PRODUCTION INVENTORY MODEL CONTROLLABLE DETERIORATION RATE AND MULTIPLE -MARKET DEMAND WITH PRESERVATION TECHNIQUE

DHARAMENDER SINGH AND BANI MUKHERJEE

Abstract: In this paper, we have discussed an inventory model for developed a deteriorating items with constant holding cost and demand rate is constant. The motive of this paper is to develop a production-inventory model of instantaneous deteriorating item multiple –market demand with preservation technology and controllable deterioration rate. Highlights of the model are solved analytically by minimizing the total inventory cost. This model is decorated by numerical example.

Keywords: Deteriorating items, Inventory, Multiple Market demand, Preservation technology.

Introduction: Inventory is the most interesting and research topic of the production and operation management. We think everyone agrees that the golden age of inventory research was in the 1950s. This was the time when conceptual and mathematical models of inventories were first formulated but basically very simple approach Harris (1915) developed first inventory model, Economic Order Quantity (EOQ), which has generalized by Wilson (1934) and he gave a formula to obtain EOQ. Many researchers have discussed inventory model to minimize the total cost and maximize the profit. Sharma, P. K. (2016), Survey of Inventory Models with Inflation. Inventory of deteriorating items first studied by Whit-in (1957).Deterioration is defined as decay or damage such that the item cannot be used for its original purposes. The effect of deterioration is very important in many inventory systems. Food item, Pharmaceuticals and radioactive substances are example of items. Mishra, V. K. (2013) developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. He, Y., & Huang, H. (2013), discussed an optimizing inventory and pricing policy for seasonal deteriorating products with preservation technology investment In this paper, we have analysed an inventory system order level lot size model for deteriorating items under constant demand and constant holding cost and improve the customer service level and increase business competitiveness. The article ends with some concluding remarks and scope of future research.

Assumptions and Notations: The mathematical model is based on the following notations and assumptions.

Notations: The manufacturer's cost parameters are as follows:

(a) c_p Unit production cost of deteriorating item.

(b) p Production rate.

(c) h_p Holding cost finished products.

(d) θ_2 Constant deterioration rate of finished products.

(e) θ_p Resultant deterioration rate $\theta_p = \theta_2 e^{-\alpha\xi}$.

(f) k_0 Setup cost.

(g) $I_i(t)$ Inventory level in the i^{th} interval $i = 1, 2, 3 \ldots u, u+1, \ldots T$.

(h) ξ Preservation technology(PT) cost is reduces to deterioration rate $\xi > 0$

Assumptions: This following criterion has been fulfilled to formulate the problem.

a. Production rate p is constant and greater demand rate.
b. A single product, a single manufacturer and multiple –markets demand.
c. The planning horizon is finite.
d. Lead time is zero or negligible.
e. Demand rate is constant and known.

Mathematical Modal and Solution: Each cycle starts with first opening market and ends with the last closing market. Demand rate is different for each market. Each cycle starts with first opening market and ends with the last closing market. Demand rate is different for each market. At the beginning of each cycle all the market's demand piles up in order time. In this model, there is no demand and no production at initial time $t = 0$, then the production starts with zero level. As production continues inventory begins to pile up continuously and fulfil the demand and up to the time T_1 when the production stopped. The inventory also declines till the levels of inventory reaches zero after time T_1 up to the time $t_u = T$ due to both of consumption and deterioration.

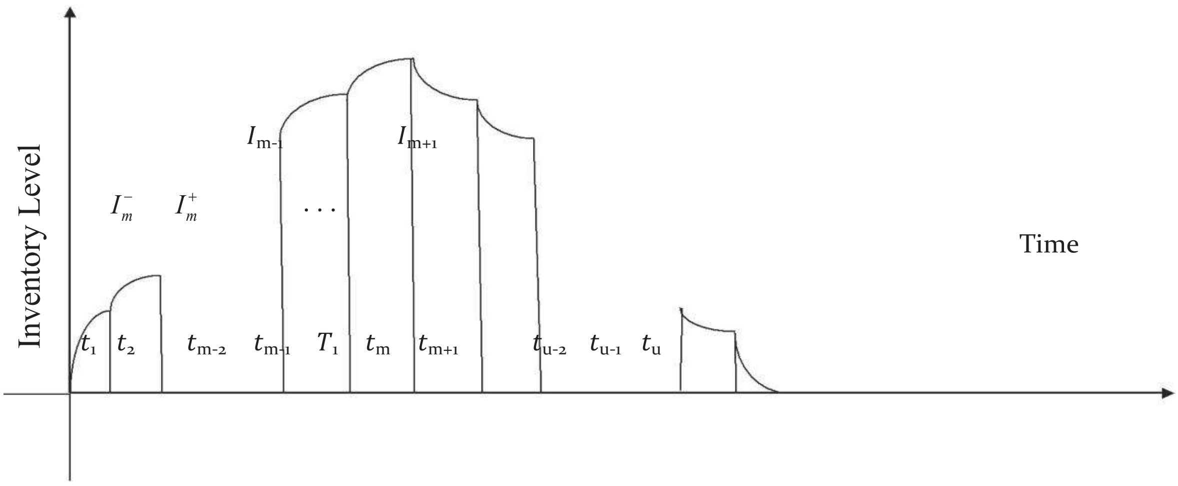

Figure 1: Geometry of the problem

Manufacture's finished products inventory model: Phase-1: The rate of change of inventory level during positive stock period $[t_{k-1}, t_k]$ and $[t_{m-1}, T_1]$ is governed by the following differential equations.

$$\frac{dI_k(t)}{dt} + \theta_p I_k(t) = p - d_k \ ; \ t_{k-1} \le t \le t_k \qquad (1)$$

k=1,2,3, . . . ,m-1

$$\frac{dI_m^-(t)}{dt} + \theta_p I_m^-(t) = p - d_m \ ; t_{m-1} \le t \le T_1 \qquad (2)$$

With boundary condition

$$I_1(0) = 0, I_k(t_{k-1}) = I_{k-1}(t_{k-1}) \ \& I_m^-(t_{k-1}) = I_{m-1}(t_{k-1}) \qquad (3)$$

Phase-2: The rate of change of inventory during positive stock period $[T_1, t_m]$ and $[t_{j-1}, t_j]$ is governed by the following differential equations.

$$\frac{dI_m^+(t)}{dt} + \theta_p I_m^+(t) = -d_m ; \ T_1 \le t \le t_m \qquad (4)$$

$$\frac{dI_j(t)}{dt} + \theta_p I_j(t) = -d_j ; t_{j-1} \le t \le t_j \qquad (5)$$

j=m+1,m+2, . . .,u

With boundary condition

$$I_{j-1}(t_{j-1}) = I_j(t_{j-1}) \ ,$$
$$I_m^+(t_{m-1}) = I_m(T_{m-1}) \ \& I_u(t_u) = 0, \qquad (6)$$

Solution of the (phase-1) from the differential equations (1) and (2) using boundary condition (3) are as follow:

$$I_k(t) = \frac{p - d_k - pe^{-\theta_p t}}{\theta_p} + \sum_{i=1}^{k} \frac{(d_i - d_{i-1})e^{-\theta_p (t - t_{i-1})}}{\theta_p} \qquad (7)$$

$$I_m^-(t) = \frac{p - d_m - pe^{-\theta_p t}}{\theta_p} + \sum_{i=1}^{m} \frac{(d_i - d_{i-1})e^{-\theta_p (t - t_{i-1})}}{\theta_p} \qquad (8)$$

Solution of the (phase-2) from the differential equations (4) and (5) using equation (6) is as follow:

$$I_m^+(t) = \frac{-d_m}{\theta_p} + \frac{d_u e^{-\theta_p (t - t_u)}}{\theta_p} - \sum_{i=m+1}^{u} \frac{\{(d_i - d_{i-1})\}}{\theta_p} e^{-\theta_p (t - t_{i-1})} \qquad (9)$$

$$I_j(t) = \frac{-d_j}{\theta_p} + \frac{d_u e^{-\theta_p (t - t_u)}}{\theta_p} - \sum_{j=j+1}^{u} \frac{\{(d_i - d_{i-1})\}}{\theta_p} e^{-\theta_p (t - t_{i-1})} \qquad (10)$$

When the production stop, which is T_1 using the condition, $I_m^-(T_1) = I_m^+(T_1)$ is as follow:

$$T_1 = \frac{1}{p}\left(A + \frac{\theta_p B}{2} \right) \qquad (11)$$

$$A = \sum_{i=1}^{u} (d_i)(t_i - t_{i-1}) \ \& \ B = \sum_{i=1}^{u} (d_i)(t_i^2 - t_{i-1}^2)$$

Equations (11) have no relation with m so if the value of t_i and d_i are known then the optimal production time T_1 can be found directly by using equation (11).

3.2. Cost Calculation of Finished Products

1. Set up cost $TS_c = k_0$ (12)

2. Holding Cost of finished products TH_p

$$\left[h_p \left(\sum_{k=1}^{m-1} I_k(t) + I_m^-(t) + I_m^+(t) + \sum_{j=m+1}^{u} I_j(t) \right) \right] = \frac{h_p B}{2} \qquad (13)$$

3. The deterioration cost of finished products

$$TD_p = c_p \left\{ pT_1 - \sum_{i=1}^{u} d_i (t_i - t_{i-1}) \right\} = \frac{c_p \theta_p B}{2} \quad (14)$$

4. Preservation technology $PT_C = \xi t_u$ (15)

Total cost of finished products $TC = TS_c + TH_p + TD_p + PT_c$

4. Objective

The objective of the study is to determine the optimal value of preservation cost $\xi*$ that minimizes the total cost TC is as follows put $\theta_p = \theta_2 e^{-\alpha\xi}$ then equation reduces to as;

$$TC = \left[k_0 + \frac{h_p B}{2} + \frac{c_p \theta_2 e^{-\alpha\xi} B}{2} + \xi t_u \right] \quad (16)$$

Differentiate with respect to ξ

$$\frac{\partial TC}{\partial \xi} = \left[-\frac{\alpha c_p B \theta_2 e^{-\alpha\xi}}{2} + t_u \right] \quad (17)$$

$$\frac{\partial^2 TC}{\partial \xi^2} = \left[\frac{\alpha^2 c_p B \theta_2 e^{-\alpha\xi}}{2} \right], \frac{\partial^2 TC}{\partial \xi^2} > 0$$

The optimal value of $\xi*$ will be calculated using Mathematica-software-9 from equation (17).

5. Numerical Analysis: Yong et al. (2010) related data for the case of a three-market situation along with the criteria of the proposition, the following data has been consider to validate the proposed model the demand rate of market one is \$160 units per week, the demand rate of market two\$280 units per week and the demand rate of market three \$120 units per, production rate is \$350 units per week. first market is from week 1 to week 8 and for second market is from week 8 to week 20 and for the third market is from week 20 to week 30.The deterioration rate of raw materials is \$0.2 units per week, finished products is \$0.3units per week, the set-up cost for production is \$600 and the holding cost of finished products is \$0.15 per week, preservation parameter $\alpha = 0.2$, the unit production cost is \$10 respectively. The optimal value of preservation technology (PT) cost ξ^* , the optimal production time T_1^*, the optimal total cost TC^**, with and without preservation technology respectively has been calculated with help of equation (17, 11&16,), shown in table -1.

With preservation technology			Without preservation technology	
ξ^*	T_1^*	Optimal Cost TC^*	T_1^*	Optimal Cost TC^*
4.97	19.29	21995.9	23.73	37372

Graphical Analysis: The graphical representation of the optimal total cost that is convexity of TC^* with respect ξ^* has been figure -2 respectively as follow:

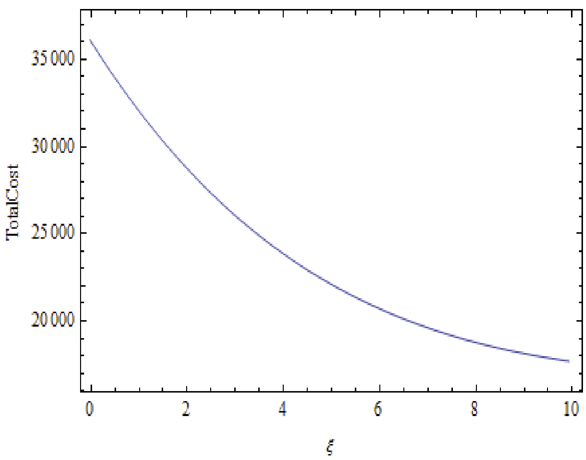

Fig-2: Graph with respect to TC^* and ξ^*

Conclusion: It is a unique opportunity to improve the profitability of a deteriorating items' manufacturer in different timing of the deteriorating items at different markets. The model is very practical for the industry in which holding cost is constant. In this paper, a method have suggested for finding the optimal production and inventory schedule for manufacturers of deteriorating items by using preservation Technology first & then the result are discussed for with & without preservation Technology. The proposed model can further be enriched by taking more realistic assumptions such as finite replenishment rate, probabilistic demand rate, etc. The obtained results designate the validity and stability/strength of the model

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