idempotent semigroup is left regular.
Let S be a left normal near – idempotent semigroup. Then $xuvuy = xu.uvy = xu^2vy = xuvy$ for all $x, y, u, v \in S$. Hence S is left regular.

3.3 Result: In a left normal near – idempotent semigroup S, $\delta = \lambda$ and $\rho = \xi$.
These follow from the fact that S is left regular near – idempotent semigroup (Theorems 2.4 and 2.6)

3.4 Lemma: Let S be a left normal near – idempotent semigroup. then
(i) $\delta = \lambda$
(ii) $\rho = \xi$
(iii) for $u, v \in S$ such that $u\ \lambda\ v$, we have $wu\ \rho\ wv$.

Proof:

(i) and (ii) follows from the fact that S is left regular
(ii) Let $u, v \in S$ such that $u \lambda v$.
Then $xuvy = xuy$ and $xvuy = xvy$ for all $x,y \in S$.
Let $w \in S$
For all $x,y \in S$
$xwuy = xwuvy$ since $u \lambda v$
$= xwvuy$ since S is left normal
$= xwvy$ since $v \lambda u$
Hence $wu \ \xi \ wv$ for all $w \in S$
But by (ii) $\xi = \rho$
Hence $wu \ \rho \ wv$.
Lemma 3.5 is the converse of lemma 3.4.
3.5 Lemma: Let S be a near – idempotent semigroup in which
(i) $\delta = \lambda$
(ii) For $u, v \in S$ such that $u \lambda v$, $wu \ \rho \ wv$ for all $w \in S$.
Then S is a left normal near – idempotent semigroup.
Proof: $vw \ \delta \ wv$ for all v, w in the near- idempotent semigroup S.
But $\delta = \lambda$. By (i) Therefore $vw \ \lambda \ wv$
Hence by (ii) $uvw \ \rho \ uwv$
But $\rho = \xi$ in a near – idempotent semigroup. Hence
$xuvwy = xuwvy$ for all $x, u, v, w, y \in S$.
Hence S is a left – normal near – idempotent semigroup.
Thus we have
3.6 Theorem: A near – idempotent semigroup S is a left – normal near idempotent semigroup if and only if
(i) $\delta = \lambda$
(ii) If $u \lambda v$ for $u, v \in S$,
then $wu \ \rho \ wv$ for all $w \in S$.
Dually, a near – idempotent semigroup S is a right normal near – idempotent semigroup if and only if
(i) $\delta = \rho$
(ii) If $u \rho v$ for $u, v \in S$, then $uw \ \lambda \ vw$ for all $w \in S$.
Conclusion: In this paper , we present several results about varieties of near idempotent semigroups. In particular we studied some special types of generalizations of a band we called left(right) regular near idempotent semi groups and left(right) normal near idempotent semi groups. Further we want to proceed some more types of near idempotent semi groups.

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T.N.Kavitha/A. Jayalakshmi/
Assitant professor/mathematics/
SCSVMVUniversity/kanchipuram/tnkmaths@gmail.com.
Professor of mathematics/SCSVMVUniversity/kanchipuram/
a_jayalakshmi@yahoo.co.in/Tamilnadu India.