SOME CONTRA - CONTINUOUS FUNCTIONS VIA SEMI OPEN AND α -OPEN SETS

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Abstract: In this paper, we apply the notions of semi – open sets and α – open sets in topological spaces to define and study contra α s - continuous function.

Keywords: Semiopen sets, α – open sets, Semicontinuity, α -continuity, Contra-continuity, Contra-semicontinuity, Contra- α -continuity.

Introduction: In 1996, Dontchev [10] introduced a new class of functions called Contra – Continuous functions. Recently Dontchev and Noiri [11] in 1999, introduced and studied among others, a new weaker form of this class of functions called Contra – semi continuous functions. They also introduced the notion of RC- Continuity [11] which is weaker than contra - continuity and stronger than β -Continuity [29]. In 1999, Jafari and Noiri [13] introduced and studied a new class of functions called Contra–Super – Continuous functions which lies between class of RC–continuous functions contra-continuous functions. In 2000, Beceren.Y and Noiri.T studied On α -pre continuous functions [3]. Jafari and Noiri, in 2001 studied on Contra- α - Continuous functions [14] and in 2002, they studied on Contra–Pre Continuous functions [15]. In 2008, Yusuf Beceren and Takashi Noiri studied on some functions defined by and Takashi Noiri studied on some functions defined by α -open and pre open sets [5].

In this paper we introduce and investigate a new class of functions called contra- α s - continuous functions which is weaker than contra – continuous functions and stronger than both contra- α -continuous functions and contra– semi continuous functions.

Preliminaries: Throughout this paper, all spaces X and Y (or (X, τ) and (Y, σ)) are topological spaces. A subset A is said to be regular open (resp. regular closed) if A = Int(Cl (A)) (resp. A = Cl(Int (A))) where Cl(A) and Int (A) denotes the closure and interior of A.

Definition 2.1 : A subset A of a space X is called

i) Semi – open [16]- if A $\subset$ Cl (Int (A)).
ii) α -open [26]- if A $\subset$ Int (Cl (Int (A))).
iii) preopen [18]- if A $\subset$ Int (Cl(A)).

The complement of pre open (resp. Semi–open, α -open,) set is said to be preclosed (resp. semi–closed, α -closed). The collection of all closed (resp. preopen semi–open, α -open) subsets of X will be denoted by C (X) (resp. PO(X), SO (X), α O(X)). It is shown in [26] that α (x) (or τ^{α}) is a topology for X and it is stronger than the given topology on X.

Definition 2.2 : The α -interior [7] of A, denoted by α Int A and is defined as the union of all α -open sets which are contained in A.

Definition 2.3 : The α -closure [19] of a subset A of X is the intersection of all α -closed sets that contain A and is denoted by α Cl(A).

Definition 2.4 : The pre-interior [20] of a subset A of X is the union of all preopen sets that are contained in A and is denoted by pInt(A).

Definition 2.5 : The pre–closure [9] of a subset A of X is the intersection of all preclosed sets that contain A and is denoted by pCl(A).

Definition 2.6 : A function f : X $\rightarrow$ Y is called

i) Pre–continuous[8]: if f^{-1}(V) is preopen in X for every open set V of Y.
ii) α -Continuous [19]: if for each open set V of Y, f^{-1} (V) $\in$ α (X, τ).
iii) Contra – Continuous [10]: if f^{-1} (V) is closed in X for each open set V of Y.
iv) Contra–Semi–Continuous [11]: If f^{-1}(v) is semi–closed in X for each open set V of Y.
v) Contra–Pre–Continuous [14]: if f^{-1} (V) is preclosed in X for each open set V of Y.
vi) Contra–semipre–continuous [24]: if f^{-1} (V) is semipreclosed in X for each open set V of Y.
vii) Contra- α -Continuous [14]: if f^{-1} (V) is α -closed in X for each open set V of Y.
viii) Contra pre-α open[jafari]: if the image of each α-open set of X is α-closed in Y.
ix) Contra pre-α closed[jafari]: if the image of each α-closed set of X is α- open in Y.

Definition 2.7: A function f : (X, τ) $\rightarrow$ (Y. ν) is said to be

i) pre–irresolute [28] function, if f^{-1}(V) is preopen set in X for every preopen subset V of Y.
ii) α - irresolute [17] function, if f^{-1} (V) is α - open set in X for every α - open subset V of Y.

Definition 2.8 : A function f : X $\rightarrow$ Y is called α -pre (in brief α p)– continuous [3] if f^{-1}(V) is preopen in X for each α -open set V of Y .

Definition 2.9: A function f : X $\rightarrow$ Y is said to be strongly precontinuous [4] if f^{-1} (V) is pre open in X for every semi–open set V of Y.

Definition 2.10: A function f : X $\rightarrow$ Y is to said to be p-continuous if inverse image of each preopen set of Y is open in X.

Definition 2.11: A function f : X $\rightarrow$ Y is said to be strongly α -continuous [2] if f^{-1}(V) is α - open in X for every semi–open set V of Y .

Definition 2.12 : A function $f : X \rightarrow Y$ is said to be M-preopen[21] (M-preclosed) if image of each preopen(resp. preclosed) set is preopen (resp.eclosed).

Definition 2.13 : A function $f : X \rightarrow Y$ is said to be

i) preopen [21] function if the image of each open set of X is preopen in Y.

ii) preclosed [9] function if the image of each closed set o X is preclosed in Y.

iii) α-open[19] function if the image of each open set of X is α-open in Y.

iv) α-closed [19] function if the image of each closed set of X is α-closed in Y.

v) pre-α-open [jafari] if the image of each α-open set of X is α-open in Y.

vi) pre-α-closed[jafari] if the image of each α-closed set of X is α-closed in Y.

Contra α -Semi–Continuous Functions: In this section we study a new class of functions called contra α semi -continuous function.

Definition 3.1 : A function $f : X \rightarrow Y$ is called contra-α semi (in brief contra α s) – continuous if the inverse image of each α -open set of Y is semi-closed in X.

Theorem 3.2: The following are equivalent for function $f : X \rightarrow Y$:

i. f is contra-αs-continuous;

ii. for every α-closed subset F of Y, $f^{-1}(F) \in SO(X)$;

iii. for each $x \in X$ and each α-closed set F of Y containing f(x), there exists a semiopen set U in X containing x such that $f(U) \subset F$.

Proof : (i)⇔(ii).

Let F be a α-closed subset of Y then Y-F is α-open subset of Y. Since f is contra-αs-continuous, $f^{-1}(Y\text{-}F) = X - f^{-1}(F)$ is semiclosed set in X. Thus, for every α-closed subset F of Y, $f^{-1}(F) \in SO(X)$.(ii)⇔(iii).Let $x \in X$ be an arbitrary point. For each α-closed set F of Y containing f(x).By (ii) $f^{-1}(F)$ is semiopen set in X. Let U= $f^{-1}(F)$, then U is semiopen set in X containing x such that $f(U) \subset F$ .In 1965, Njastad [26] shown that $\alpha O(x)$ (or τ^{α}) is a topology for X and it is stronger than the given topology on X.

Theorem 3.3 :A function $f : (X, \tau) \rightarrow (Y,\sigma)$ is contra-αs-continuous if and only if $f : (X, \tau^{\alpha}) \rightarrow (Y, \sigma^{\alpha})$ is contra-αs-continuous.

Proof : Obvious.

Theorem 3.4 : If $f : X \rightarrow Y$ is a surjective M-semiopen and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is contra-αs-continuous then g is contra-αs-continuous.

Proof : Let U be any α-closed set in Z. Since gof is contra-αs-continuous.$(gof)^{-1}(U)= f^{-1}(g^{-1}(U))$ is semiopen in X. Since f is surjective M-semiopen. $f(f^{-1}(g^{-1}(U)))= g^{-1}(U)$ is semiopen in Y. Therefore, g is contra-αs-continuous.

Theorem 3.5 : If $f : X \rightarrow Y$ is a surjective M-semiclosed and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is contra-αs-continuous then g is contra-αs continuous.

Proof : Similar to the above Theorem 3.4. We define the following.

Definition 3.6 : A function $f : X \rightarrow Y$ is said to be

i) αs-open if the image of each α-open set of X is semiopen in Y.

ii) αs-closed if the image of each α-closed set of X is semiclosed in Y.

iii) Contra-αs-open if the image of each α-open set of X is semiclosed in Y.

iv) Contra-αs-closed if the image of each α-closed set of X is semiopen in Y.

Theorem 3.7 : If $f : X \rightarrow Y$ is a surjective αs-open and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is α-irresolute then g is contra-αs-continuous.

Proof : Let U be any α-open set in Z. Since gof is α-irresolute. $(gof)^{-1}(U)= f^{-1}(g^{-1}(U))$ is α-open in X. Since f is surjective αs-open. $f(f^{-1}(g^{-1}(U)))= g^{-1}(U)$ is semiopen in Y. Therefore, g is contra-αs-continuous.

Theorem 3.8 : If $f : X \rightarrow Y$ is a surjective αs-closed and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is α-irresolute then g is contra-αs-continuous.

Proof : Similar to the above Theorem 3.7.

Theorem 3.9 : If $f : X \rightarrow Y$ is a surjective semiopen and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is contra-continuous then g is contra-semi-continuous.

Proof : Let U be any closed subset in Z. Since gof is contra-continuous,

$(gof)^{-1}(U)= f^{-1}(g^{-1}(U))$ is open in X. Since f is surjective semiopen. $f(f^{-1}(g^{-1}(U))) = g^{-1}(U)$ is semiopen in Y. Therefore, g is contra-semi-continuous.

Theorem 3.10 : If $f : X \rightarrow Y$ is a surjective semi-α-closed and $g : Y \rightarrow Z$ is a function such that $gof : X \rightarrow Z$ is contra-α-irresolute then g is contra-α-irresolute.

Proof : Let U be any α-open set in Z. Since gof is contra-α-irresolute,$(gof)^{-1}(U)= f^{-1}(g^{-1}(U))$ is α-closed in X. Since f is surjective semi-α-closed. $f(f^{-1}(g^{-1}(U)))= g^{-1}(U)$ is α-closed in Y. Therefore, g is contra-α-irresolute.

Theorem 3.11 : Let X,Y and Z be three topological spaces and let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions with $gof: X \rightarrow Z$ is a αs-open functions. Then,

i) If f is α-irresolute and surjective, then g is αs-open.

ii) If g is semiirresolute and injective, then f is αs-open.

Proof : (i) Let V be an arbitrary α-open set in Y. Since gof is αs-open and f is surjective then $g(V)=gof(f^{-1}(V))$ is a semiopen in Z. This shows that g is a αs-open function.

(ii) Since g is injective, We remark that $f(A)=g^{-1}[g(f(A))]$ for every subset A of X. Let U be any arbitrary α-open set in X, then by hypothesis, gof(U) is a semi-open set in Z. We have $f(U)=g^{-1}(gof(U))\in SO(Y)$ which implies that f(U) is semiopen set in Y. Hence, f is a αs-open function.

Theorem 3.12: For a function f: X $\rightarrow$Y the following are equivalent:

i) f is contra-αs-open.

ii) For every subset B of Y and for every α- closed subset F of X with $f^{-1}(B) \subseteq F$, there exists a semi-open subset U of Y with $B\subseteq U$ and $f^{-1}(U) \subseteq F$.

iii) For every $y\in$ Y and for every α- closed subset F of X with $f^{-1}(y) \subseteq F$, there exists a semi-open subset U of Y with $y\in$ U and $f^{-1}(U) \subseteq F$.

Proof: (i)$\rightarrow$(ii). Let B be a subset of Y and let F be a α-closed subset of X with $f^{-1}(B) \subseteq F$. Put $U=[f(F^c)]^c$. Since f is contra semi-α open, then U is a semi-open set of Y and since $f^{-1}(B) \subseteq F$. We have $f(F^c) \subseteq B^c$ and hence $B\subseteq U$. More over $f^{-1}(U)=[f^{-1}(f(F^c))]^c \subseteq (F^c)^c=F$. (ii) $\rightarrow$(iii). It is sufficient to put B={y}. (iii)$\rightarrow$(i). Let A be a α-open subset of X. Then let $y\in[f(A)]^c$ and let $F=A^c$. By (iii) there exists a semi-open subset U_y of Y with $y \in U_y$ and $f^{-1}(U_y) \subseteq F$. Then we see that $y\in U_y\subseteq [f(A)]^c$. Hence $[f(A)]^c =\cup\{U_y : y\in[f(A)]^c\}$ is pre-open. Therefore f(A) is semi-closed subset in Y. This shows that f is contra αs-open.

Theorem 3.13: For a function f: X $\rightarrow$Y the following are equivalent:

i) f is contra αs -closed.

ii) For every subset B of Y and for every α-open subset U of X with $f^{-1}(B) \subseteq U$, there exists a semi-closed subset F of Y with $B \subseteq F$ and $f^{-1}(F) \subseteq U$.

Proof: Proof is easy and hence omitted.

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