VAGUE RELATION WITH MAX- MIN COMPOSITION TECHNIQUE IN MEDICAL DIAGNOSIS

MARIAPRESENTI. L, AROCKIARANI. I

Abstract: In this paper we propose an new max-min composition technique to diagnose the symptom of the disease using generalized interval valued vague intuitionistic relation.

Keywords: Vague set, Interval valued vague set, generalized interval valued vague relation, vague composite relations, max- min composition, Algorithm, case study.

Introduction: In the real world treatment depends on proper diagnosis. Patient may express their feelings of discomfort by linguistic variables. Sometimes doctors cannot properly diagnose the disease and cannot start treatment by only considering the information provided by patient. In order to deal with the problem, Zadeh[7,8] in 1969 proposed an application of fuzzy set on medical science fields and Sanchez[4] in 1979 proposed medical diagnosis by using composite fuzzy relation. In 1993 Gau and Buehrer [2] introduced the concept of vague set which was the generalization of fuzzy set with truth membership and false membership function. The vague set theory has been investigated by many authors and has been applied in different fields. In this paper we present an application of generalized interval valued vague relation with max min composition technique to diagnose the deficiency disease caused by vitamins.

Preliminaries:

Definition 2.1: [6] Let X be a non-empty set. A fuzzy set A drawn from X is defined as $A=\{<x,\mu_A(x)>: x\in X\}$, where $\mu_A : X \rightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 2.2: [3] Let [I] be the set of all closed subintervals of the interval [0,1] and $\mu=[\mu_L,\mu_U]\in[I]$, where μ_L and μ_U are the lower extreme and the upper extreme, respectively. For a set X, an IVFS A is given by equation $A=\{<x,\mu_A(x)>/x\in X\}$ where the function $\mu_A : X\rightarrow[I]$ defines the degree of membership of an element x to A, and $\mu_A(x)=[\mu_{AL}(x),\mu_{AU}(x)]$ is called an interval valued fuzzy number.

Definition 2.3: [2] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U\rightarrow[0,1]$ and

(ii) A false membership function $f_A : U\rightarrow[0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x)+f_A(x)\le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x),1-f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x)\le\mu(x)\le 1-f_A(x)$ .The vague set A is written as $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle/u\in U\}$ where the interval $[t_A(x),1-f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.4:[1] Let A and B be VSs of the form $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle/x\in X\}$ and $B=\{\langle x,[t_B(x),1-f_B(x)]\rangle/x\in X\}$ Then

$A\subseteq B$ if and only if $t_A(x)\le t_B(x)$ and $1-f_A(x)\le 1-f_B(x)$ for all $x\in X$

A=B if and only if $A\subseteq B$ and $B\subseteq A$

$$A^c=\{\langle x,f_A(x),1-t_A(x)\rangle/x\in X\}$$

$$A\cap B=\{\langle x,\min(t_A(x),t_B(x)),\min(1-f_A(x),1-f_B(x))\rangle/x\in X\}$$

$$A\cup B=\{\langle x,(t_A(x)\vee t_B(x)),(1-f_A(x)\vee 1-f_B(x))\rangle/x\in X\}$$

For the sake of simplicity, we shall use the notation $A=\langle x,t_A,1-f_A\rangle$ instead of $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle/x\in X\}$.

Definition 2.5:[5] An interval valued vague sets $\widetilde{A}^V$ over a universe of discourse X is defined as an object of the form $\widetilde{A}^V=\{<x_i,[T_{\widetilde{A}^V}(x_i),F_{\widetilde{A}^V}(x_i)]>,x_i\in X\}$ where $T_{\widetilde{A}^V}:X\rightarrow D([0,1])$ and $F_{\widetilde{A}^V}:X\rightarrow D([0,1])$ are called " truth membership function" and "false membership function" respectively and where D[0,1] is the set of all intervals within [0,1], or in other word an interval valued vague set can be represented by $\widetilde{A}^V=<[(x_i),[\mu_1,\mu_2],[\nu_1,\nu_2]]>,x_i\in X$ where $0\le\mu_1\le\mu_2\le 1$ *and* $0\le\nu_1\le\nu_2\le 1$. For each interval valued vague set $\widetilde{A}^V$, $\pi_{1\widetilde{A}^V}(x_i) = 1 - \mu_{1\widetilde{A}^V}(x_i) - \nu_{1\widetilde{A}^V}(x_i)$ and are called degree of hesitancy of x_i in $\widetilde{A}^V$ respectively.

Analysis of the problem: Vitamins in your diet plays a vital role in maintaining good health. In the present lifestyle of processed foods and fatty diet, people are prone to have one or many vitamin deficiency. This can also happen due to an improper digestive system. Hence, one should consume the proper amount of vitamins or take a supplement. Deficiency of essential vitamins usually is a result of poor dietary habits like low intake of fruits and vegetables. Vitamin deficiency is a very serious problem, and should be treated by natural foods or dietary supplements.

In this section we present an application of generalized interval valued vague relation with max-min composition technique in the diagnosis of deficiency disease. In a given pathology, suppose S is a set of symptoms, P a set of patients. We define a generalized interval valued vague relation A from the set of symptoms to the set of disease D(i.e., on S x D) which reveals the degree of interval valued truth function and interval valued false membership function between symptoms and disease. Then we also define a generalized interval valued vague relation B from the set of patients to the set of symptoms (i.e., on P x S) which reveals the degree of interval valued truth function and interval valued false membership function between symptoms and disease. The methodology involves mainly the following jobs:

Determination of symptoms, Formulation of medical knowledge based on generalized interval valued vague relations and determination of medical diagnosis on the basis of composition of generalized interval valued vague relations.

Definition 3.1: A generalized interval valued vague relation(GIVR) is defined as a generalized interval valued vague subsets of X xY, having the form $A = \{< (x,y), [t_{AL}(x,y), t_{AU}(x,y)], [1 - f_{AL}(x,y), 1 - f_{AU}(x,y)] >: x \in X, y \in Y\}$ where $[t_{AL}, t_{AU}]: X \times Y \to [0,1]$, $[f_{AL}, f_{AU}]: X \times Y \to [0,1]$ represents the degree of interval valued truth and interval valued false membership function.

Definition 3.2: Let B $\in$ GIVR(X x Y) and A $\in$ GIVR (Y x Z), then we define two vague composite relations on X x Z, denoted by A ◦ B and defined by

$$A \circ B = \{< (x,z), [t_{AL\circ BL}(x,z), t_{AU\circ BU}(x,z)], [1 - f_{AL\circ BL}(x,z), 1 - f_{AU\circ BU}(x,z)] >: x \in X, z \in Z\}$$

where $t_{AL\circ BL}(x,z) = \underset{y\in Y}{Max}\{\underset{x\in X}{Min}\{t_{BL}(x,y), t_{AL}(y,z)\}$,

$$t_{AU\circ BU}(x,z) = \underset{y\in Y}{Max}\{\underset{x\in X}{Min}\{t_{BU}(x,y), t_{AU}(y,z)\}$$

$$1 - f_{AL\circ BL}(x,z) = \underset{y\in Y}{Max}\{\underset{x\in X}{Min}\{1 - f_{BL}(x,y), 1 - f_{AL}(y,z)\}\},$$

$$1 - f_{AU\circ BU}(x,z) = \underset{y\in Y}{Max}\{\underset{x\in X}{Min}\{1 - f_{BU}(x,y), 1 - f_{AU}(y,z)\}\}$$

represents the degree of truth and false function

Algorithm of the problem: Step 1: Compute the composition C of generalized interval valued vague relation A and B, i.e., Compute C = A ◦ B.

Step 2: Compute E, where $E = [t_{Al}(P,D), 1 - f_{AL}(P,D)], [t_{AU}(P,D), 1 - f_{AU}(P,D)]$

Step3: Find $\{Min\ [t_{AL}(P,D), 1 - f_{AL}(P,D)]\},\ Min\ [t_{AU}(P,D), 1 - f_{AU}(P,D)]\}$

Step4: Find $Max\ \{\frac{Min\{t_{AL}(P,D), 1 - f_{AL}(P,D)\},\ Min\{t_{AU}(P,D), 1 - f_{AU}(P,D)]}{2}\}$

then we conclude that the patients P_i is suffering from the diagnosis D_j (i.e., i= 1,2,3,4 & j=1,2,3,4,5)

Case study: Let us consider four patients P={P_1,P_2,P_3,P_4} and the set of symptoms S={ Brain damage, Body weakness, Muscles and bone weakness, Hair loss, Vision problem}. The generalized interval valued vague relation B(P→S) is given as in Table 1. Let the set of diseased D={ Beri beri, Pellagra, Biotine, Rickets, Night blindness}. The generalized interval valued vague relation A (S→D) is given as in Table 2.

Table 1: Generalized interval valued vague relation B(P→S)

B	Brain damage	Body weakness	Muscles and bone	Hair loss	Vision problem

			weakness		
P_1	[0.0,0.2][0.6,0.7]	[0.3,0.4][0.5,0.6]	[0.1,0.2][0.4,0.5]	[0.2,0.3][0.3,0.4]	[0.3,0.4][0.5,0.6]
P_2	[0.3,0.4][0.5,0.6]	[0.1,0.2][0.6,0.7]	[0.2,0.3][0.3,0.4]	[0.4,0.5][0.6,0.8]	[0.1,0.2][0.7,0.8]
P_3	[0.0,0.1][0.5,0.6]	[0.3,0.4][0.4,0.5]	[0.7,0.8][0.8,0.9]	[0.1,0.2][0.5,0.8]	[0.2,0.3][0.3,0.4]
P_4	[0.1,0.3][0.6,0.7]	[0.3,0.4][0.5,0.6]	[0.8,0.9][0.8,0.9]	[0.0,0.1][0.5,0.8]	[0.3,0.4][0.5,0.6]

Table 2: Generalized interval valued vague relation A(S→D)

A	Beri beri	Pellagra	Biotin	Rickets	Night blindness
Brain damage	[0.3,0.4][0.7,0.8]	[0.1,0.3][0.5,0.6]	[0.3,0.4][0.6,0.7]	[0.0,0.1][0.5,0.6]	[0.2,0.3][0.6,0.8]
Body weakness	[0.1,0.2][0.5,0.6]	[0.4,0.5][0.7,0.8]	[0.1,0.2][0.3,0.4]	[0.0,0.1][0.7,0.8]	[0.1,0.2][0.5,0.6]
Muscles and bone	[0.2,0.4][0.7,0.8]	[0.6,0.7][0.6,0.7]	[0.7,0.8][0.8,0.9]	[0.2,0.3][0.7,0.9]	[0.1,0.4][0.3,0.5]
Hair Loss	[0.4,0.5][0.5,0.8]	[0.1,0.3][0.3,04]	[0.3,0.4][0.7,0.8]	[0.2,0.3][0.4,0.6]	[0.1,0.3][0.4,0.5]
Vision problem	[0.1,0.2][0.7,0.8]	[0.4,0.5][0.6,0.7]	[0.1,0.2][0.4,0.5]	[0.2,0.3][0.4,0.6]	[0.0,0.1][0.3,0.5]

Table 3: The Max- Min composition C of A∘B

C=A∘B	Beri beri	Pellagra	Biotin	Rickets	Night blindness
P_1	[0.2,0.3][0.6,0.7]	[0.3,0.4][0.5,0.6]	[0.2,0.3][0.6,0.7]	[0.2,0.3][0.5,0.6]	[0.1,0.3][0.6,0.7]
P_2	[0.4,0.5][0.7,0.8]	[0.2,0.3][0.6,0.7]	[0.3,0.4][0.6,0.8]	[0.2,0.3][0.6,0.7]	[0.2,0.3][0.5,0.6]
P_3	[0.2,0.4][0.5,0.6]	[0.6,0.7][0.6,0.7]	[0.7,0.8][0.8,0.9]	[0.2,0.3][0.7,0.9]	[0.1,0.4][0.5,0.6]
P_4	[0.2,0.4][0.6,0.7]	[0.6,0.7][0.6,0.7]	[0.7,0.8][0.8,0.9]	[0.2,0.3][0.7,0.9]	[0.1,0.3][0.6,0.7]

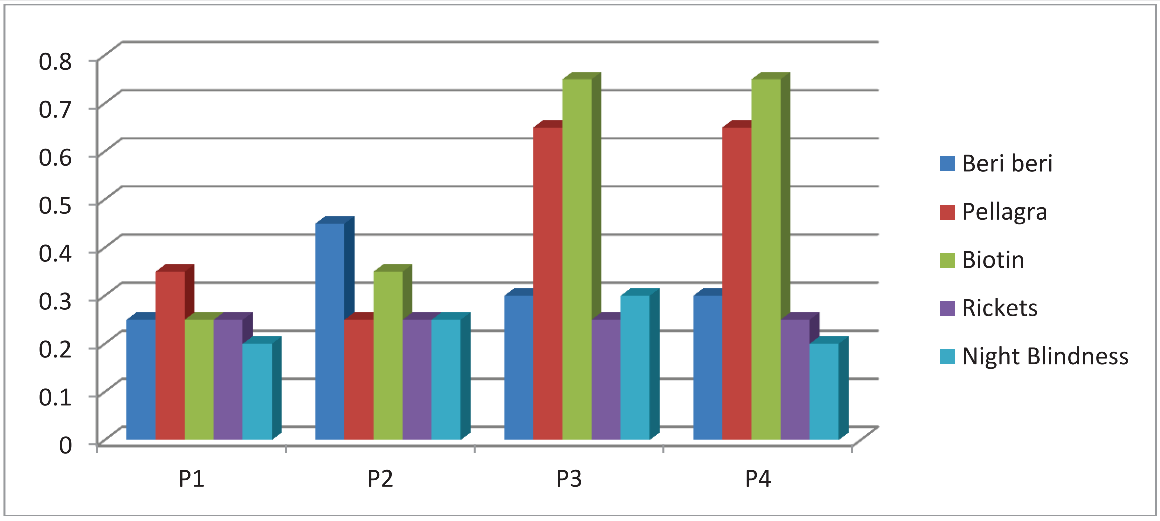
Table 4: $\{Min\ [t_{AL}(P,D),1-f_{AL}(P,D)]\},\ Min\ [t_{AU}(P,D),1-f_{AU}(P,D)]\}$

E	Beri beri	Pellagra	Biotin	Rickets	Night blindness
P_1	[0.2,0.3]	[0.3,0.4]	[0.2,0.3]	[0.2,0.3]	[0.1,0.3]
P_2	[0.4,0.5]	[0.2,0.3]	[0.3,0.4]	[0.2,0.3]	[0.2,0.3]
P_3	[0.2,0.4]	[0.6,0.7]	[0.7,0.8]	[0.2,0.3]	[0.1,0.4]
P_4	[0.2,0.4]	[0.6,0.7]	[0.7,0.8]	[0.2,0.3]	[0.1,0.3]

Table 5: $Max\ \{\frac{Min\{t_{AL}(P,D),1-f_{AL}(P,D)\},\ Min\{t_{AU}(P,D),1-f_{AU}(P,D)]}{2}\}$

E	Beri beri	Pellagra	Biotin	Rickets	Night blindness
P_1	0.25	**0.35**	0.25	0.25	0.2
P_2	**0.45**	0.25	0.35	0.25	0.25
P_3	0.3	0.65	**0.75**	0.25	0.3
P_4	0.3	0.65	**0.75**	0.25	0.2

Conclusion: Finally $Max\ \{\frac{Min\{t_{AL}(P,D),1-f_{AL}(P,D)\},\ Min\{t_{AU}(P,D),1-f_{AU}(P,D)]}{2}\}$ gives the result of the medical diagnosis in the table 6, we see that the maximum value of P_1 is 0.35 , P_2 is 0.45 and P_3 & P_4 is 0.75. This concludes that P_1 suffers from Pellagra, P_2 suffers from Beri beri and P_3 & P_4 suffers from Biotin deficiency. This is represented by the chart diagram as follows:



References:

1. Borumandsaeid. A and Zarandi. A., Vague set theory applied to BM- Algebras. International journal of algebra, 5, 5 (2011), 207-222.
2. Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.
3. Hye Mi Choi, Gil Seong mun and Jeong yong ahn, A medical diagnosis based on interval valued fuzzy sets, Biomedical engineering: Applications, Basis and communication, 24(2012), 349-354.
4. Sanchez. E, Medical diagnosis and composite fuzzy relations, Advances in fuzzy set theory and applications, Gupta. M. M, Ragade. R. K and R. R. Yager (eds), Elsevier science Ltd, 1979.
5. Sharma. M. K, Similarity measure using interval valued vague sets in multiple criteria decision making, IOSR journal of mathematics(IOSR-JM), 11(2015), 46-55.
6. Yao. J. F and Yao. J. S, Fuzzy decision making for medical diagnosis based on fuzzy number and compositional rule of inference, Fuzzy sets and systems, 120 (2001), 351-366.
7. Zadeh. L. A, Biological applications of the theory of fuzzy sets and systems, Proc. of an international symposium on biocybernetics of the central nervous system, (1969), 199-206.
8. Zadeh. L. A, Fuzzy sets, Information and control, 8 (1965), 338-353.

Mariapresenti.L, Arockiarani. I
Nirmala college for women, Coimbatore- 641018, Tamilnadu, India.