CORRELATION MEASURE FOR VAGUE MULTI SETS AND ITS APPLICATION IN BRICK SELECTION

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Abstract: A new approach for Multiple Criteria Decision Making problem is presented in this paper together with a correlation coefficient defined for vague multi sets. Correlation coefficient for vague multi set is used for selection of alternatives. An illustrative numerical example for quality of brick selection is solved to show the effectiveness of the proposed model.

Keywords: Vague multi sets, correlation measure, brick selection.

Introduction: Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. Gau and Buehrer [5] introduced the concept of vague sets as a generalization of vague set. As the vague set took the truth membership and false membership degree into account, and has more ability to deal with uncertain information than traditional fuzzy set, lots of scholars pay attentions to the research of vague sets. Chen [3] and Hong [6] presented new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. Fuzzy correlation has captured the attention of many researchers in recent days. Chiang and Lin [4] studied the correlation of fuzzy sets. Solairaju [8] have applied correlation coefficient of interval valued vague set to decision making problems. An element of a multi fuzzy set can occur more than once with possibly the same or different membership values. The multi-set theory was formulated first by Yager [11] as generalizations of the concept of set theory. Several authors from time to time made a number of generalizations of the multi set theory.

In this study, we have introduced correlation coefficient for vague multi sets and derived some of their properties. and also present an application of correlation measure of vague multi set to decision making model for quality of brick selection for construction field.

Preliminaries: Definition 2.3[12]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{< x : \mu_A(x) >: x \in X\}$, where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A

Definition 2.2[11]: Let X be a nonempty set. A Fuzzy Multiset (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp multi sets drawn from the unit interval [0, 1]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multi sets drawn from [0, 1].

Definition 2.3[5]: A vague set A in the universe of discourse U is a pair $[t_A, f_A]$ where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are the mappings (called truth membership function and false membership function respectively) where $t_A(x)$ is a lower bound of the grade of membership of x derived from the evidence for x and $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \le 1$ for all $x \in U$.

Definition 2.4[5]: The interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, and it is denoted by $V_A(x)$. That is $V_A(x) = [t_A(x), 1 - f_A(x)]$.

Definition 2.3[5]: A vague set A of U with $t_A(x) = 1$ and $f_A(x) = 0 \quad \forall x \in U$, is called the unit vague set of U.

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Definition 2.4[5]: Let Abe a non-empty set and the vague set A and B in the form $A = \{\langle x, t_A(x), 1 - f_A(x)\rangle : x \in X\}$, $B = \{\langle x, t_B(x), 1 - f_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$

(ii) $A \cup B = \max\{t_A(x), t_B(x)\}$ and $\max\{1 - f_A(x), 1 - f_B(x)\}$.

(ii) $A \cap B = \min\{t_A(x), t_B(x)\}$ and $\min\{1 - f_A(x), 1 - f_B(x)\}$.

(iv) $\overline{A} = \{\langle x, f_A(x), 1 - f_A(x)\rangle : x \in X\}$.

Definition 2.5: Let X be a nonempty set. A vague multi sets (VMS) A in X is characterized by two functions namely count truth membership function t_C and count false membership function f_C such that $t_C : X \to Q$ and $f_C : X \to Q$ where Q is the set of all crisp multi sets in [0, 1]. Hence, for any $x \in X$, $t_C(x)$ is the crisp multi sets from [0, 1], whose truth membership sequence is defined as $(t_A^1(x), t_A^2(x), ...t_A^j(x))$ and the corresponding false membership sequence is defined as $(1-f_A^1(x), 1-f_A^2(x), ...1-f_A^j(x))$ such that $0 \le t_A^i(x) + f_A^i(x) \le 1,\ \forall x \in X$ and $i = 1,2,3,.....j$. Therefore, A VMS is given by $A = \{< x, (t_A^1(x), t_A^2(x)).......t_A^p(x)),\ (1-f_A^1(x), 1-f_A^2(x), ...1-f_A^p(x)) > / x \in X\}$.

Definition 2.6: The cardinality of the truth membership function $t_C(x)$ and false membership function $1-f_C(x)$ is the length of a element x in a VMS A denoted as ξ, defined as $\xi = |t_C(x)| = |1-f_C(x)|$. If A, B, C are the VMS defined on X, then their cardinality $\xi = Max\{\xi(A), \xi(B), \xi(C)\}$.

3. Correlation coefficient for two vague multi sets

Definition 3.1: Let $X = \{x_1, x_2, x_2, ...x_n\}$ be the finite universe of discourse and $A = \{< t_A^j(x_i), 1-f_A^j(x_i) > | x_i \in X\}$, $B = \{< t_B^j(x_i), 1-f_B^j(x_i) > | x_i \in X\}$ be two vague multi sets consisting truth and false membership functions. Then the correlation coefficient of A and B

$$\rho_{VMS}(A, B) = \frac{C_{VMS}(A, B)}{\sqrt{C_{VMS}(A, A) * C_{VMS}(B, B)}} \quad ....(1)$$

Where

$$C_{VMS}(A, B) = \frac{1}{\xi} \sum_{j=1}^{\xi} \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ t_A^j(x_i) t_B^j(x_i) + 1 - f_A^j(x_i) 1 - f_B^j(x_i) \right\} \right]$$

$$C_{VMS}(A, A) = \frac{1}{\xi} \sum_{j=1}^{\xi} \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ t_A^j(x_i) t_A^j(x_i) + 1 - f_A^j(x_i) 1 - f_A^j(x_i) \right\} \right]$$

$$C_{VMS}(B, B) = \frac{1}{\xi} \sum_{j=1}^{\xi} \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ t_B^j(x_i) t_B^j(x_i) + 1 - f_B^j(x_i) 1 - f_B^j(x_i) \right\} \right]$$

Proposition 3.2: The defined correlation measure between Vague multi set A and B satisfies the following properties:

$(i)\ 0 \le \rho_{VMS}(A, B) \le 1$

$(ii)\ \rho_{VMS}(A, B) = 1$ *if and only if* $A = B$

$(iii)\ \rho_{VMS}(A, B) = \rho_{VMS}(B, A)$

Proof:

(i) $0 \le \rho_{VMS}(A, B) \le 1$

As the truth and false membership function of the vague multi set lies between 0 and 1, $\rho_{VMS}(A, B)$ also lies between 0
and 1

(ii) $\rho_{VMS}(A, B) = 1$ if and only if A=B

a. Let the two vague multi sets A and B (i.e A=B). Hence for any
$t_A^j(x_i) = t_B^j(x_i),\ 1 - f_A^j(x_i) = 1 - f_B^j(x_i)$ then

$$C_{VMS}(A, A) = C_{VMS}(B, B) = \frac{1}{\xi} \sum_{j=1}^{\xi} \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ t_A^j(x_i) t_A^j(x_i) + 1 - f_A^j(x_i) 1 - f_A^j(x_i) \right\} \right] \text{ and}$$

$$C_{VMS}(A, B) = \frac{1}{\xi} \sum_{j=1}^{\xi} \left[\frac{1}{n} \sum_{i=1}^{n} \left\{ t_A^j(x_i) t_B^j(x_i) + 1 - f_A^j(x_i) 1 - f_B^j(x_i) \right\} \right]$$

$$= \frac{1}{\xi}\sum_{j=1}^{\xi}\left[\frac{1}{n}\sum_{i=1}^{n}\left\{t_A^j(x_i)t_A^j(x_i)+1-f_A^j(x_i)1-f_A^j(x_i)\right\}\right]$$

$= C_{VMS}(A,A)$

Hence

$$\rho_{VMS}(A,B)=\frac{C_{VMS}(A,B)}{\sqrt{C_{VMS}(A,A)*C_{VMS}(B,B)}}=\frac{C_{VMS}(A,A)}{\sqrt{C_{VMS}(A,A)*C_{VMS}(A,A)}}=1$$

b. Let the $\rho_{VMS}(A,B)=1$. Then, the unit measure is possible only if

$$\frac{C_{VMS}(A,B)}{\sqrt{C_{VMS}(A,A)*C_{VMS}(B,B)}}=1$$ this refers that

$t_A^j(x_i)=t_B^j(x_i),\ 1-f_A^j(x_i)=1-f_B^j(x_i)$ for all i, j values. Hence A=B.

(iii) If $\rho_{VMS}(A,B)=\rho_{VMS}(B,A)$, it obvious that

$$\frac{C_{VMS}(A,B)}{\sqrt{C_{VMS}(A,A)*C_{VMS}(B,B)}}=\frac{C_{VMS}(B,A)}{\sqrt{C_{VMS}(A,A)*C_{VMS}(B,B)}}=\rho_{VMS}(B,A)$$ as

$$C_{VMS}(A,B)=\frac{1}{\xi}\sum_{j=1}^{\xi}\left[\frac{1}{n}\sum_{i=1}^{n}\left\{t_A^j(x_i)t_B^j(x_i)+1-f_A^j(x_i)1-f_B^j(x_i)\right\}\right]$$

$$=\frac{1}{\xi}\sum_{j=1}^{\xi}\left[\frac{1}{n}\sum_{i=1}^{n}\left\{t_B^j(x_i)t_A^j(x_i)+1-f_B^j(x_i)1-f_A^j(x_i)\right\}\right]$$

$=C_{VMS}(B,A)$.

Illustrative Example For Brick Selection: Suppose that the administration of an authority is going to construct a building. For this purpose it is necessary to collect quality bricks from various brick fields. After initial screening, four types of bricks (alternatives) A_1, A_2, A_3, A_4 remain for further selection. A selection committee is formed with decision makers or experts D_1, D_2, D_3. Five criteria of bricks obtained from experts opinions, namely solidity C_1, size and shape C_2, strength of brick C_3, cost of brick C_4 and carrying cost C_5 are considered for selection criteria. Selection process is divided into three phases represented by vague multi sets.

Table 1: The relation between experts and criteria

Q	C_1	C_2	C_3	C_4	C_5
D_1	[0.5,0.7] [0.3,0.5] [0.1,0.6]	[0.3,0.7] [0.3,0.9] [0.5,0.8]	[0.2,0.7] [0.6,0.9] [0.1,0.8]	[0.5,0.6] [0.3,0.8] [0.4,0.6]	[0.1,0.5] [0.5,0.8] [0.1,0.3]
D_2	[0.3,0.4] [0.4,0.7] [0.5,0.6]	[0.3,0.8] [0.2,0.3] [0.5,0.7]	[0.6,0.7] [0.1,0.2] [0.6,0.8]	[0.5,0.6] [0.1,0.6] [0.2,0.8]	[0.4,0.8] [0.1,0.5] [0.6,0.7]
D_3	[0.8,0.9] [0.5,0.6] [0.7,0.8]	[0.2,0.6] [0.1,0.2] [0.1,0.5]	[0.2,0.7] [0.3,0.4] [0.1,0.7]	[0.4,0.8] [0.6,0.7] [0.2,0.6]	[0.3,0.7] [0.4,0.6] [0.4,0.8]

Table 5: The relation between criteria and Alternatives

	A_1	A_2	A_3	A_4
C_1	[0.4,0.5]	[0.5,0.8]	[0.1,0.3]	[0.6,0.9]
C_2	[0.1,0.3]	[0.7,0.8]	[0.2,0.9]	[0.7,0.8]
C_3	[0.6,0.8]	[0.2,0.5]	[0.5,0.6]	[0.5,0.7]
C_4	[0.2,0.5]	[0.6,0.7]	[0.4,0.7]	[0.6,0.8]
C_5	[0.3,0.5]	[0.2,0.7]	[0.3,0.8]	[0.2,0.6]

Using equation 1, the correlation measure between table 1 and table 2 is presented in table 3

Table 3: The correlation measure between experts and Alternatives

	A_1	A_2	A_3	A_4
D_1	0.7515	0.9274	0.9141	**0.9354**
D_2	0.8758	0.8544	0.9216	**0.9391**
D_3	0.8948	0.9012	0.8336	**0.9068**

The highest correlation measure from the table 3 gives the result of most appropriate brick. Here the most appropriate brick is A_4.

Conclusion: In this study, we defined the correlation coefficient of vague multi sets and presented an application of correlation measure in decision making problem for brick selection. Therefore, in future the proposed approach can be used for dealing with multi-criteria decision making problems such as project evaluation, manufacturing system and many other areas of management decision problems.

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