THERMOSOLUTAL INSTABILITY OF THE RIVLIN-ERICKSEN ROTATION FLUID IN AN ANISOTROPIC POROUS MEDIA

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Abstract: In this paper,the thermosolutal instability of Rivlin-Ericksen rotating fluid in an anisotropic porous medium is considered in the presence of uniform vertical rotation is considered. For the case of stationary convection, the stable solute gradient and rotation have stabilizing effects on the system, whereas, the medium permeability has stabilizing (or destabilizing) effect on the system under certain condition. The visco-elasticity effects disappear for stationary convection. The stable solute gradient. rotation, porosity and visco-elasticity introduce oscillatory modes in the system which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Keywords: Rivlin-Ericksen rotating fluid, anisotropic porous medium, thermosolutal.

Introduction: The theoretical and experimental results of the onset of thermal instability (Benard convection) in a fluid layer under varying assumptions of hydrodynamics has been treated in detail by **Chandrasekhar** (1981) in his celebrated monograph. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by **Veronis** (1965).The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The thermosolutal convection problems arise in oceanography, limnology and engineering.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Two classes of such fluids are **Rivlin-Ericksen**(1955) fluid and **Walters**(1960) (model B') fluid. **Walters**(1960) has proposed the constitutive equations for such elastico-viscous fluids. The mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5 gram of polymer per liter behaves very nearly as the **Walters** (model B') visco-elastic fluid and which is proposed by **Walters**(1962). **Rivlin-Ericksen**(1955) have proposed a theoretical model for another type of elastico-viscous fluids and used in agriculture, communication appliances and in biomedical applications. **Joshi**(1976) has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Recently, **Sharma et al**.(2000) have studied the Hall effect on the thermal instability of Rivlin-Ericksen fluid. **Sharma and Kango**(1999) have studied the thermal convection in Rivlin-Ericksen elastico-viscous fluids in porous medium in hydromagnetics. In all the above studies, the gravity field is assumed to be constant. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Rivlin-Eicksen fluid motion is replaced by the resistance term $\left[-\frac{1}{\left(k_x,k_y,k_z\right)}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the viscosity and visco-elasticity of the Rivlin-Eicksen fluid, $\left(k_x,k_y,k_z\right)$ is anisotropic effect and **q** is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil science, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (**McDonnel**,1978). In many astrophysical situations, the effect of rotation on thermosolutal convection in porous medium is also important.

Srivastava and Singh (1988) studied the unsteady flow of a dusty elastico-viscus Rivlin-Ericksen fluid through channels of different cross-sections in the presence of time-dependent pressure gradient. **Garg et.al.** (1994) have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid and the effect of suspended particles on the thermal instability of Rivlin-Ericksen elastico-viscous fluid.

A layer of Rivlin-Ericksen elastico-viscous, compressible fluid heated and soluted from below in the presence of suspended particles (fine dust) is considered by **Sharma and Sharma** (2004).For stationary convection, the Rivlin-Ericksen elastico-viscous fluid behaves like Newtonian fluid and the stable solute gradient and compressibility postpone the onset of convection whereas the suspended particles hasten the onset of convection.

The stable solute gradient introduces oscillatory modes in the system which were non-existent in its absence. Keeping in mind the importance in geophysics, soil physics, astrophysics, ground-water hydrology and various applications mentioned above, the thermosolutal instability of a Rivlin-Ericksen fluid in an anisotropic porous medium in the presence of uniform vertical rotation has been considered in the present paper.

Formulation of the Problem: Consider an infinite, horizontal, incompressible Rivlin-Erickson fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_0, ρ_d and C_0 and at the upper surface z = d are T_d, ρ_0 and C_d respectively, and that a uniform temperature gradient $\beta\left(=\left|dT/dz\right|\right)$ and uniform solute gradient $\beta'\left(=\left|dc/dz\right|\right)$ are maintained. The gravity field $\mathbf{g}(0,0,-g)$, and a uniform vertical rotation $\mathbf{w}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an anisotropic and homogeneous porous medium of porosity $\in$ and medium permeability $\left(k_x,k_y,k_z\right)$.

Let $p,\rho,T,C,\alpha,\alpha',g$ and $\mathbf{q}(u,v,w)$ denote respectively the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and fluid velocity. The equation expressing the conservation of momention, mass, temperature, solute mass concentration and equation of state of Rivlin – Ericksen fluid are

$$\frac{1}{\in}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla P+\mathbf{g}\left(1+\frac{\delta p}{\rho_0}\right)$$

$$-\frac{1}{\left(k_x,k_y,k_z\right)}\left(V+V'\frac{\partial}{\partial t}\right)\mathbf{q}+\frac{2}{\in}(\mathbf{q}\times\omega), \qquad ...(1)$$

$$\nabla.\mathbf{q}=0, \qquad ...(2)$$

$$E\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T=\kappa\nabla^2 T, \qquad ...(3)$$

$$E'\frac{\partial C}{\partial t}+(\mathbf{q}.\nabla)C=\kappa'^2\nabla^2 C \qquad ...(4)$$

and $$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)+\alpha'\left(C-C_0\right)\right], \qquad ...(5)$$

where the suffix zero refers to values at the reference level $z=0$ and in writing equation (1), use has been made of the Boussinesq approximation. The Kinematic viscosity V, Kinematics viscoelasticity V', the thermal diffusivity κ and solute diffusivity κ' are all assumed to be constant. $E=\in+(1+\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat. $\rho_s, C_s; \rho_0$ and C_i denote the density and heat capacity of solid (porous matrix) material and fluid respectively.

The steady state solution is

$$\mathbf{q}=(0,0,0), \qquad T=-\beta z+T_0, \qquad C=-\beta' z+C_0$$

$$\rho=\rho_0\left(1+\alpha\beta z-\alpha'\beta' z\right). \qquad ...(6)$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady solution, and let $\delta p,\delta\rho,\theta,\gamma$ and $\mathbf{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentrations C and velocity **q**(0,0,0) . The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and solute concentrations, is given by

$$\delta\rho=-\rho_0\left(\alpha\theta-\alpha'\gamma\right) \quad . \qquad ...(7)$$

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla \delta p) - \mathbf{g}(\alpha\theta - \alpha'\gamma)$$

$$-\frac{1}{(k_x, k_y, k_z)}\left(V + V'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{2}{\in}(\mathbf{q}\times\omega), \qquad ...(8)$$

$$\nabla.\mathbf{q} = 0, \qquad ...(9)$$

$$E\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta \qquad ...(10)$$

and $$E'\frac{\partial \gamma}{\partial t} = \beta' w + \kappa' \nabla^2 \gamma . \qquad ...(11)$$

The Dispersion Relation:m Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, \zeta] = [W(z), \Theta(z), \Gamma(z), Z(z)]\exp(ik_x x + ik_y y + nt), \qquad ...(12)$$

where k_x and k_y are the wave numbers is x- and y- directions respectively , $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number, and n is the growth rate which is, in general, a complex constant.

$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ stands for the z- component of vorticity.

Expressing the coordinates x, y, z in the new unit of length d and letting $d^* = \kappa d, \sigma^* = \frac{nd^2}{v}$, $p^*_1 = \frac{V}{k}, q^* = \frac{V}{k^1}, F^* = \frac{V'}{k}, p_\ell^* = \frac{k}{d^2}, p_\ell^z = \frac{k_z}{d^2}$ and $D^* = \frac{d}{dz}$, equations (8) - (11), with the help of expression (12), in non-dimensional form become

$$\frac{\sigma}{\in}(D^2 - a^2)W + (1 + F\sigma)\left(\frac{1}{p_\ell^*}\right)D^2W - (1 + F\sigma)\left(\frac{1}{p_\ell^z}\right)a^2W$$

$$+\frac{ga^2d^2}{V}(\alpha\Theta - \alpha'\Gamma) + \frac{2\Omega d^3}{\in V}DZ = 0, \qquad ...(13)$$

$$\left[\frac{2\sigma}{\in} + \frac{1}{p_\ell^*}(1 + \sigma F) + \frac{1}{p_\ell^z}(1 + \sigma F)\right]Z = \left(\frac{4\Omega d}{\in v}\right)DW, \qquad ...(14)$$

$$(D^2 - a^2 - Ep_1\sigma)\Theta = -\left(\frac{\beta d^2}{\kappa}\right)W \qquad ...(15)$$

and $$(D^2 - a^2 - E'q\sigma)\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right)W . \qquad ...(16)$$

Consider the case where both boundaries are free as well as maintained at constant temperatures and solute concentrations. The appropriate boundary conditions, with respect to which equations (13)-(16) must be solved, are (**Chandrasekher**, 1981)

$$W = D^2W = 0, \Theta = 0, \Gamma = 0, DZ = 0 \text{ at } z = 0 \text{ and } 1 \qquad ...(17)$$

The case of two free boundaries is little artificial but it enables us to find analytical solution and to make some qualitative conclusions. This is most appropriate for stellar atmospheres (**Spiegel** 1965). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \qquad ...(18)$$

where W_0 is a constant.

Eliminating Θ, Γ and Z between equations (13)-(16) and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$R_1 = \frac{(1+x)}{x}\left[\frac{i\sigma_1}{\in} + i\sigma_1 F\right](1+x+iEp_1\sigma_1) + \frac{1}{x}\left(\frac{1}{R}+\frac{1}{S}x\right)(1+x+iEp_1\sigma_1)$$
$$+T_{A_1}\frac{(1+x)(1+x+iEp_1\sigma_1)}{x\left[(1+x)\frac{i\sigma_1}{\in}+\left(\frac{1}{R}+\frac{x}{S}\right)+i\sigma_1 F(1+x)\right]} + S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)},$$

where ...(19)

$$\left.\begin{aligned} R_1 &= \frac{g\alpha\beta d^4}{V\kappa\pi^4}, S_1 = \frac{g\alpha'\beta' d^4}{V\kappa'\pi^4}, T_{A_1} = \frac{16\Omega^2 d^4}{V^2\pi^4} = \left(\frac{4\Omega d^2}{V\pi^2}\right)^2 \\ x &= \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}, R = \pi^2 P_\ell^*, S = P_\ell^z \pi^2. \end{aligned}\right] \quad ...(20)$$

Equation (19) is the required dispersion relation for examining the effects of rotation, medium permeability, kinematic viscoelasticity and stable solute gradient on thermoslutal instability of Rivlin-Ericksen rotating fluid in a porous medium.

The Stationary Convection : When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (19) reduces to

$$R_1 = \frac{(1+x)}{x}\left(\frac{1}{R}+\frac{1}{S}x\right) + T_{A_1}\frac{(1+x)^2}{\left(\frac{1}{R}+\frac{1}{S}x\right)x} + S_1, \quad ...(21)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1, T_{A_1}, R and S. The parameter F accounting for the visco-elasticity effect disappears for the stationary convection .

To investigate the effects of stable solute gradient, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dS_1}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dR}$ and $\frac{dR_1}{dS}$ analytically. Equation (21) yields

$$\frac{dR_1}{dS_1} = 1, \quad ...(22)$$

which implies that the stable solute gradient has a stabilizing effect on the thermosolutal convection. The adverse solute gradient has destabilizing effect on the system since $\frac{dR_1}{dS_1}$ then becomes negative.

Equation (21) also yields

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)^2}{\left(\frac{1}{R}+\frac{1}{S}x\right)x}. \quad ...(23)$$

The rotation, therefore, has always a stabilizing effect on the thermosolutal instability of Rivlin – Ericksen rotation fluid in porous medium.

It is evident from (21) that

$$\frac{dR_1}{dR} = \frac{1}{R^2}\frac{(1+x)}{x} + \frac{T_{A_1}(1+x)^2}{\left(\frac{1}{R}+\frac{1}{S}x\right)^2 x}\left(\frac{1}{R^2}\right) + 0 = \frac{1}{R^2}[1+x]\left[\frac{T_{A_1}(1+x)}{\left(\frac{1}{R}+\frac{1}{S}x\right)^2 x} - \frac{1}{x}\right] \quad ...(24)$$

$$\frac{dR_1}{dS}=\frac{(1+x)}{x}\left[0+\frac{(-x)}{S^2}\right]+\frac{T_{A_1}(1+x)^2}{\left(\frac{1}{R}+\frac{1}{S}x\right)^2 x}\frac{x}{S^2}=\frac{1}{S^2}(1+x)\left[\frac{T_{A_1}(1+x)}{\left(\frac{1}{R}+\frac{1}{S}x\right)^2}-1\right].$$

$$\ldots(24)''$$

In the absence of rotation $\left(T_{A_1}\to 0\right), \frac{dR_1}{dR}$ and $\frac{dR_1}{dS}$ is given by

$$\frac{dR_1}{dR}=-\frac{(1+x)}{x.R^2}, \qquad \ldots(25)$$

$$\frac{dR_1}{dS}=-\frac{(1+x)}{S^2}, \qquad \ldots(25)$$

which is always negative. The medium permeability, therefore, has a destabilizing effect on the thermosolutal instability of a fluid in the absence of rotation. In the presence of rotation, the system is unstable or stable if

$$T_{A_1}<(\text{or}>)\frac{(1+x)}{R^2}$$

and $T_{A_1}<(\text{or}>)\frac{(1+x)}{S^2}$. ...(26)

Stability of the System and Oscillatory:

Modes: Multiplying equation(13) by W*, the complex conjugate of W, and using equations (14) - (16) together with the boundary conditions (17), we obtain

$$\left[\frac{\sigma}{\in}+\frac{1}{p_\ell^*}(1+\sigma F)\right]I_1' +\left[\frac{\sigma}{\in}+\frac{1}{p_\ell^z}(1+\sigma F)\right]I_1'' +\left(\frac{g\alpha'\kappa' a^2}{V\beta'}\right)\left[I_4+E'q\sigma^* I_5\right]$$
$$+\frac{d^2}{2}\left[\left\{\frac{\sigma^*}{\in}+\frac{1}{p_\ell^*}(1+\sigma^* F)\right\}+\left\{\frac{\sigma^*}{\in}+\frac{1}{p_\ell^z}(1+\sigma^* F)\right\}\right]I_6$$
$$-\left(\frac{g\alpha\kappa a^2}{V\beta}\right)\left[I_2+Ep_1\sigma^* I_3\right]=0, \qquad \ldots(27)$$

Where

$$\left.\begin{aligned}
I_1' &= \int_0^1 |DW|^2 dz,\\
I_1'' &= \int_0^1 a^2|W|^2 dz,\\
I_2 &= \int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\\
I_3 &= \int_0^1\left(|\Theta|\right)^2 dz,\\
I_4 &= \int_0^1\left(|D\Gamma\right)^2+a^2|\Gamma|^2 dz,\\
I_5 &= \int_0^1\left(|\Gamma|\right)^2 dz,\\
I_6 &= \int_0^1\left(|z|^2\right)dz.
\end{aligned}\right\}. \qquad \ldots(28)$$

The integrals $I_1', I_1'', I_2, I_3, I_4, I_5, I_6$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of (27), we obtain

$$\left[\left(\frac{1}{\in}+\frac{F}{P_\ell^*}\right)I_1' + \left(\frac{1}{\in}+\frac{F}{P_\ell^z}\right)\left(I_1''\right) + \frac{g\alpha'\kappa' a^2}{V\beta'}.E'qI_5 \right.$$

$$\left. + \frac{d^2}{2}\left[\left(\frac{1}{\in}+\frac{F}{P_\ell^*}\right)+\left(\frac{1}{\in}+\frac{F}{P_\ell^z}\right)\right]I_6 - \frac{g\alpha\kappa a^2}{V\beta}Ep_1I_3\right]\sigma_r$$

$$= -\left[\frac{I_1'}{P_\ell^*}+\frac{I_1''}{P_\ell^z}+\frac{g\alpha'\kappa' a^2}{V\beta'}I_4 + \frac{d^2}{2}\left(\frac{1}{P_\ell^*}+\frac{1}{P_\ell^z}\right)I_6 - \frac{g\alpha\kappa a^2}{V\beta}I_2\right], \qquad \ldots(29)$$

and

$$\left[\left(\frac{1}{\in}+\frac{F}{P_\ell^*}\right)I_1' + \left(\frac{1}{\in}+\frac{F}{P_\ell^z}\right)I_1'' - \frac{g\alpha'\kappa' a^2}{V\beta'}E'qI_5 \right.$$

$$\left. - \frac{d^2}{2}\left\{\left(\frac{1}{\in}+\frac{F}{P_\ell^*}\right)+\left(\frac{1}{\in}+\frac{F}{P_\ell^z}\right)\right]I_6 + \frac{g\alpha\kappa a^2}{V\beta}EP_1I_3\right]\sigma_i = 0. \qquad \ldots(30)$$

It is evident from (29) that σ_r is positive or negative. The system is, therefore stable or unstable. It is clear from (30) that σ_i may be zero or non – zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation, stable gradient and visco-elasticity, which were non-existent in their absence.

The Case of Over stability: Here we discuss the possibility of whether instability may occur as over stability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (19) admits solutions with σ_1 real.

Equating the real and imaginary parts of equation (19) and eliminating R_1 between them, we obtain

$$A_2\sigma_1^4 + A_1\sigma_1^2 + A_0 = 0$$

or

$$A_2C_1^2 + A_1C_1 + A_0 = 0, \qquad \ldots(31)$$

where $C_1 = \sigma_1^2$, $1/p = c/b$, $b = 1 + x$, $c = \left(\frac{1}{R}+\frac{1}{S}x\right)$,

$$A_2 = b\left\{\frac{c}{b}(1+\in F)\right\}^2 E'^2.q^2\left[b(1+\in F)+\frac{c}{b}.\in Ep_1\right] \qquad \ldots(32)$$

$$A_1 = \left\{\left[\left(1+\frac{c}{b}\in F\right)\left(1+\frac{c}{b}2\in F\right)\right]b^4 + \left[\frac{c}{b}\in p_1\left(1+\in F\left(2+\frac{c}{b}\in F\right)\right)\right]b^3\right.$$

$$+\left[\in^2\frac{c^2}{b^2}E'^2q^2\left(1+\frac{c}{b}\in F\right)\right]b^2 + \left[\in^2 E'^2q^2\left(\frac{c^3}{b^3}\in FP_1 - T_{A_1}\left(1+\frac{c}{b}\in F\right)\right)\right]$$

$$\left.+\in(b-1)S_1\left(1+\in F\frac{c}{b}\right)\left(Ep_1 - E'q\left\{1+\frac{c}{b}\in F\right\}\right)b + \left[\frac{c}{b}\in^3 T_{A_1}Ep_1E'^2q^2\right]\right\} \qquad \ldots(33)$$

$$A_0 = \in^2 b\left\{\left[\frac{c^2}{b^2}\left(1+\in F\frac{c}{p}\right)\right]b^3 + \left[\in p_1E\frac{c^3}{b^3}T_{A_1}\left\{1+\in F\frac{c}{p}\right\}\right]b^2\right.$$

$$\left.+\left[\frac{\in c}{b}T_{A_1}Ep_1\right]b + \left[\frac{\in c^2}{b^2}(b-1)S_1(Ep_1 - E'q)\right]\right\}. \qquad \ldots(34)$$

Since σ_1 is real for overstability, both the value of $C_1(\sigma_1^2)$ are positive. Equation (31) is quadratic in C_1 and does not allow any of its roots to be positive if

$$\left.\begin{aligned} &Ep_1 > \frac{b^3}{c^3}.\frac{T_{A_1}}{\in}\left(1+\frac{c}{b}\in F\right), \\ &E_1p_1 > E'q\left(1+\frac{c}{b}\in F\right) \\ \text{and}\quad &Ep_1 > E'q \end{aligned}\right], \qquad ...(35)$$

which imply that

$$\kappa < \frac{\in EV^3d^2}{4\Omega^2k_1\left(\pi^2k_1^2+\in V'd^2\right)} \text{ and } E'_\kappa\left(1+\frac{\in V'd^2}{\pi^2k_1^2}\right) < E_{\kappa'} \qquad ...(36)$$

Thus $\kappa < \frac{\in EV^3d^2}{4\Omega^2k_1\left(\pi^2k_1^2+\in V'd^2\right)}$ and $E'_\kappa\left(1+\frac{\in V'd^2}{\pi^2k_1^2}\right) < E_{\kappa'}$

are the sufficient conditions for the non-existence of overstability, the violation of which dose not necessarily imply the occurrence of overstability.

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