A NOTE ON NEAR-SEMIRINGS

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Abstract : Near-rings are the algebraic systems with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. Semirings are the algebraic systems which are closed and associative under two operations, usual addition, multiplication, and which satisfy both distributive laws. Semirings abound in the mathematical world around us. Indeed, the first mathematical structure we encounter, the set of natural numbers with the usual operations of addition and multiplication of integers is an example of a semiring. A near-semiring (or seminear-ring) S is an algebraic system with two binary operations usual addition and usual multiplication such that S forms a semigroup with respect to both the operations, and satisfies the right distributive law. In this Paper we consider the algebraic system near-semiring which is generalization of both a nearring and a semiring. The theory of near-semirings has enormous applications to automata theory, projective geometry and nonlinear interpolation theory.

Keywords: near-semiring, s-ideal, insertion of factors property.

Introduction:

Definition 1.1 : A near-semiring S is said to have an absorbing zero 0 if $a + 0 = 0 + a = a$ and $a \cdot 0 = 0 = 0 \cdot a$ for all $a \in S$.

Definition 1.2 : A subset I of a near-semiring S is a right (respectively, left) s-ideal if

(i) $x + y \in I$, for all $x, y \in I$.

(ii) $xr \in I$ (right s-ideal), $rx \in I$ (left s-ideal) for all $x, y \in I$ and $r \in S$.

Definition 1.3 : A near-semiring S with absorbing zero is said to have insertion of factors property (IFP, in short) if $a, b \in S$, $ab = 0 \Rightarrow asb = 0$ for all $s \in S$.

Definition 1.4: s-ideal I is said to have IFP if it satisfies, for all $a, b, s \in N$, $ab \in I \Rightarrow asb \in I$.

Definition 1.5 : Let S be near-semiring with absorbing zero. For an element $s \in S$, we define the set $(0: s) = \{x \in S \mid xs = 0\}$ called the annihilator of s in S.

Definition 1.6 : Let S be a near-semiring with absorbing zero.

(i) S is said to have left weak commutativity if $xrs = rxs$ for all $x, r, s \in S$.

(ii) s-ideal I is regular if for each $a \in I$, there exists $s \in S$ s uch that $asa \in I$.

Result 1.7 : The following assertions are equivalent:

(i) S has the IF- property,

(ii) for all $s \in S$, $(0 : s)$ is a left s-ideal, and it is a right s-ideal if S has left week commutativity.

(iii) for all $I \subseteq S$, $(0 : I)$ is an s-ideal.

Proof: **(i) ⇒ (ii):**

Take $x, y \in (0: s)$. Then $xs = 0$ and $ys = 0$.

Now $(x + y)s = xs + ys$ (by right distributivity)

$= 0 + 0 = 0.$

Take $s_1 \in S$, $x \in (0: s)$. This implies $xs = 0$. Since S has IFP, $xrs = 0$ for all $r \in S$.

In particular, $xs_1s = 0$. This implies $xs_1 \in (0: s)$. Therefore, $(0: s)$ is a left s-ideal.

Also, for any $s_1 \in S$, $x \in (0: s)$, by the above we get $xrs = 0$ for all $r \in S$.

In particular, $xs_1s = 0$. Since S has left weak commutativity, we have $s_1xs = 0$.

This implies $s_1x \in (0: s)$. Therefore,$(0: s)$ is a right s-ideal.

(ii) ⇒ (iii):

Suppose for all $s \in S$, $(0: s)$ is a left and a right s-ideal of S.

To show $(0: I)$ is a s-ideal. Take $x, y \in (0: I)$. This implies $xs = 0 = ys$ for all $s \in I$.

Now $(x + y)s = xs + ys = 0$. This is true for all $s \in I$.

That is, $(x + y)I = 0 \Rightarrow x + y \in (0: I)$.

Also, for any $s_1 \in S$, $x \in (0: I)$, we get $xI = 0$.

This implies $xs = 0$ for all $s \in I \Rightarrow x \in (0: s)$.

Since $(0: s)$ is an s-ideal, we have $xs_1 \in (0: s) \Rightarrow xs_1s = 0$ for all $s \in I$.

This implies that $xs_1I = 0 \Rightarrow xs_1 \in (0: I)$. Hence $(0: I)$ is an s-ideal.

(iii) ⇒ (i):

Suppose $ab = 0$. This implies $a \in (0: b) = (0: \{b\})$.

Since $(0: \{b\})$ is an s-ideal, we have $as \in (0: \{b\})$ for all $s \in S$.

This implies $asb = 0$ for all $s \in S$.

Hence S has IFP.

Definition 1.8: A non-empty fuzzy subset μ (that is, $\mu(x) \neq 0$ for some $x \in S$) of a near-semiring S is called a fuzzy s-ideal if it satisfies

(i) $\mu(x + y) \geq \min \{(\mu(x), \mu(y)\}$

(ii) $\mu(xy) \geq \max \{\mu(x), \mu(y)\}$

Definition 1.9 : Let μ be a fuzzy subset of a near-semi ring S. Then the set defined by

$$\mu_t = \{x \in S \mid \mu(x) \geq t,\ t \in [0,1]\}.$$

is called the level subset of S with respect to μ.

Definition 1.10 : Let μ be a fuzzy s-ideal of S.

(i) μ is said to have fuzzy IFP if $\mu(asb) \geq \mu(ab)$ for all a, b, s ∈ S.

(ii) μ is said to be fuzzy regular for each a ∈ S, there exists s ∈ S such that

$\mu(asa) \geq \mu(a)$.

Proposition 1.11 : Let μ be s-ideal of a near-semiring S with absorbing zero.

(i) μ has fuzzy IFP if and only if μ_t has IFP for all t ∈ (0, 1)

(ii) μ is regular if and only if μ_t is regular for all t ∈ (0, 1)

Proof: (i) Suppose μ has IFP.

Let t ∈ (0, 1). Clearly, μ_t is a s-ideal.

Take a, b ∈ S. Let ab ∈ μ_t. Then $\mu(ab) \geq t$. Since μ has IFP,

we have $\mu(asb) \geq \mu(ab) \geq t$ for all s ∈ S.

Therefore , asb ∈ μ_t. Hence μ_t has IFP.

Conversely,

suppose μ_t has IFP for all t ∈ (0, 1).

Let a, b ∈ S. Since S is a near-semiring, we have ab ∈ S. Let $\mu(ab) = t$.

Then ab ∈ μ_t.

Since μ_t has IFP, we have asb ∈ μ_t. That is, $\mu(asb) \geq t = \mu(ab)$.

Therefore, μ has IFP.

(ii) Suppose μ is regular. Let t ∈ (0, 1).

Take a ∈ μ_t. Then $\mu(a) \geq t$.

Since μ is regular there exists s ∈ S such that $\mu(asa) \geq \mu(a) \geq t$.

this implies asa ∈ μ_t.

Therefore, μ_t is regular.

Conversely,

suppose that μ_t is regular for all t ∈ (0, 1).

Let a ∈ S and $\mu(a) = t \Rightarrow a \in \mu_t$.

Since μ_t is regular, there exists s ∈ S such that asa ∈ μ_t.

This implies that $\mu(asa) \geq t = \mu(a)$.

Therefore μ is regular.

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