DUST-ACOUSTIC SOLITARY WAVES IN DUSTY PLASMA WITH TWO-TEMPERATURE IONS AND DUST CHARGE FLUCTUATIONS.

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Abstract: The effect of variable dust charge, with two temperature ions on small-amplitude dust-acoustic solitary waves and arbitrary-amplitude dust-acoustic solitary waves are investigated. Reductive perturbation method is employed to study the small but finite amplitude dust acoustic waves. It is found that small amplitude solitary waves with Φ > o and Φ < o both exist. Variation of critical Mach number and Sagdeev potential are investigated in case of arbitrary amplitude dust-acoustic waves. In this case we also found that arbitrary amplitude dust-acoustic waves with Φ > o and Φ < o both exist. The properties of the dust-acoustic solitary waves accounting for dust grain charge fluctuations are examined.

Introduction: The study of the dynamics of dust contaminated plasmas has recently received considerable interest not only due to their occurrence in space and laboratory plasmas, but also for its vital role in understanding different collective processes in space physics as well as in laboratory plasmas.A lot of studies of the dust-acoustic waves assumed a constant charge on the dust grains modifies the existing plasma wave spectra, whereas the dust charge dynamics introduces new eigen-modes in dusty plasma. Since the dust charge variation with parameters such as electrostatic plasma potential, electron and ion densities would influence the collective characteristics of the plasma, the effect of dust charge variation is of crucial importance in understanding dusty plasma waves.

The dust acoustic waves have been theoretically predicted by Rao et *al.*[1] in a multicomponent collisionless dusty plasma whose constituents are the electrons, ions and negetively charged dust grains. Mamun et *al.*[2] have used the quasi-potential to study the nonlinear dust acoutic waves. The highly charged massive dust grains present in a dusty plasma may exhibit charge fluctuations due to a variety of intrinsic plasma charging mechanisms. W.F.El-Taibany and I. Kourakis[3] studied the oblique modulational instability of dust acoustic(DA) waves in an unmagnetised warm dusty plasma with nonthermal ions and taking into account dust grain charge variation. The effects of dust temperature,dust charge variation, ion deviation from Maxwellian equilibrium (nonthermality) and constituent species, concentration on the modulational instability of DA waves are examined. They found that these parameters modify significantly the oblique modulational instability domain in the k-θ plane. Y − N. Nejoh[4] investigated the effect of the dust charging and the influence of the ion density and temperature on electrostatic nonlinear ion waves in a dusty plasma having trapped electrons by numerical calculation. It has been shown that the characteristics of the dust charge number sensitively depend on the electrostatic potential, Mach number, trapped electron temperature, ion density and temperature . An increase of the ion temperature decreases the dust charging rate and the propagation speed of ion waves. It turns out that a decrease of the trapped electron temperature increases the charging rate of dust grains. He also found that the existence of ion waves sensitively depends on the ion to electron density ratio. Xie et*al.*[5]derived dust acoustic solitons with varying dust charge and they showed that only rarefactive solitons (and solitary waves) exist when the Mach number lies within an appropriate regime, depending on the system parameters. The effect of the temperature of low temperature ions on the system parameters, e.g., angular frequency, group velocity,dispersion coefficient, is investigated by El − Labany et *al.*[6]on a cold plasma system consisting of dust grains with varying dust charge, electrons, and two species of ions with different temperatures.

Basic Equations: We consider a four-component collisionless unmagnetised dusty plasma system consisting of electrons, massive dust charge grains carrying negetive charge and two temperature ions, taking the charge fluctuation into account. The dynamics of low phase velocity one dimensional DAS waves is governed by

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d\ u_d\ )}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d\ \frac{\partial (u_d\ )}{\partial x} = \frac{\partial \Phi}{\partial x} \qquad (2)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = n_d + \mu_e n_e - \mu_{il} n_{il} - \mu_{ih} n_{ih} \qquad (3)$$

where the subscripts e,d,ih and il are denote the electrons, dust grains, high and lowtemperature ions respectively.We define the effective temperature $\boldsymbol{T_{eff}}$ by the relation

$$\boldsymbol{T_{eff}} = Z_{d0}n_{d0}\left(\frac{\boldsymbol{n_{e0}}}{\boldsymbol{T_e}} + \frac{\boldsymbol{n_{il0}}}{\boldsymbol{T_{il}}} + \frac{\boldsymbol{n_{ih0}}}{\boldsymbol{T_{ih}}}\right)^{-1},$$

where $\boldsymbol{T_e}, \boldsymbol{T_{il}}, \boldsymbol{T_{ih}}$ and n_{so}(s=e,d,il,ih) are the temperatures of electrons, temperature of low temperature ions, temperature of high temperature ions and unperturbed density of species 's' respectively. Here $n_d$is the particle number density normalised by n_{do}, u_d is the dust fluid velocity normalised by dust-acoustic speed C_d $=\left(\frac{Z_{d0}\,T_{eff}}{m_d}\right)^{1/2}$and Φ is the electrostatic wave potential normalised by T_{eff}/e. The time and space variables are in units of the dust plasma period$\left(\omega_{pd}\right)^{-1} = \left(m_d/4\pi {Z_{d0}}^2\, n_{d0}\, e^2\right)^{1/2}$and the Debye length λ_{Dd}=$\left(T_{eff}/4\pi Z_{d0}\, n_{d0}\, e^2\right)^{1/2}$respectively.

The distribution of ions and electrons are assumed to have Boltzmann distribution and in dimensionless form they can be expressed as[5]

μ_e=$n_{e0}/(Z_{d0}n_{d0})$, μ_{il}=$n_{il0}/(Z_{d0}n_{d0})$, μ_{ih}=$n_{ih0}/(Z_{d0}n_{d0})$.

The normalized electron and ion number densities are respectively

$$n_e = e^{(\beta_1 s\phi)} \quad (4)$$

$$n_{il} = e^{(-s\phi)} \quad (5)$$

$$n_{ih} = e^{(-\beta s\phi)} \quad (6)$$

Where

$$\beta_1 = \boldsymbol{T_{il}}/\boldsymbol{T_e}, \beta_2 = \boldsymbol{T_{ih}}/\boldsymbol{T_e}, \beta = \boldsymbol{T_{il}}/\boldsymbol{T_{ih}}, \ s=\frac{Z_{d0}\, n_{d0}\, \boldsymbol{T_e T_{ih}}}{n_e\, \boldsymbol{T_{il}T_{ih}}+n_{il0}\, \boldsymbol{T_e T_{ih}}+n_{ih0}\, \boldsymbol{T_e T_{il}}}\ .$$

Let $\delta_1 = \frac{n_{il0}}{n_{e0}}$ and $\delta_2 = \frac{n_{ih0}}{n_{e0}}$.

So we find that

$$\mu_e=1/(\delta_1+\delta_2\text{-}1),\ \mu_{il}=\delta_1/(\delta_1+\delta_2\text{-}1),\ \mu_{ih}=\delta_2/(\delta_1+\delta_2\text{-}1),\ s=\frac{\delta_1+\delta_2-1}{\delta_1+\beta\delta_2+\beta_1}.$$

Where equations (1)-(6) are analysed for two cases. First we study small but finite amplitude dust acoustic solitary waves for which we use the reductive perturbation method and later we focus on arbitrary amplitude dust acoustic solitary waves for which we use the Sagdeev potential approach.

Small-amplitude DAS waves: In this section, we study the small but finite amplitude DAS waves by employing the reductive perturbation and stretched coordinates

$\xi = \epsilon^{1/2}(x-\lambda t)$, $\eta = \epsilon^{3/2}t$

where ϵ is a smallness and λ is the soliton speed (normalized by C_d. We can then expand the variables n_d, u_d and ϕ about the unperturbed states in power series of ϵ as

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2\, n_d^{(2)} + \epsilon^3\, n_d^{(3)} + \ldots \quad (7)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2\, u_d^{(2)} + \epsilon^3\, u_d^{(3)} + \ldots \quad (8)$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\, \phi^{(2)} + \epsilon^3\, \phi^{(3)} + \ldots \quad (9)$$

Substituting (4)-(9) into the equations (1) - (3) and equating the coefficients of same power of ϵ. To lowest order in ϵ, equations (1) – (3) give

$$u_d^{(1)} = -\phi^{(1)}/\lambda,\ n_d^{(1)} = -\phi^{(1)}/\lambda^2$$

and $\lambda = 1/\sqrt{s(\beta_1\,\mu_e + \mu_{il} + \beta\mu_{ih})}$.

Now equating the next higher order in ϵ, we obtain a set of equations

$$-\lambda\frac{\partial n_d^{(2)}}{\partial\xi} + \frac{\partial u_d^{(2)}}{\partial\xi} + \frac{\partial n_d^{(1)}}{\partial\eta} + \frac{\partial}{\partial\xi}\left(u_d^{(1)}\, n_d^{(1)}\right) = 0 \quad (10)$$

$$-\lambda\frac{\partial u_d^{(2)}}{\partial\xi} + u_d^{(1)}\frac{\partial u_d^{(1)}}{\partial\xi} + \frac{\partial u_d^{(1)}}{\partial\eta} = \frac{\partial\phi^{(2)}}{\partial\xi} \quad (11)$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = n_d^{(2)} + \mu_e\left[\beta_1 s\phi^{(2)} + \frac{1}{2}\beta_1^2 s^2\{\phi^{(1)}\}^2\right] + \mu_{il}\left[s\phi^{(2)} - \frac{1}{2}s^2\{\phi^{(1)}\}^2\right] + \mu_{ih}\left[\beta s\phi^{(2)} - \frac{1}{2}\beta^2 s^2\{\phi^{(1)}\}^2\right] \quad (12)$$

Combining the equations (10) – (12), we obtain

$$\frac{\partial\phi^{(1)}}{\partial\eta} + a_s\,\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + b_s\,\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0 \quad (13)$$

Which is the KdV equation with the coefficients

$$a_s = \frac{\lambda^3}{2}\left\{(\mu_{il}\, s^2 + \mu_{ih}\,\beta^2 s^2 - \mu_e\,\beta_1^2 s^2) - \frac{3}{\lambda^4}\right\} \quad (14)$$

$$b_s = \frac{\lambda^3}{2} \quad (15)$$

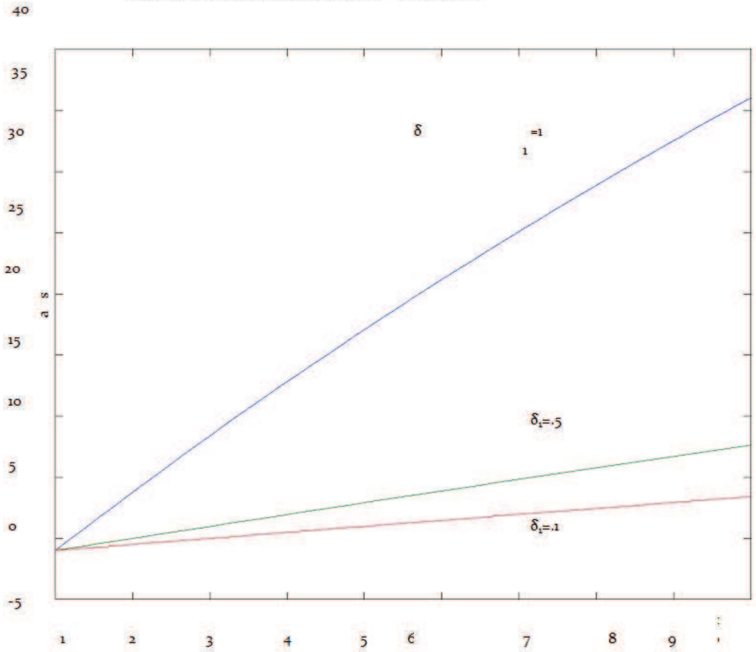
The stationary solution of the KdV equation (13) is obtained transforming the independent variables ξ and η to $\zeta = (\xi - c\eta)$ and $\eta = \eta$, where c is a constant speed normalized by C_d and the boundary conditions for localized perturbations, $\phi \to 0$, $\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} \to 0$ at $\zeta \to \pm\infty$. Accordingly the solution of the equation (13) is of the form[7]

$$\phi^{(1)} = \frac{3C}{a_s}\,\text{sech}^2\left[\frac{\xi - c\eta}{\sqrt{4b_s/C}}\right] \quad (16)$$

Where the amplitude and the width are given by $\frac{3C}{a_s}$ and $\sqrt{4b_s/C}$ respectively. Equation (16) shows that as C>o, the small amplitude solitary waves with ϕ > o exists if $a_s > 0$ and small amplitude solitary waves with ϕ < o exists if $a_s < 0$. *Now*,

$$a_s = \frac{[(\delta_1+\delta_2-1)\{\delta_1+\delta_2\,\beta^2-\beta_1^2\}-3(\delta_1+\beta\delta_2+\beta_1^2)]}{2(\delta_1+\beta\delta_2+\beta_1)^2} \quad (17)$$

Figure 1: Variation of a_s against δ_2 for β = .001, β_1=.0001



Arbitrary amplitude DAS waves: To study time independent arbitrary amplitude DAS waves we take all the dependent variables depend only on a single variable ξ = (x-Mt), where ξ is normalized by $(\lambda)_{Dd}$ and the mach number M= (solitary wave)/C_d.The boundary conditions are $\phi \to$o, $n_d \to 1$, $u_d \to 0$ at $\xi \to \pm\infty$. The equations (1) – (3) reduce to

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0 \quad (18)$$

Where the Sagdeev potential V (ϕ) is given by

$$V(\phi) = \frac{\mu_{il}}{s}\left(1-exp^{-s\phi}\right) + \frac{\mu_{ih}}{\beta s}\left(1-exp^{-\beta s\phi}\right) + \frac{\mu_{ih}}{\beta_1 s}\left(1-exp^{-\beta_1 s\phi}\right) + M^2\left[1-\left(1+\frac{2\phi}{M^2}\right)^{1/2}\right] \quad (19)$$

It is clear from the equation (19) that V ($\phi) = d\,V(\phi)/d(\phi) = 0$ at ϕ =o. Therefore, solitary wave solutions of the equation (18) *exist*[7] if

(i) $\left(\frac{d^2V}{d\phi^2}\right)_{\phi=0} < 0$ *so that the* fixed point at the origin is unstable and

(ii) $\left(\frac{d^2V}{d\phi^2}\right)_{\phi=0} > (<)0$ for solitary waves with $\phi > (<)0$. the nature of these solitary waves, whose amplitude tends to zero as the Mach number M tends its critical value, can be obtained by expanding the Sagdeev potential V(ϕ) to third order in a Taylor series in ϕ. The critical Mach number is that which corresponds to the vanishing of the quadratic term. At the same time, if the cubic term is negative, there is a potential well on the negative side and if the cubic term is positive, there is a potential well on the positive side. Therefore, by expanding the

Sagdeev potential V(ϕ) around the origin, the critical Mach number at which the second derivative changes sign can be obtained as

$$M_c = \sqrt{\frac{\delta_1+\delta_2-1}{\delta_1+\beta\delta_2+\beta_1}} \qquad (20)$$

At this critical value of M the cubic term V(ϕ) can be expressed as

$$\frac{[(\delta_1+\delta_2-1)\{\delta_1+\beta^2\delta_2-{\beta_1}^2\}-3(\delta_1+\beta\delta_2+\beta_1)^2]}{3(\delta_1+\beta\delta_2+\beta_1)^2} \qquad (21)$$

Which reveals that (i) arbitrary amplitude solitary waves with $\phi >$ o exist if cubic term is positive and (ii) arbitrary amplitude solitary waves with $\phi < 0$ exist if cubic term is negative. The fig.2 shows the variation of the critical Mach number M_c against δ_2 for different values of δ_1. The figure shows that the critical Mach number decreases with δ_1 and increases with δ_2.

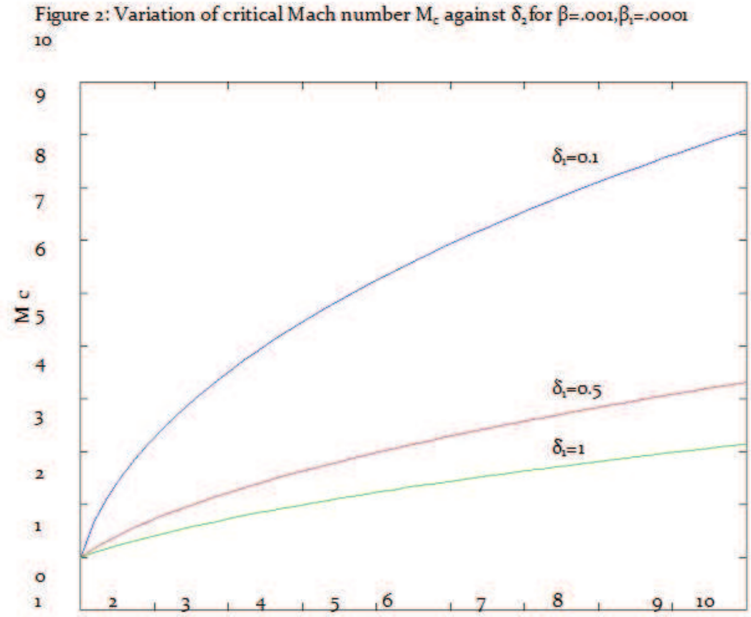
It is of interest to examine whether or not there exist upper limit of M for which dust-acoustic solitary waves with $\phi >$ o and $\phi < 0$ exist. The upper limit of M can be found by the condition V (ϕ_c) ≥ 0, *where* $\phi_c = -M^2/2$ is the minimum value of ϕ for which the dust number density n_d is real. Thus, the upper limit of M is that value of M for which S_m=o, where

$$S_m = \frac{\mu_{il}}{s} + \frac{\mu_{ih}}{\beta s} + \frac{\mu_e}{\beta_1 s} + M^2 - \frac{\mu_{il}}{s} e^{\frac{M^2 s}{2}} + \frac{\mu_{ih}}{\beta s} e^{\frac{M^2 \beta s}{2}} - \frac{\mu_e}{\beta_1 s} e^{\frac{M^2 \beta_1 s}{2}}$$

Fig. 3(a) to 3(d) shows the variation of S_m against M for different values of δ_1. It is clear from the figures that the upper limit of M increases with δ_1.

We have also analysed the Sagdeev potential V(ϕ) for different values of M. The behavior of the Sagdeev potential V (ϕ) for different values of M. The behavior of the Sagdeev potential V(ϕ) are shown in Fig. 4. We see that the dust acoustic solitary waves with $\phi >$ o and $\phi < 0$ both exist when Mach number M lies between .5 and 1.5 .

Figure 2: Variation of critical Mach number M_c against δ_2 for β=.001,β_1=.0001



Effect of dust charge fluctuations: We consider a four-component collisionless unmagnetised dusty plasma system consisting of electrons, massive dust charge grains carrying negative charge and two temperature ions, taking the charge fluctuation into account. The basic equations describing the system in dimensionless form are given *by*[5]

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d\ u_d\ )}{\partial x} = 0 \qquad (22)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial (u_d\ )}{\partial x} = Z_d \frac{\partial \Phi}{\partial x} \qquad (23)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = Z_d n_d + \mu_e n_e - \mu_{il} n_{il} - \mu_{ih} n_{ih} \qquad (24)$$

Where Z_d is normalized by its equilibrium value Z_{d0}.Equations (4), (5),(6),(21),(22) and (23) are completed by the charging equation, which can be expressed in terms of normalized variables as[6]

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial t}\right) Z_d = - (I_e + I_{il} + I_{ih}) \quad (25)$$

Figure 3.a – 3.b : Variation of S_m against M for δ_2=10, β_1=.05,β = .001

Figure 3.a

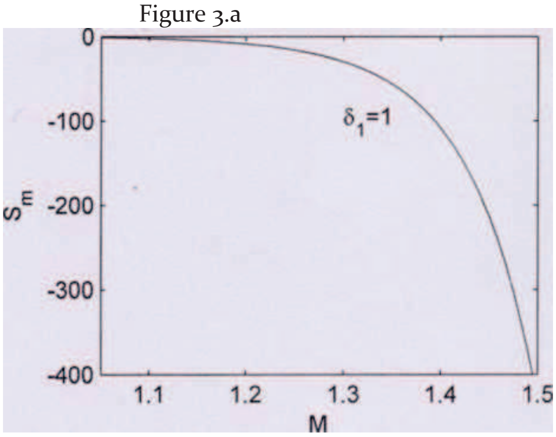


Figure 3.b

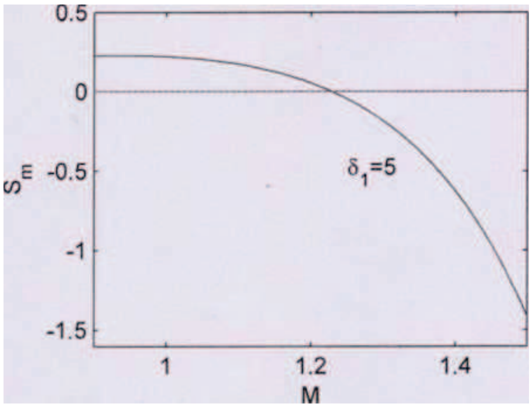


Figure 3.c

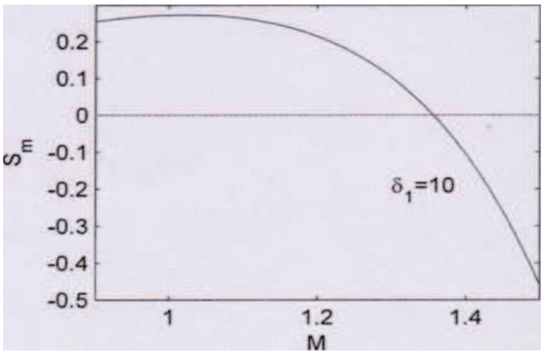
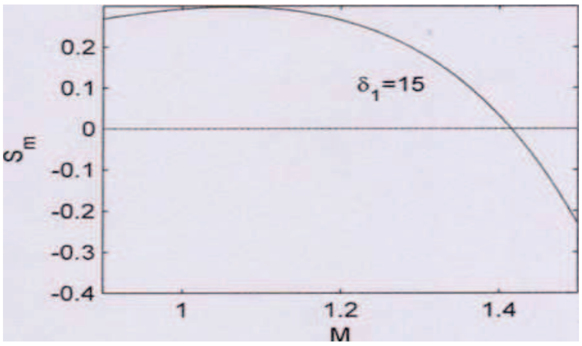


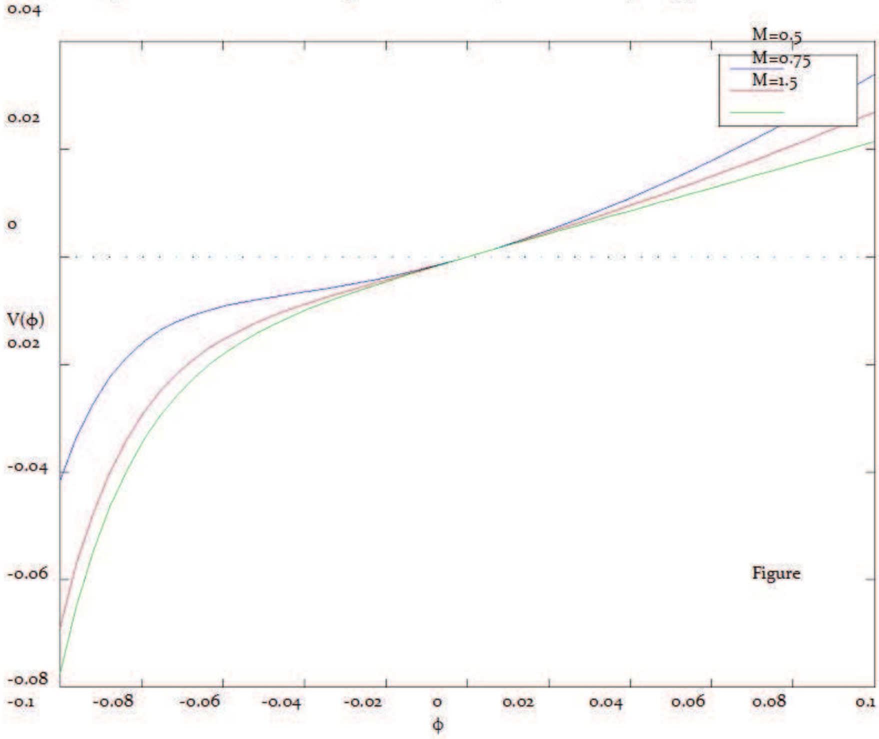
Figure 3.d



The charging currents originate from the electrons and ions reaching the dust grain surface. The characteristic time for dust motion is of the order of tens of milliseconds for micrometersized grains, while the dust charging time is typically of the order of tens of nanoseconds[5,6,7]. Therefore, on the hydrodynamic time scale, the dust charge can quickly reach local equilibrium at which the currents from the electrons and ions to the dust are balanced. The current balance equation reads[5,6,9]

$$I_e + I_{il} + I_{ih} \approx 0 \quad (26)$$

Figure 4:The behaviour of the Sagdeev Potential V(ϕ) for δ_1=0.1,δ_2=10,β_1=0.05,β=0.001



Assuming that the streaming velocities of electrons and ions are much smaller than their thermal velocities, according to the well-known orbit motion-limited probe model, the noemalised currents of electrons and the low and high temperature ions for spherical grains of radius r_d are given by

$$I_e = -\pi {r_d}^2 n_e \, \mu_e \, e \left(\frac{8T_e}{\pi m_e}\right)^{\frac{1}{2}} \exp\left(\frac{e\phi_d}{T_e}\right) \quad (27)$$

$$I_{ih} = -\pi {r_d}^2 n_{ih} \, \mu_{ih} \, e \left(\frac{8T_{ih}}{\pi m_i}\right)^{\frac{1}{2}} \left(1 - \frac{e\phi_d}{T_{ih}}\right) \quad (28)$$

$$I_{il} = -\pi {r_d}^2 n_{il} \, \mu_{il} \, e \left(\frac{8T_{il}}{\pi m_i}\right)^{\frac{1}{2}} \left(1 - \frac{e\phi_d}{T_{il}}\right) \quad (29)$$

Where ϕ_d is the dust charge grain potential with respect to the plasma potential and

$$\phi_d = -\frac{Z_d e}{r_d} \quad (30)$$

Using the current balance equation (26) as well as substituting n_e, n_{ih}, n_{il} We have

$$e^{-s\phi}\delta_1\sqrt{\beta_1}\left(1 - \frac{e\phi_d}{T_{il}}\right) + e^{-s\phi\beta}\delta_2\sqrt{\beta_2}\left(1 - \frac{e\phi_d}{T_{ih}}\right) - e^{-\beta_1 s\phi}\sqrt{\mu_i}\exp\left(\frac{e\phi_d}{T_e}\right) = 0 \quad (31)$$

Where,

$$\mu_i = \frac{m_i}{m_e}.$$

Expanding the exponential term and keeping only the first two terms in the expansion, we have

$$\phi = \frac{A - B\phi_d}{C - D\phi_d} \quad (32)$$

Where

$$A = \left(\delta_1\sqrt{\beta_1} + \delta_2\sqrt{\beta_2} - \sqrt{\mu_i}\right) \quad 33)$$

B= $\left(\frac{e\delta_1\sqrt{\beta_1}}{T_{il}} + \frac{e\delta_2\sqrt{\beta_2}}{T_{ih}} + \frac{e\sqrt{\mu_i}}{T_e}\right)$ (34)

C= $\left(s\delta_1\sqrt{\beta_1} + s\beta\delta_2\sqrt{\beta_2} + s\beta_1\sqrt{\mu_i}\right)$ (35)

D= $\left[s\left\{\frac{e\delta_1\sqrt{\beta_1}}{T_{il}} + \frac{e\beta\delta_2\sqrt{\beta_2}}{T_{ih}} - \frac{e\beta_1\sqrt{\mu_i}}{T_e}\right\}\right]$ (36)

To reduce the equations (22) – (24) into a single equation, we introduce a dimensionless $function$ [7]

$\psi = -\int_0^{\phi} Z_d \, d\phi$ (37)

Using the equations (30) and (32), the equation (37) can be expressed as

$\psi = \frac{AD-BC}{D^2}\left[\frac{r_d}{e}\ln\left(1+\frac{DZ_d e}{Cr_d}\right) - \frac{DZ_d r_d}{Cr_d + DZ_d e}\right]$ (38)

To study the arbitrary amplitude time-independent Dust acoustic solitary waves, we make all dependent variables depend only on a single variable ξ by using the transformation

ξ = $\left(1-\frac{2\psi}{M^2}\right)^{1/2}$ (39)

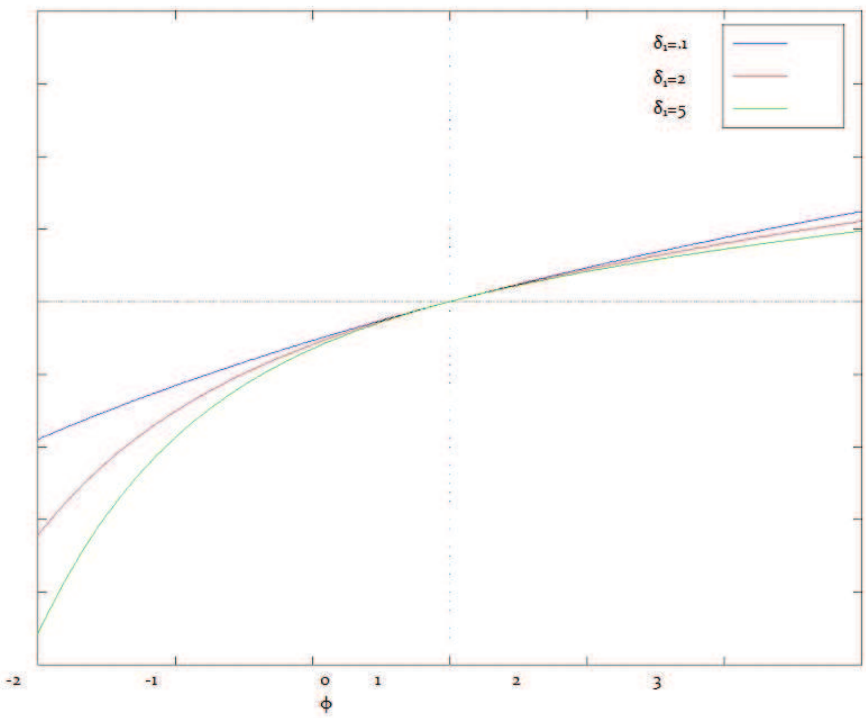
Substituting the equations (4), (5), (6) and (38) into the equation (24) and integrating the resulting equation we obtain

$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2$ + V (ϕ, Z_d) =0 (40)

Where the Sagdeev potential V (ϕ, Z_d) is of the form

V (ϕ, Z_d)= $\frac{\mu_e}{\beta_1}(1-e^{\beta_1\phi}) + \frac{\mu_{ih}}{\beta}(1-e^{\beta\phi}) + \mu_{il}(1-e^{-\phi}) + M^2\left[\left(1-\frac{2\psi}{M^2}\right) - 1\right]$ (41)

: behaviour of the Sagdeev Potential V(ϕ)for δ_2 =10,T_{eil} =1.6x10^{-11}erg,T =1.6x10^{-14}erg,T_{ih} =14.4x10^{-14}erg,M=.5,r_d =5



:Solitary wave solutions of equation (40) for δ_2=10,T_e=1.6x10^{-11}erg, T_{il}=1.6x10^{-14}erg,T_{ih}=14.4x10^{-14}erg,M=.5,r_d

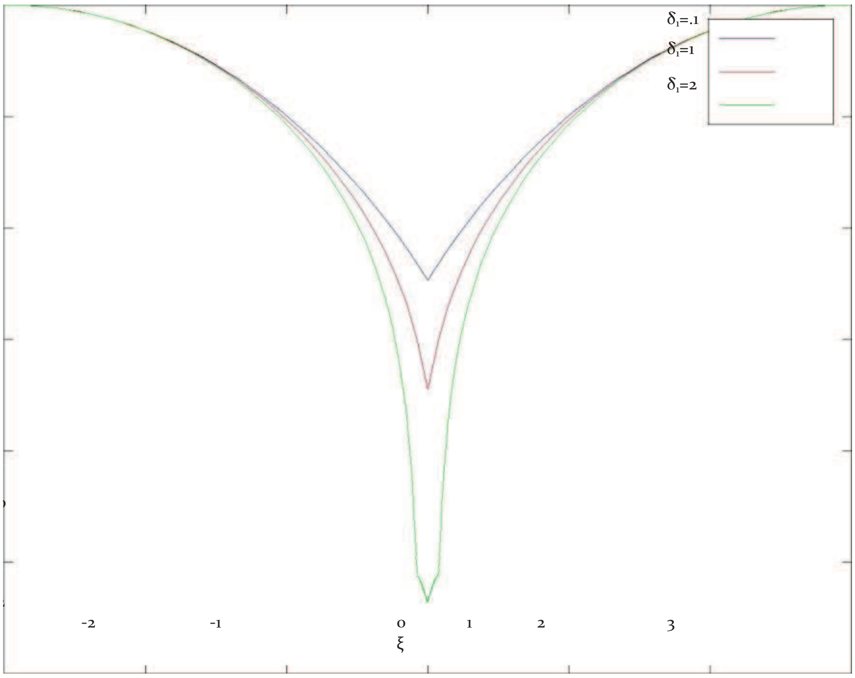


Fig. 5 shows the variation of the Sagdeev potential V (ϕ, Z_d) against ϕ for different values of δ_1. We notice that the Dust –acoustic solitary waves accounting for dust charge fluctuations exist both for $\phi > 0$ *and* $\phi < 0$.

The soliton profiles, that exist in a dusty plasma with fluctuating charges on the dust grain surface, are obtained by numerically integrating equation (40) and the structure of the profiles are typical inverted bell shaped as shown in fig. 6.

Conclusions: In this paper, I have studied the effects of variable dust charge on the small but finite amplitude DAS waves and arbitrary amplitude DAS waves in dusty plasmas having two temperature ions. It ia found that small but finite amplitude DAS waves with $\phi > 0$ *and* $\phi < 0$ both exist. Furthermore, as the dust fluid speed u_0 increases, the amplitude of the DAS waves increases but their width decreases. In case of time independent arbitrary amplitude DAS waves, the dust-acoustic solitary waves with $\phi > 0$ *and* $\phi < 0$ both exist when the Mach number lies between 0.5 and 1.5 . The effect of dust grain charge fluctuation on the arbitrary amplitude dust acoustic waves are also investigated and the structure of the DAS profiles are obtained.

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