EFFECT OF CORIOLIS FORCE ON BRINKMAN-BÉNARD CONVECTION WITH LTNE EFFECTS

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Abstract: In the paper, the effects of Coriolis force and LTNE on onset of Brinkman-Bénard convection and on heat transport are investigated. Free-free, isothermal boundaries are considered for investigation. The linear boundary eigen value problem yields the critical Rayleigh number which is computed numerically and presented. The inter-phase heat transfer coefficient advances the onset of convection and ratio of thermal diffusivities delays the onset. LTNE effect ceases for large values of inter-phase heat transfer coefficient both in the presence and absence of rotation.

Introduction: Study of thermoconvection in rotating porous media has engineering applications that includes rotating machinery, industrial and chemical applications. The applications are discussed in detail by Bejan [1] and Nield and Bejan [8]. Many researchers have investigated thermo convection in rotating porous media in the absence/presence of modulation (Bejan [1], Bhadauria et al. [2], Bhadauria and Khan [3], Desaive et al. [4], Govender [5], Malashetty et al. [6], Malashetty and Swamy [7], Nield and Bejan [8], Shivakumara and Dhananjaya [10], Vadasz [11], Vanishree and Siddheshwar [12]). Niiler and Bisshop [9] studied the effect of Coriolis force on onset of convection in a shallow layer by considering free-free, isothermal boundaries and rigid-rigid, isothermal boundaries. Desaive et al., [4] studied thermal stability of rotating saturated porous media by neglecting centrifugal force. The above studies considered local thermal equilibrium (LTE) between the solid and liquid phases. Vanishree and Siddheshwar [12] studied linear and non-linear stability analyses of rotating nanofluid-saturated porous medium with local thermal non-equilibrium (LTNE) effects by considering the effects of Brownian motion and thermophoresis in bidisperse porous medium. In this paper we investigate onset of Brinkman-Bénard convection and amount of heat transport in a liquid-saturated rotating porous medium with LTNE effects. Free- free, isothermal boundaries are considered for investigation.

Mathematical formulation: Consider an infinite, horizontal, liquid-saturated porous layer of thickness d heated from below. The system is rotating about z axis with constant angular speed $\vec{\Omega}$ as shown in Fig 1. Here we consider two equations of energy one each for the liquid and solid phases which are coupled by means of the inter-phase heat transfer coefficient. The dimensionless governing equations for studying steady, two-dimensional Brinkman-Bénard convection in the case of local thermal non-equilibrium between liquid and solid phases in dimensionless form are:

$$\Lambda\nabla^4\Psi-\sigma^2\nabla^2\Psi-Ra_l\frac{\partial\Theta_l}{\partial X}+\sqrt{Ta}\frac{\partial V}{\partial Z}=0, \quad (1)$$

$$-\frac{\partial\Psi}{\partial X}+\nabla^2\Theta_l+H(\Theta_s-\Theta_l)+\frac{\partial\Psi}{\partial X}\frac{\partial\Theta_l}{\partial Z}-\frac{\partial\Psi}{\partial Z}\frac{\partial\Theta_l}{\partial X}=0, \quad (2)$$

$$\nabla^2\Theta_s+\gamma H(\Theta_l-\Theta_s)=0, \quad (3)$$

$$\Lambda\nabla^2V-\sigma^2V-\sqrt{Ta}\frac{\partial\Psi}{\partial Z}=0, \quad (4)$$

where

$$\Lambda=\frac{\mu'}{\mu}\text{(Brinkman number)},\ \sigma^2=\frac{d^2}{K}\text{(Porous parameter)},$$

$$Ra_l=\frac{\rho_l(\rho c_p)_l\alpha g d^3\Delta T}{\phi\kappa_l\mu}\text{(Rayleigh number)},\ H=\frac{hd^2}{\phi\kappa_l}\text{(Interphase heat transfer coefficient)},$$

$$\gamma=\frac{\phi\kappa_l}{(1-\phi)\kappa_s}\text{(Ratio of thermal conductivities)},\ \text{Ta}=\left(\frac{2\rho\Omega d^2}{\phi\kappa_l}\right)^2\text{(Porous Taylor number)}.$$

A. Marginal stability - stationary convection: The boundary conditions for solving Eqns. (1)-(4) are:

$$\Psi=D^2\Psi=\Theta_l=\Theta_s=0\quad\text{at } Z=\pm\frac{1}{2}. \quad (5)$$

To study the linear stability, we use the linearized version of Eqns. (1)-(4). We seek the solutions of Eqns. (1)-(4) in the form

$$\left.\begin{aligned}&\Psi(X,Z)=A\sin(kX)\sin\left(\pi Z+\frac{\pi}{2}\right),\ \Theta_l(X,Z)=B\cos(kX)\sin\left(\pi Z+\frac{\pi}{2}\right),\\&\Theta_s(X,Z)=C\cos(kX)\sin\left(\pi Z+\frac{\pi}{2}\right),\ V(X,Z)=D\sin(kX)\cos\left(\pi Z+\frac{\pi}{2}\right),\end{aligned}\right\}. \quad (6)$$

Following the classical procedure, we get Rayleigh number as

$$Ra_l=\frac{\delta^2\left(\delta^2+H+\gamma H\right)(\delta^2\Lambda+\sigma^2)}{k^2(\delta^2+\gamma H)}\left(1+\frac{\pi^2 Ta}{\delta^2(\delta^2\Lambda+\sigma^2)}\right), \quad (7)$$

where $\delta^2=k^2+\pi^2$. In order to study the amount of heat transfer we perform non-linear stability analysis in the next section.

Weakly non linear stability analysis

A minimal mode double Fourier series solution which describes steady finite amplitude convection in Newtonian liq

$$\left.\begin{aligned}&\Psi(X,Z)=A\sin(k_cX)\sin\left(\pi Z+\frac{\pi}{2}\right), V(X,Z)=D_1\sin(k_cX)\cos\left(\pi Z+\frac{\pi}{2}\right),\\&\Theta_l(X,Z)=B_1\cos(k_cX)\sin\left(\pi Z+\frac{\pi}{2}\right)+B_2\sin\left(2\pi Z+\pi\right),\\&\Theta_s(X,Z)=C_1\cos(k_cX)\sin\left(\pi Z+\frac{\pi}{2}\right)+C_2\sin\left(2\pi Z+\pi\right),\end{aligned}\right\}.$$

Following the classical procedure, we get the amplitudes as given below:

$$\left.\begin{aligned}&A_1^2=4\pi^2\frac{\delta^2\delta_2^2\left(\gamma H+H+4\pi^2\right)}{\delta_1^2\left(\gamma H+4\pi^2\right)}(r-1),\ B_1=-\frac{\delta_1^2 k}{\delta^2 r\delta_2^2}A,\\&C_1=-\frac{\gamma Hk}{\delta^2 r\delta_2^2}A,\ B_2=\frac{1}{\pi}\left(1-\frac{1}{r}\right),\ C_2=\frac{\gamma H}{\pi(4\pi^2+\gamma H)}\left(1-\frac{1}{r}\right),\\&D_1=-\frac{\pi\sqrt{T}}{\delta^2\Lambda+\sigma^2}A,\ r=\frac{Ra_l}{Ra_{l_c}}\end{aligned}\right\}. \quad (8)$$

The Nusselt number is defined as follows:

$$Nu=\frac{\text{Amount of heat transfer by (Conduction+Convection)}}{\text{Amount of heat trasfer by Conduction}}. \quad (9)$$

The Nusselt number for liquid and solid phases are respectively given by:

$$Nu_l=1+2\pi B_2,\ Nu_s=1+2\pi C_2. \quad (10)$$

Using B_2 and C_2 from Eq. (8) in Eq. (10), we get

$$Nu_l=1+2\left(1-\frac{1}{r}\right),$$

$$Nu_s=1+2\frac{\gamma H}{4\pi^2+\gamma H}\left(1-\frac{1}{r}\right). \quad (11)$$

The weighted-average Nusselt number, Nu_w, for stationary mode of convection evaluated at lower boundary $Z=-\frac{1}{2}$ for a single wavelength is given by

$$Nu_w=\phi Nu_l+(1-\phi)Nu_s=1+2\left(\frac{4\pi^2\phi+\gamma H}{4\pi^2+\gamma H}\right)\left(1-\frac{1}{r}\right).$$

Results and discussion: We first discuss the results of the linear stability analysis followed by that on the non-linear one. From Fig 2. we find that Ra_{lc} increases with increasing the values of H and with decreasing values of γ. For large values of H, values of the critical Rayleigh number matches with that of LTE case. Thus we may infer that LTNE effect ceases for large values of H. The above results are in the case of LTNE in the presence/absence of rotation. From Fig 3, it is observed that Ra_{lc} increases with increase in the values of Taylor number, Ta, which leads to a decrease in amount of heat transfer.

The plots of Nu_w versus Ra_l for different values of various parameters are presented in Fig. 4 - 6 and these plots depict the fact that Nu_w decreases with individual and collective increases of Λ, σ, H, Ta where as γ shows opposite effect to that of others.

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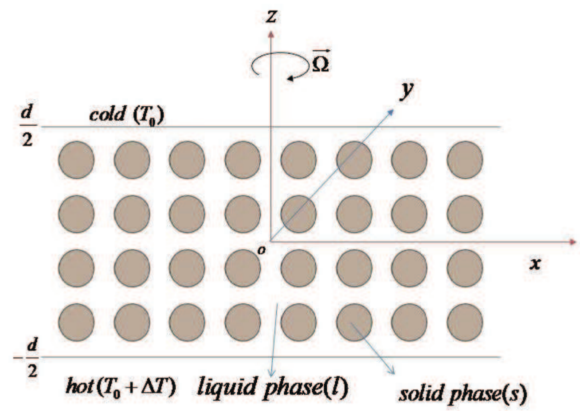

Fig 1. Schematic of physical configuration

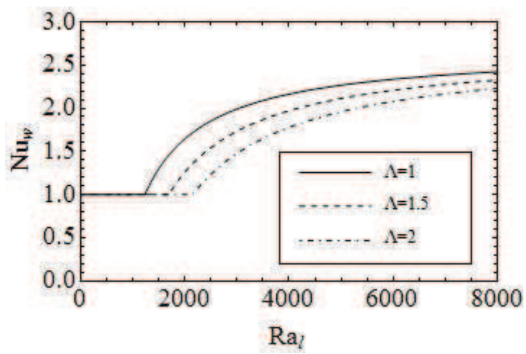

Fig 4. Variation of Nu_w with Ra_l, for different values of Λ and fixed values of σ^2=5, γ=1, H=10.

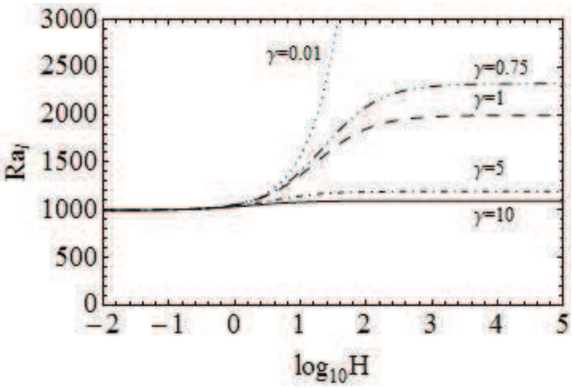

Fig 2. Variation of Ra_l with H for different values of γ and fixed values of Λ=1, σ^2=5, Ta=10.

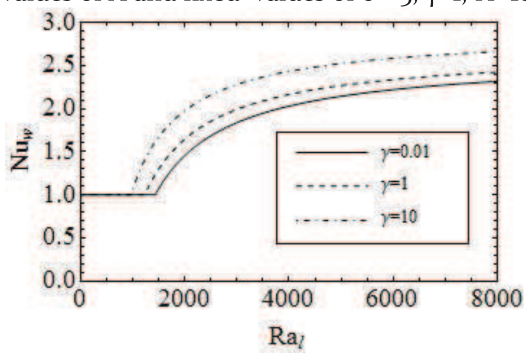

Fig 5. Variation of Nu_w with Ra_l, for different values of γ and fixed values of Λ=1, σ^2=5, H=10, Ta=10

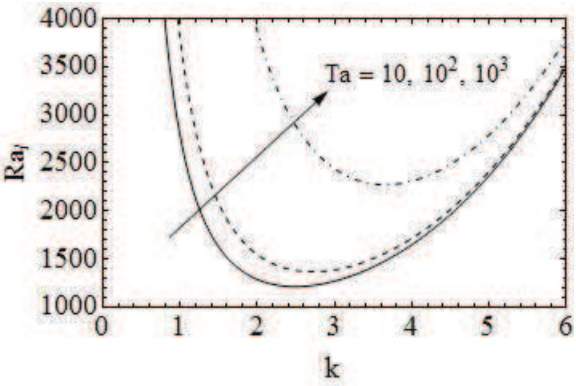

Fig 3. Variation of Ra_l with k, for different values of Ta and fixed values of Λ=1, σ^2=5, γ=1, H=10.

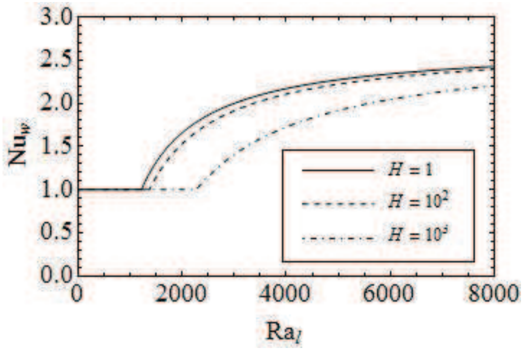

Fig 6. Variation of Nu_w with Ra_l, for different values of H and fixed values of of Λ=1, σ^2=5, γ=1, Ta=10.

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