DIFFERENTIAL TRANSFORMATION METHOD FOR FINGERO-IMBIBITION PHENOMENA ARISING IN DOUBLE PHASE FLOW THROUGH HOMOGENEOUS POROUS MEDIA

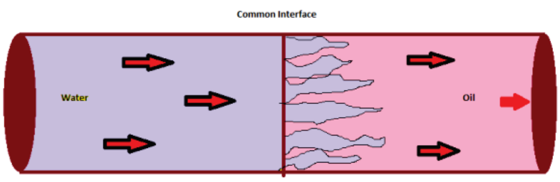
N.D. PATEL, RAMAKANTA MEHER

Abstract: In this paper, Differential Transform Method is used here to study the saturation for homogenous porous media. A detailed discussion of saturation for these phenomena by using Differential Transform Method and a comparison analysis is done with the results obtained by variational iteration method to check the accuracy of the results .Comparison study for both methods and interpretation of results has been done by using software.

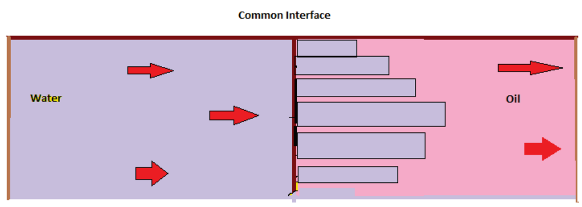
Keywords: Differential Transform Method, Non-Linear Partial Differential equation, Oils Recovery, Porous Media.

Introduction: Analytical and numerical simulation of the problems arising in oil-water displacement has become a predictive tool in oil industry. In oil recovery process, oil is produced by simple natural decompression without any pumping effort at the wells. This stage is referred to as primary recovery, and it ends when a pressure equilibrium between the oilfield and the atmosphere occurs. Primary recovery usually leaves 70%–85% of oil in the reservoir. To recover part of the remaining oil, a fluid (usually water) is injected into some wells (injection wells) while oil is produced through other wells (production wells).This process serves to maintain high reservoir pressure and flow rates. It also displaces some of the oil and pushes it toward the production wells. This stage of oil recovery is called secondary recovery process. It is a very well-known physical fact that when a fluid having greater viscosity flowing through a porous medium is displaced by another fluid of lesser viscosity then, instead of regular displacement of whole front, protuberance takes place which shoot through the porous medium at a relatively very high speed, and fingers have been developed during this process as shown in Figure 1.This phenomenon is called fingering or instability phenomenon. In the statistical treatment of the fingers only average cross-sectional area occupied by the fingers is considered while the size and shape of the individual fingers are neglected. Many researchers have discussed this phenomenon from various view points. Sheideger and Johnson have discussed the statistical behaviour of fingering in homogeneous porous media without capillary pressure [11]. A.P.Verma has examined the behaviour of fingering in a displacement process through heterogeneous porous media with capillary pressure and pressure dependent phase densities [2]. Mehta has used special relation with capillary pressure and he used singular perturbation technique to find its solution [7].Pradhan et al. have discussed the solution of instability phenomenon by finite element method [6].Meher et al. discussed the solution of instability phenomenon arising in double phase flow through porous medium with capillary pressure using exponential self similar solutions technique [5].In the In this paper, we investigate the applicability of Differential Transform method to the nonlinear partial differential equation arising in Fingero-Imbibition phenomena in double phase flow through porous media.Also obtained solution was compared with VIM solution.

Mathematical Formulation: As shown in Figure 1, a well developed fingers flow is furnished on account of uniform water injection into the oil saturated isotropic, homogeneous porous medium. The schematic presentation of fingers is expressed in Figure 2. Our particular interest in the present investigation is to develop a mathematical model by considering the mass flow rate of oil and water and discuss the fingering phenomenon analytically by using Differential Transform method.



Fiq-1



Fiq-2

The seepage velocity of water (injected fluid) (v_w) and oil (native fluid) (v_o) is given by Darcy's law

$$v_w = -\frac{k_w}{\mu_w} K \frac{\partial p_w}{\partial x} \quad (1)$$

$$v_o = -\frac{k_o}{\mu_o} K \frac{\partial p_o}{\partial x} \quad (2)$$

Where k_w and k_o are relative permeability, p_w and p_o are pressures and μ_w and μ_o are kinematics viscosities (which are constants) of the wetting phase and non- wetting phase respectively and K is the permeability of homogeneous porous medium.

The equations of continuity of two phases are given as

$$\frac{\partial(\phi\rho_w S_w)}{\partial t} + \frac{\partial \rho_w v_w}{\partial x} = q_w \quad (3)$$

$$\frac{\partial(\phi\rho_w S_o)}{\partial t} + \frac{\partial \rho_o v_o}{\partial x} = q_o \quad (4)$$

Where q_w and q_o are the constant mass flow rate of water and oil. ρ_w and ρ_w are density of water and oil, ϕ is the porosity of the medium.

From the definition of phase saturation, it is evident that

$$S_o + S_w = 1 \quad (5)$$

Analytical Relationships: For definiteness we assume the following relationships

Capillary Pressure: The capillary pressure p_c defined as the pressure discontinuity of the flowing phases across their common interface, is a function of the phase saturation. It may be written as

$$p_c = p_o - p_w \quad (6)$$

$$p_c = -\beta S_w \quad (7)$$

The negative sign indicates that the direction of injected liquid is opposite to capillary pressure, (Scheidegger)

Relative Permeability and Phase Saturation : For definiteness of the mathematical analysis the standard relation between phase saturation and relative permeability, given by (Scheidegger and Johnson 1961) is used here. It is given by,

$$k_w = S_w \quad (8)$$

$$k_o = S_o = 1 - S_w \quad (9)$$

Equation of Motion for Saturation: The equation of motion for saturation can be obtained by substituting the value of v_w and v_o

From (1) and (2) in equation (3) and (4) , we get

$$\frac{\partial(\phi\rho_w S_w)}{\partial t} = q_w + \frac{\partial[\rho_w \frac{k_w}{\mu_w} K \frac{\partial p_w}{\partial x}]}{\partial x} \quad (10)$$

$$\phi\frac{\partial(\phi\rho_o S_o)}{\partial t} = q_o + \frac{\partial}{\partial x}[\rho_o \frac{k_o}{\mu_o} K \frac{\partial p_o}{\partial x}] \quad (11)$$

Eliminating $\frac{\partial p_w}{\partial x}$ from (6) and (10) we get

$$\frac{\partial(\phi\rho_w S_w)}{\partial t} = q_w + \frac{\partial[\rho_w \frac{k_w}{\mu_w} K (\frac{\partial p_o}{\partial x} - \frac{\partial p_c}{\partial x})]}{\partial x} \quad (12)$$

Combining (11) and (12) and using (15) we get

$$0 = (\frac{q_w}{\rho_w} + \frac{q_o}{\rho_o}) +$$

$$\frac{\partial}{\partial x}[(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})\frac{\partial p_o}{\partial x} - K\frac{k_w}{\mu_w}\frac{\partial p_c}{\partial x}] \quad (13)$$

Integrating above with respect to x,

$$c = (\frac{q_w}{\rho_w} + \frac{q_o}{\rho_o})x + (K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})\frac{\partial p_o}{\partial x}$$

$$-K\frac{k_w}{\mu_w}\frac{\partial p_c}{\partial x} \quad (14)$$

Where c is a constant of integration.

on simplifying,

$$\frac{\partial p_o}{\partial x} = \frac{c}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})} +$$

$$\frac{K\frac{k_w}{\mu_w}}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})}\frac{\partial p_c}{\partial x} - \frac{(\frac{q_w}{\rho_w} + \frac{q_o}{\rho_o})x}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})} \quad (15)$$

Substituting the value of (15) in (12),

$$\phi\frac{\partial(\phi S_w)}{\partial t} = \frac{q_w}{\rho_w} + \frac{\partial}{\partial x}[\frac{k_w}{\mu_w}K(\frac{c}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})} +$$

$$\frac{K\frac{k_o}{\mu_o}}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})}\frac{\partial p_c}{\partial x} - \frac{(\frac{q_w}{\rho_w} + \frac{q_o}{\rho_o})x}{(K\frac{k_w}{\mu_w} + K\frac{k_o}{\mu_o})}) - \frac{\partial p_c}{\partial x})] \quad (16)$$

The value of the pressure of native liquid

$p_o = \bar{p} + \frac{1}{2}(p_c)$ implies

$$\frac{\partial p_o}{\partial x} = \frac{1}{2}(\frac{\partial p_c}{\partial x}) \quad (17)$$

Where $\overline{p}=\frac{p_o+p_w}{2}$ be the Mean pressure and is constant.

Thus from (17) and (14) we get

$$c=(\frac{q_w}{\rho_w}+\frac{q_o}{\rho_o})x+(K\frac{k_w}{\mu_w}+K\frac{k_o}{\mu_o})\frac{1}{2}(\frac{\partial p_c}{\partial x})-K\frac{k_w}{\mu_w}\frac{\partial p_c}{\partial x}$$

Substituting the value of c in (16) and on simplification we have

$$\frac{\partial(\phi S_w)}{\partial t}=\frac{q_w}{\rho_w}+\frac{\partial}{\partial x}[-K\frac{k_w}{2\mu_w}\frac{\partial p_c}{\partial x}] \qquad (18)$$

Using (8) and (7) in (18) and after some simplification, we get

$$\phi\frac{\partial(S_w)}{\partial t}=\frac{q_w}{\rho_w}+(\frac{K\beta}{2\mu_w})\frac{\partial}{\partial x}[S_w\frac{\partial Sw}{\partial x}]$$

or

$$\frac{\partial(S_w)}{\partial t}=\frac{q_w}{\phi\rho_w}+(\frac{K\beta}{2\phi\mu_w})\frac{\partial}{\partial x}[S_w\frac{\partial Sw}{\partial x}]$$

where porosity ϕ and permeability K are treated as constant for homogeneous porous medium.

Considering the dimensionless variables,

$X=\frac{x}{L},T=\frac{K\beta t}{2\mu_w\phi L^2}$ in above we get,

$$\frac{\partial(S_w)}{\partial T}=A+\frac{\partial}{\partial X}[S_w\frac{\partial Sw}{\partial X}]$$

Where $A=\frac{2\mu_w L^2 q_w}{K\beta\rho_w}$

Which is desired non-linear partial differential equation.

In order to solve above completely the following specific initial and boundary conditions are considered:

$$\mathrm{S}_w(\mathrm{X},0)=\mathrm{f}(\mathrm{X})$$

$$\mathrm{S}_w(0,\mathrm{T})=\mathrm{f}_1(\mathrm{T})$$

$$\mathrm{S}_w(\mathrm{L},\mathrm{T})=\mathrm{f}_2(\mathrm{T})$$

Differential Transform Method: The Differential transform method (DTM) was first introduced by Zhou [4] in 1986 to solve linear and nonlinear initial/boundary value problems generally arising in electrical circuit analysis. This method constructs an analytical solution in the form of series solution with x and t.

Infect differential transform method has been developed for solving various types of differential and integral equations

Definition: Differential Transform of function w(x,y) is defined as follows:

W(k,h)= $\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial^k x\partial^h y}[w(x,y)]_{(0,0)}\right]$

Where w(x,y)is original function and W(k,h) is the transformed function, which is also known as the T-function.

In this paper , the lowercase and uppercase letters represent the original and transformed function respectively.

The inverse differential transform of W(k,h)is defined as :

w(x,y)= $\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k y^h$

Combing above equation we have,

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial^k x\partial^h y}[w(x,y)]_{(0,0)}\right]x^k y^h$$

Important theorems

Theorem : 1.If w(x,y)=u(x,y)+v(x,y) then W(k,h) =U(k,h)+V(k,h).

Theorem: 2.If w(x,y)=c.u(x,y)) then W(k,h) =c.U(k,h).

Theorem: 3. If w(x,y)= $\frac{\partial u(x,y)}{\partial x}$ then then W(k,h) =(k+1).U(k+1,h).

Theorem:4 If w(x,y)= $\frac{\partial u(x,y)}{\partial y}$ then

W(k,h) =(h+1).U(k,h+1).

Theorem: 5 If w(x ,y)= $\frac{\partial^{r+s}u(x,y)}{\partial x^r\partial y^s}$ then

W(k,h)
(k+1)(k+2).....(k+r).(h+1)(h+2)...(h+s)U(k+r,h+s).

Theorem: 6. If w(x,y)=u(x,y)v(x,y) then W(k,h) = $\sum_{r=0}^{k}\sum_{s=o}^{h}U(r,h-s)V(k-r,s)$.

Theorem: 7. If w(x,y)= $\frac{\partial u(x,y)}{\partial x}\ \frac{\partial u(x,y)}{\partial y}$ then

$$\mathrm{W}(\mathrm{k},\mathrm{h})=\sum_{r=0}^{k}\sum_{s=o}^{h}(r+1)(k-r+1)U(r+1,h-s)V(k-r+1,s)$$

.

Theorem: 8*. If w(x,y)=* $x^m y^n$ *then*

W(k,h) $=\delta(k-m.h-n)=\delta(k-m)\delta(h-n)$.

Where $\delta(k-m)=\begin{cases}1 & k=m\\0 & k\neq m\end{cases}$ and

$$\delta(h-n)=\begin{cases}1 & h=n\\0 & h\neq n\end{cases}$$

3. Application of DTM to Porous Medium Equation

Consider a porous medium equation

$$\frac{\partial S_w}{\partial T}=A+\left(\frac{\partial S_w}{\partial X}\right)^2+u\frac{\partial^2 S_w}{\partial X^2} \qquad (19)$$

with initial condition

$S_w(X,0)=0.01X^2$ Where A=0.68 (20)

The Differential transform of eq.(19) can be written as

$$(h+1)S_w(k,h+1)=\delta(h)\delta(k)A+$$

$$\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)S_w(r+1,h-s)S_w(k-r+1,s)$$

$$+\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(k-r+2)S_w(r,h-s)S_w(k-r+2,s) \qquad (21)$$

Using the initial conditions (20) in eq.(21) , we have,

$$S_w(0,0)=0 \quad S_w(1,0)=0 \quad S_w(2,0)=0.01$$

$$S_w(3,0)=0 \quad S_w(4,0)=0$$

$$S_w(5,0)=0 \quad S_w(6,0)=0 \qquad (22)$$

Now by substituting the results (22) in eq.(21) ,it obtains

$S_w(0,1)=0.68$ $S_w(1,1)=0$ $S_w(2,1)=0.0006$

$S_w(0,2)=0.0068$ $S_w(1,2)=0$ $S_w(2,2)=0.000036$

$S_w(0,3)=0.00031733$ $S_w(1,3)=0$

$S_w(2,3)=0.00000216$ $S_w(0,4)=0.00001587$

Hence the approximate solution of eq. (19) up to 4 degree terms can be written as

$$S_w(X,T)=0.01X^2+0.68T+0.0006X^2T+$$
$$0.0068T^2+0.000036X^2T^2+0.00031733T^3$$
$$+0.00000216X^2T^3+0.00001587T^4+\ldots$$

The graphical representation of the same has been shown in Figures 3. and 4. From figures 3. and 4., it is observed that saturation of injected water increases with the space variable X and time variable T. This resembles well with the physical phenomenon of the problem.

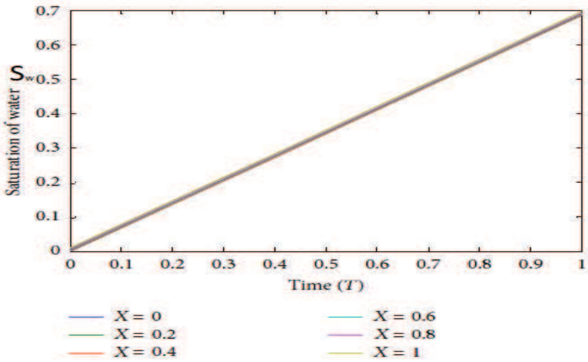


Figure-3:The plot of time (T) versus saturation of water (Sw) for different values of distance (X).

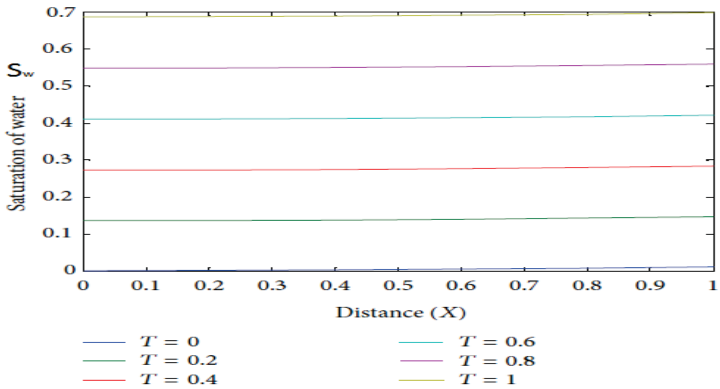


Figure 1.4: The plot of distance (X) versus saturation of water (Sw) for different values of time (T).

References:

1. Arikoglu, A., Ozkol, I. "Solution of difference equations by using differential transformation method" Applied Mathematics andComputation,174 (2006): 1216-1228.
2. A. P. Verma, "Statistical behaviour of fingering in a displacement process in heterogeneous porous medium with capillary pressure," *Canadian Journal of Physics*, vol. 47, no. 319, 1969.
3. Abdel-Halim Hassan, I.H., Ertürk, V. "Solutions of Different Types of the linear and Nonlinear Higher-Order Boundary Value Problems by DifferentialTransformation Method" European Journalof Pure and Applied Mathematics,2, no.3 (2009):426-447.
4. Chen, C.K. "Application of differential transformation to Eigen value problems", Applied Mathematics and Computation, 79.no.2-3 (1996):173-188.
5. Meher, Mehta(2010).Instability phenomenon arising in double phase flow through porous medium with capillary pressure, *Int. J. of Appl. Math and Mech.* 7 (15): 97-112, 2011.
6. Mehta,M.N. (1978).An asymptotic expansion in fluid flow through porous media;Ph.D.Thesis;S.G.University,Surat(India).
7. Mehta, M.N. (2008).Analytical approximate expression for primary imbibition front in homogeneous porous media, International J. of Math. Sci. & Engg. Appls. (IJMSEA) Vol. 2 No. 2, pp. 155-162.
8. Mehta, M.N.and Joshi M.S. (2009). Solution by group invariant method of instability phenomenon with power law nonlinearity International J. of Math. Sci. & Engg. Appls. (IJMSEA), Vol. 3 No. III, pp. 291-303
9. Mirzaee, F. "Differential Transform Method for Solving Linear and Nonlinear Systems of Ordinary Differential Equations "Applied Mathematical Sciences, 5, no.70 (2011):3465 - 3472.
10. Mistry,P.R., Pradhan,V.H., and Desai,K.R.,
11. Mathematical Model and Solution for Fingering Phenomenon in Double Phase Flow through Homogeneous Porous Media, The Scientific World Journal Volume 2013(2013), Article ID 470174: 1-7
12. Scheidegger, A.E. (1960). The Physics of Flow through Porous Media, the Macmillan Co., New York.
13. Soltanalizadeh, B. "Application of differential transformation method for numerical analysis of Kawahara equation" Australian Journal of Basic and Applied Sciences,5, no.12 (2011): 490-495.

X\T	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM
	0	0	0.2	0.2	0.4	0.4	0.6	0.6	0.8	0.8	1	1
0	0	0	0.136275	0.13627	0.273108	0.273109	0.410517	0.410519	0.548515	0.548521	0.6871	0.6871
0.2	0.0004	0.0004	0.136679	0.136679	0.273518	0.273519	0.410932	0.410934	0.548935	0.548941	0.6875	0.6876
0.4	0.0016	0.0016	0.137894	0.137894	0.274748	0.274748	0.412176	0.412178	0.550196	0.550202	0.6888	0.6888
0.6	0.0036	0.0036	0.139918	0.139918	0.276797	0.276797	0.414251	0.414253	0.552297	0.552303	0.6909	0.691
0.8	0.0064	0.0064	0.142752	0.142752	0.279666	0.279666	0.417156	0.417158	0.555238	0.555244	0.6939	0.6939
1	0.01	0.01	0.146396	0.146396	0.283354	0.283355	0.42089	0.420892	0.559019	0,559025	0.6978	0.6978

* * *

N. D. Patel/ *Assistant Professor* /GEC,Gandnhinagar
Ramakanta Meher /*Assistant Professor*/ *SVNIT, Surat-395007, India.*