LINEAR AND NON-LINEAR ANALYSES OF BRINKMAN- BÈNARD CONVECTION IN A NANOLIQUID-SATURATED ENCLOSURE

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Abstract: The paper deals with the study of linear and non-linear Brinkman-Bènard convection of a nanoliquid in a rotating enclosure. The momentum equation is modified by incorporating Coriolis and centrifugal effects. The linear stability analysis is based on normal mode technique. It is observed that rotation delays the onset of convection and hence decreases the heat transport. It is further observed that nanoparticles advance the onset of convection and hence enhance heat transport.

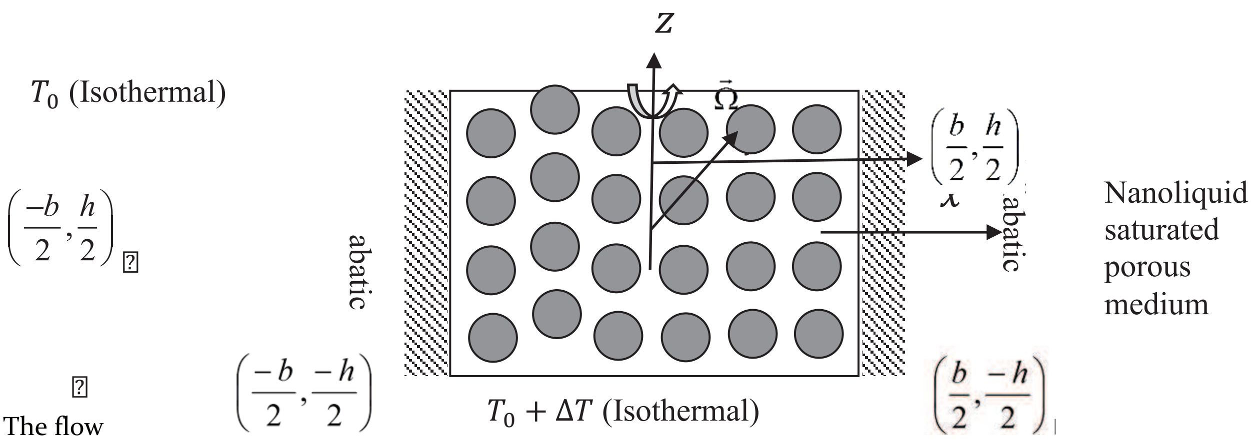
Introduction: Nanoliquids refers to a mixture of base liquid with very small amount of nanoparticles which may be metallic or metallic oxide, or nanotubes. It was Choi [1] who first coined the term nanoliquid. In recent years, researchers have shown considerable interest in the study of nanoliquids due to their applications in automotive industries, energy saving, nuclear reactors, cancer therapy and other allied fields.

Heat transfer problems in the presence of porous media have various engineering applications, such as geothermal energy recovery, ground water pollution, thermal energy storage and this has been studied by Nield and Kuznetsov [2].

A detailed account of thermal instability in Newtonian fluid, under various assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [7]. Thermal instability of nanoliquids in the presence of rotation has been investigated by Yadav et al. [4]. The thermal instability of nanoliquids saturating a porous medium has been studied by Agarwal et al. [3]. Review of literature shows that there is no similar study in an enclosure.

In the present paper the effect of rotation on Brinkman-Bènard convection in nanoliquid-saturated porous enclosure is studied using single-phase model [6]. The nanoliquid considered in the problem is water-copper saturating a 30% glass fiber reinforced polycarbonate porous matrix. The thermophysical properties of nanoliquids were collected from literature [6, 8] and phenomenological laws [6] and mixture theory [6, 8] were used for the same.

Mathematical formulation: The physical configuration of the flow problem is as shown below. The porous enclosure is rotated about z-axis with angular velocity $\vec{\Omega}$ uniformly and is acted upon by gravity.



The flow is governed by the Brinkman model with effects of Coriolis force and centrifugal acceleration included where centrifugal term is absorbed in pressure term of the linear momentum equation. An appropriate single-phase equations with suitable correction for a rotating porous medium is used [6].

The non-dimensional form of governing equations are:

$$- Ra_{ne} A^4 a^2 \frac{\partial \theta}{\partial X} + \wedge a \nabla_A^4 \psi - \sigma^2 a A^2 \nabla_A^2 \psi + a \sqrt{Ta}\, A^2 \frac{\partial V}{\partial Z} = 0 \quad (1)$$

$$Ma \nabla_A^2 \theta - A \frac{\partial \psi}{\partial X} + A \frac{\partial(\psi, \theta)}{\partial(X, Z)} = 0 \quad (2)$$

$$\frac{1}{\phi} A \frac{\partial(\psi, V)}{\partial(X, Z)} + a \wedge \nabla_A^2 V - a\sigma^2 A^2 V - a\sqrt{Ta} A^2 \frac{\partial \psi}{\partial Z} = 0 \text{, (3)}$$

where $Ra_{ne} = \frac{(\rho\beta)_{ne} g \Delta T b^3}{\mu_{ne}\alpha_{ne}}$, $A = \frac{h}{b}$, $\wedge = \frac{\mu'_{ne}}{\mu_{ne}}$, $a = \frac{\alpha_{ne}}{\alpha_{bl}}$, $\sigma^2 = \frac{b^2}{K}$, $Ta = \left(\frac{2\Omega b^2 \rho_{ne}}{\phi\mu_{ne}}\right)^2$, $M = \frac{(\rho C_p)_{ne}}{(\rho C_p)_{nl}}$.

Linear Stability Analysis: Since the principle of exchange of stabilities is valid we consider marginal stationary state. In order to study onset of convection a linear stability analysis is performed by considering linear terms in eqs. (1-3) and the normal mode solutions assumed are as follows:

$$\left.\begin{aligned} \psi(X,Z) &= \psi_0 \sin\left[\pi\left(X+\frac{1}{2}\right)\right]\sin\left[\pi\left(Z+\frac{1}{2}\right)\right] \\ \theta(X,Z) &= \theta_0 \cos\left[\pi\left(X+\frac{1}{2}\right)\right]\sin\left[\pi\left(Z+\frac{1}{2}\right)\right] \\ V(X,Z) &= V_0 \sin\left[\pi\left(X+\frac{1}{2}\right)\right]\cos\left[\pi\left(Z+\frac{1}{2}\right)\right] \end{aligned}\right\} \text{, (4)}$$

Subjected to boundary condition:

$$\psi = \frac{\partial^2\psi}{\partial Z^2} = \frac{\partial V}{\partial Z} = \theta = 0 \; at \; z = \frac{-1}{2}, \frac{1}{2}; \; \psi = \frac{\partial^2\psi}{\partial X^2} = V = \frac{\partial\theta}{\partial X} = 0 \; at \; z = \frac{-1}{2}, \frac{1}{2}.$$

Following classical procedure, we obtain expression for critical Rayleigh number in the form

$$Ra_{nec} = \frac{M\delta_A^4(\wedge\delta_A^2 + \sigma^2 A^2)}{A^5\pi^2} + \frac{TaM\delta_A^2}{A(\wedge\delta_A^2 + \sigma^2 A^2)} \text{. (5)}$$

Linear stability analysis predicts only the onset of convection. To study the heat transport we perform local non-linear stability analysis in the next section.

Local non-linear stability analysis: Using the truncated representation of Fourier series, a nonlinear stability

$$\left.\begin{aligned} & B^* - A^* + D^* = 0 \\ & rA^* - A^*C^* - B^* = 0 \\ & A^*B^* - b_1 C^* = 0 \\ & \frac{1}{Q_1} D^* + \frac{Ta\pi^2 Q_1 A^4}{\delta_A^6} A^* = 0 \end{aligned}\right\}$$

analysis is performed and the following algebraic equations are obtained.,

where $r = \frac{Ra_{ne}}{Ra_{nec}}$, $b_1 = \frac{4\pi^2}{\delta_A^2}$, $Q_1 = \frac{M\delta_A^6}{Ra_{ne} A^5 \pi^2}$.

Nusselt number expression: Heat transport is quantified by Nusselt number, Nu_{ne}, as follows:

$$Nu_{ne} = \frac{Heat\ transport\ by\,(conduction + convection)}{Heat\ transport\ by\ convection} = 1 + 2\frac{k_{ne}}{k_{be}}\left[1 - \frac{1}{r}\right].$$

Results and Discussion:

Sl. No.	A	Λ	σ^2	χ	Ta	Ra_{nec}	Nu_{ne}
1	0.8	1.2	10	0.06	100	2011.93	2.4116
2	1	1.2	10	0.06	100	1287.19	2.7539
3	1.2	1.2	10	0.06	100	990.809	2.8938

4	1	1	10	0.06	100	1149.91	2.8187
5	1	1.4	10	0.06	100	1425.97	2.6883
6	1	1.2	0	0.06	100	944.142	2.9159
7	1	1.2	5	0.06	100	1113.67	2.8358
8	1	1.2	10	0	100	1286.5	2.4855
9	1	1.2	10	0.1	100	1287.65	2.9525
10	1	1.2	10	0.06	0	1232.87	2.8765
11	1	1.2	10	0.06	500	1504.47	2.7695
12	1	1.2	10	0.06	1000	1776.07	2.6626

Table 1: Variation of Ra_{nec}, Nu_{ne} by varying one parameter and fixing the others ,i.e., A = 0.8, 1, 1.2; Λ = 1, 1.2, 1.4; σ^2 = 0, 5, 10; χ = 0, 0.06, 0.1, Ta = 0, 500, 1000 respectively.

Brinkman-Bènard convection in rotating nanoliquid-saturated porous enclosure is studied using single-phase model. Linear stability analysis is made with the help of normal mode technique. All the thermophysical properties are calculated by using phenomenological laws and mixture theory [6, 8] in the case of a water-copper nanoliquid saturating 30% reinforced glass fiber polycarbonate porous medium. From table 1 it is clear that onset of convection is delayed with individual and collective increases in Λ, σ^2 and Ta which implies that Λ, Brinkman number and Taylor number (rotation) have stabilizing effect on convection. It is further observed that onset of convection is advanced with increase in values of A and χ which implies that these parameters destabilize the system. It is also observed that among shallow, square, tall enclosures onset of convection is fast in a tall enclosure.

Weakly non-linear stability analysis is done by considering truncated Fourier series with two terms. A system of algebraic equations is obtained and Nu_{ne} is calculated. From table 1 it is clear that heat transport decreases with individual and collective increases in Λ, σ^2 and Ta. It is further observed that heat transport is enhanced with increase in values of A and χ.

Conclusion:

1. $Ra_{ne}^{\chi \neq 0} < Ra_{ne}^{\chi = 0}$ and $Ra_{ne}^{Ta\neq 0} < Ra_{ne}^{Ta=0}$, for all values of Ta.
2. Onset of convection is delayed due to the presence of porous medium.
3. Rotation delays onset of convection leading to decrease in the heat transport.
4. Among shallow, square and tall enclosures it is found that tall enclosure enhances heat compared to shallow and square enclosure.

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