LATTICE & NEAR-RING STRUCTURE ON RECURSIVE LANGUAGES

P.Vijaya Vani[1]

Abstract: In this paper after ascertaining ($Ł_R$, $\leq$) is a lattice where $Ł_R$ is the class of Recursive languages on which $\vee$ and $\wedge$ are defined as $L_1 \vee L_2 = L_1 \cup L_2$ and $L_1 \wedge L_2 = L_1 \cap L_2$, lattice ordered finite Turing Machine has been defined and identified the language sets with fuzzy logic and are named ℓ-recursive languages. Further the closure properties of ℓ-recursive languages are verified i.e., ℓ-recursive languages are closed under union, concatenation and kleene closure and are not closed under intersection and hence complementation. Further it is proved that the ℓ-recursive languages are distributive. Since ℓ-recursive languages are not closed under intersection and complementation, ℓ-recursive languages are not Boolean Algebras and hence not a Generalized Boolean Algebra.

In the direction of M.Holcombe [16] who established a connection between Automata and Near-rings, in this chapter an attempt is made to identify the near ring structure in Turing Machines. In this process Semi Turing Machine, Group Semi Turing Machine Sub Near-ring of the Turing Machine are defined and the properties of the transition function are discussed. Further if M is a homomorphic group semi turing machine then the sub near-ring N(M) is a Near-ring N, is proved and it is also confirmed that N(M) is generalized distributively generated near ring with identity. Further for every near ring N with identity there is some group semi turing machine M with $N \cong N(M)$ is shown. Also for a near ring N there exists a group semi turing machine M with $N \cong N(M)$ iff (1). (N, +) is abelian. (2). N has an identity, (3). There is some $d \in N_d$ such that N_B is generated by {1,d} are verified.

Keywords: turing machine, ℓ-recursive languages, generalized boolean algebra, group semi turing machine, near-ring.

1. PRELIMINARIES

Definition 1.1 A binary operation $*$ on [B, Γ^*] is said to be turing operation if it satisfies :

(a) For all $L \in$ [B, Γ^*], $\Gamma * L = L$.

(b) $L_1 * L_2 \leq L_3 * L_4$ whenever $L_1 \leq L_3$ and $L_2 \leq L_4$ i.e $*$ is non decreasing.

(c) For any $L_1, L_2, L_3 \in$ [B, Γ^*], $(L_1 * L_2) * L_3 = L_1 * (L_2 * L_3)$

i.e $*$ is associative.

(d) For any $L_1, L_2 \in$ [B, Γ^*], $L_1 * L_2 = L_2 * L_1$ i.e $*$ is commutative.

ISBN - 978-93-81583-55-5

Definition 1.2 A lattice ordered Turing machine (L T M) is a machine $M = (Q, \Sigma, \Gamma, \delta, I, B, F, *)$ where

Q is a non-empty finite set, called the set of states of M.

Σ is a finite set, called the input alphabet of M.

Γ is a finite set, called the set of allowable tape symbols of M

B is a designated symbol of Γ, called the blank symbol

$\delta : Q \times \Gamma \times Q \times \Gamma \times \{L, R\} \to [B, \Gamma^*]$ is a function called the fuzzy transition function of M, where L and R are reserved symbols denoting Left and Right direction moves respectively

$I : Q \to [B, \Gamma^*]$ is a fuzzy subset of Q called the fuzzy initial state set of M.

$F : Q \to [B, \Gamma^*]$ is a fuzzy subset of Q called the fuzzy final state set of M.

Definition 1.3 An Instantaneous Description (I D) of the lattice ordered Turing machine M is given by δ (q, a, p, b, D)). Here $q \in Q$ is the current state of M ; a is the tape symbol being scanned, p is the next state after a is scanned, b is the rewritten tape symbol and D = {L, R} stands for a direction (L for left and R for right).

Definition 1.4 A move from an ID I_1 to another ID I_2 denoted by $\vdash_M$. $\vdash_M$ is defined as $x, y \in \Gamma^*$, $a, b, c \in \Gamma$ and $p, q \in Q$.

$$I_1 \vdash_M I_2 = \begin{cases} \delta(q, a, p, b, R) & \text{if } I_1 = xqay,\ I_2 = xbpy \\ \delta(q, a, p, b, L) & \text{if } I_1 = xcqay,\ I_2 = xpcby \\ \delta(q, B, p, b, R) & \text{if } I_1 = xq,\ I_2 = xbp \\ \delta(q, B, p, b, L) & \text{if } I_1 = xcq,\ I_2 = xpcb \\ B & \text{otherwise} \end{cases}$$

Definition 1.5 A lattice ordered Turing Machine is said to halt if it enters a state q, scanning a symbol a and there is no move i.e. $\delta(q, a, p, b, D) = B$ for any $p \in Q$, $b \in \Gamma$ and $D \in \{L, R\}$.

We also say that $\delta(q, a)$ is not defined if $\delta(q, a, p, b, D) = B$ for any $p \in Q$, $b \in \Gamma$ and $D \in \{L, R\}$.

Definition 1.6 The language accepted by a lattice ordered Turing machine LTM, M = (Q, Σ, Γ, δ, I , B, F, $*$) denoted by L(M) is the set L(M) = {s/ s $\in \Sigma^*$ and (I(Q)s $\vdash^*$ xpy) $*$ F(p) : p $\in$ Q, xy $\in$ Γ^*} and is called ℓ-language.

Note 1.7 Two lattice ordered Turing machines LTM's M_1 and M_2 are said to be equivalent if they accept same ℓ-language i.e. $L(M_1) = L(M_2)$.

Definition 1.8 A ℓ-language L(M) is said to be ℓ-recursively enumerable language if it is accepted by some lattice ordered Turing machine i.e L = L(M).

Furthermore if for any input s $\in \Sigma^*$, the lattice ordered turing machine M always halts i.e. M halts on input s then we say L(M) is a ℓ-recursive language.

2. CLOSURE PROPERTIES OF ℓ-RECURSIVE LANGUAGES

Theorem 2.1 Both ℓ-recursively enumerable languages and ℓ-recursive languages are closed under union, concatenation and kleene closure.

Proof: Claim 1 : Union of ℓ-recursive languages is ℓ-recursive

Let L_1 be L (M_1) for LTM M_1 = (Q_1, Σ_1, Γ_1 ,δ, I_1 ,B, F_1 , $*$)
and L_2 be L (M_2) for LTM M_2 = (Q_1, Σ_1, Γ_1 ,δ, I_2 ,B, F_2 , $*$).
If ℓ_1 and ℓ_2 are ℓ-recursive languages L_1 and L_2 then $\ell_1 + \ell_2$ denotes $L_1(M) \vee L_2(M)$ = $L_1 \cup L_2$ = L (M) for LTM M = (Q, Σ , $\Gamma_1 \cup \Gamma_2$,δ , I , B, F, $*$), where
Q = $Q_1 \cup Q_2 \cup \{q_0 , f_0\}$, $\Sigma = \Sigma_1 \cup \Sigma_2$
I : Q $\rightarrow$ [B, Γ^*] defined as I(q) = I_1 , q $\in$ Q_1 ; I(q) = I_2 , q $\in$ Q_2
F: Q $\rightarrow$ [B, Γ^*]defined as F(q) = F_1, q $\in$ Q_1 ; F(q) = F_2, q$\in$ Q_2 and δ is defined as
δ(q, a, p, b, D) = δ_1 (q, a, p, b, D) for q, p $\in$ Q_1.
δ(q, a, p, b, D) = δ_2 (q, a, p, b, D) for q, p $\in$ Q_2.
δ (q, a, p, b, D) = B otherwise
δ (I , ε, {q_1, q_2}, ε, D) = {I_1, I_2 } $\quad$ $q_1 \in Q_1$, $q_2 \in Q_2$.
δ ({f_1, f_2} , ε, F, ε, D) = {F_1, F_2 } $\quad$ $f_1 \in Q_1$, $f_2 \in Q_2$.
So $L_1 \cup L_2$ is also ℓ-recursive.
Thus Union of ℓ-recursive languages is a ℓ-recursive language.
Claim 2 : Concatenation of ℓ-regular languages is ℓ-regular.
Let L_1 be L (M_1) for LTM M_1 = (Q_1, Σ_1, Γ_1 ,δ, I_1 ,B, F_1 , $*$) and L_2 be L (M_2) for LTM M_2 = (Q_1, Σ_1, Γ_1 ,δ, I_2 ,B, F_2 , $*$).
If ℓ_1 and ℓ_2 are ℓ-recursive languages L_1 and L_2 then $\ell_1\ell_2$ denotes $L_1(M)L_2(M)$ = L_1 L_2 = L (M) for LTM M = (Q, Σ , $\Gamma_1 \cup \Gamma_2$,δ , I , B, F, $*$), where
Q = $Q_1 \cup Q_2 \cup \{q_0 , f_0\}$,
If ℓ_1 and ℓ_2 are ℓ-recursive expressions denoting ℓ-recursive languages L_1 and L_2 then $\ell_1\ell_2$ denotes $L_1(M)L_2(M)$ = L_1L_2 = L (M) for LTM
M = (Q, Σ , $\Gamma_1 \cup \Gamma_2$,δ , I_1 , B, F_2, $*$), where δ is δ (q, a, p, b, D) = δ_1 (q, a, p, b, D) for q, p in $Q_1 - F_1$ & a, b in $\Sigma_1 \cup \{\varepsilon\}$, δ (F_1 , ε, D) = I_2, δ (q, a, p, b, D) = δ_2 (q, a, p, b, D) for q, p in Q_2 and a, b in $\Sigma_2 \cup \{\varepsilon\}$

So L_1L_2 is also ℓ-recursive. Thus Concatenation of ℓ-recursive languages is ℓ-recursive.
Hence ℓ-recursive languages are closed under Concatenation.
Claim 3 : Kleene Closure of ℓ- recursive languages is ℓ- recursive.
Let L_1 be L (M_1) for LTM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, I_1, B, F_1, *)$
If ℓ_1 is a ℓ- recursive expression denoting ℓ- recursive language L_1 then ℓ_1^* denotes $L_1^* = L = L(M)$ for LTM $M = (Q_1 \cup \{q_0, f_0\}, \Sigma_1, \Gamma_1, \delta, I_1, B, F_1, *)$, where $I : Q \to [B, \Gamma_1^*]$, $F : Q \to [B, \Gamma_1^*]$ and δ is given by $\delta(I, \varepsilon, D) = \delta(F_1, \varepsilon, D) = \{I_1, F_1\}$
$\delta(q, a, p, b, D) = \delta_1(q, a, p, b, D)$ for q in $Q_1 - F_1$ & a, b in $\Sigma_1 \cup \{\varepsilon\}$.
So $L = L_1^*$ is ℓ-recursive. Thus Kleene Closure of ℓ-recursive languages is ℓ- recursive.

Theorem 2.2 The class of ℓ-recursive languages is not closed under complementation.

Proof: Let L be L (M) for LTM $M = (Q, \Sigma, \Gamma, \delta, I, B, F, *)$

and let $L \subseteq \Gamma^*$. In a contrary way to prove the class of ℓ-recursive languages is closed under complementation we construct a LTM

$M^1 = (Q^1, \Sigma^1, \Gamma^1, \delta^1, I^1, B, F^1, *)$ where $F^1 = Q - F$

where $F : Q \to [B, \Gamma^*]$ and since $[\phi, \Gamma^*]$ is a universal interval $F^1 = \phi$.

$\therefore$ $\phi \to [B, \Gamma^*]$ is not defined and F^1 is not a fuzzy final state set. $\therefore M^1$ is not a lattice ordered finite automaton.

Thus the class of ℓ-regular languages is not closed under complementation.

Corollary 2.3 The class of ℓ-recursive languages is not closed under intersection.

Proof: Since closure under intersection follow from closure under union and complementation, ℓ-recursive languages are not closed under intersection.

Theorem 2.4 Let Ł be the class of ℓ-recursive languages. Then the lattice Ł is distributive.

Proof: Now we show that the lattice Ł is a distributive lattice.

Claim : $L_1 \wedge (L_2 \vee L_3) = (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$
Consider $L_1 \wedge (L_2 \vee L_3) = L_1 \cap (L_2 \vee L_3)$
$= L_1 \cap (L_2 \cup L_3)$
$= (L_1 \cap L_2) \cup (L_1 \cap L_3)$
$= (L_1 \wedge L_2) \cup (L_1 \wedge L_3)$
$= (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$
Therefore $L_1 \wedge (L_2 \vee L_3) = (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$

Thus ℓ-recursive language is a distributive lattice.

Theorem 2.5 Let Ł be the class of ℓ-recursive languages. Then the lattice Ł is distributive.

Proof: Now we show that the lattice $Ł_\ell$ is a distributive lattice.
Claim : $L_1 \wedge (L_2 \vee L_3) = (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$
Consider $L_1 \wedge (L_2 \vee L_3) = L_1 \cap (L_2 \vee L_3)$
$= L_1 \cap (L_2 \cup L_3)$
$= (L_1 \cap L_2) \cup (L_1 \cap L_3)$
$= (L_1 \wedge L_2) \cup (L_1 \wedge L_3)$
$= (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$
Therefore $L_1 \wedge (L_2 \vee L_3) = (L_1 \wedge L_2) \vee (L_1 \wedge L_3)$

Thus ℓ-recursive languages is a distributive lattice.

Theorem 2.6 Let Ł be the class of ℓ-recursive languages. Then the distributive lattice Ł is not a complemented distributive lattice and not a generalized Boolean algebra.

Proof: By the above theorems , ℓ-recursive languages Ł is not a complemented distributive lattice and follows Ł is not a generalized Boolean algebra.

3. NEAR-RING STRUCTURE OF RECURSIVE LANGUAGES

Definition 3.1 A semi Turing machine is a 4-tuple M= (Q, Γ, B, δ) where Q is a nonempty finite set called the set of states of M. Γ is a finite set called the set of allowable tape symbols of M. B is a designated symbol of Γ called the blank symbol. $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{ L , R \}$ called the next move function of M where L and R are reserved symbols denoting left and right directions moves respectively.

Definition 3.2 If Q is a group with respect to addition we call M a group semi turing machine and denote by GSTM.

Note 3.3 For $q \in Q$ and $a \in \Gamma$, δ(q, a) is the new state obtained from the old state q by means of the input a.

Definition 3.4 If M=(Q, Γ, B, δ) is a semi turing machine, the collection of mapping δa from Q to Q are for each $a \in \Gamma$, are given by $q\delta a = \delta(q, a)$. Hence δa describes the transition of the input a on the state set Q of M.

Definition 3.5 If the input $a_1 \in \Gamma$ is followed by the input a_2 the semi turing machine "moves" from the state $q \in Q$ first into $q\delta a_1$ and then into $(q\delta a_1)\delta a_2$.

Definition 3.6 Extending Γ to the free monoid Γ* over Γ consist of all finite strings of Γ including the Blank string B and we get $\delta a_1 a_2 = \delta a_1 \delta a_2$ i.e the map

ISBN - 978-93-81583-55-5

$a \rightarrow \delta a$ is a monomorphism from Γ^* into the transformation monoid over Q with $\delta B = id_Q$.

Remark 3.7 In the case of group semi turing machine's we observe the superposition $\delta a_1 + \delta a_2$ defined point wise of two simultaneous inputs $a_1, a_2 \in \Gamma$. Hence we consider $\{\delta a / a \in \Gamma\} \cup \{ \delta B \}$ and all of its sums and products (juxtaposition) for the structure of a near-ring.

Definition 3.8 Let M=(Q, Γ, B, δ) be a group semi turing machine. The sub near-ring N(M) of map(Q) generated by id_Q and all δa's ($a \in \Gamma$) is called the syntactic near ring of M. Thus N(M) is always a near-ring with identity.

Definition 3.9 Let Q and Γ be additive groups with B and δ a homomorphism from the direct product Q x Γ. We then call M=(Q, Γ, B, δ) a homomorphic GSTM.

Because of $q\delta a = \delta(q, a) = \delta((q, B) + (B, a))$

$= \delta(q, B) + \delta(B, a)$

$= q\delta B + B\delta a$

We get $\delta a = \delta B + \delta^1 a$, where δB is a homomorphism (i.e. a distributive element in N(Q),while $\delta^1 a$ is the map with constant value Bδa.

If no input can move the Blank state i.e if $B\delta a = B$ for all $a \in \Gamma$ then N(M) obviously is a distributively generated near ring. Counting of sum of powers of δB (which are endomorphisms). We also get a distributively generated near ring if δ is additive in the first component.

Note 3.10 For homomorphic group semi turing machine's by induction

$\delta a_1 a_2 a_3 \ldots\ldots\ldots a_n = \delta^n B + (\delta^1 a_1 \delta^{n-1} B + \ldots\ldots\ldots + \delta^1 a_{n-1}\, \delta_B + \delta^1 a_n)$ where the map in brackets is constant. Each power $\delta^n B$ is a homomorphism.

Theorem 3.11 Let M=(Q, Γ, B, δ) be a homomorphic group semi turing machine then $N(M) = \{\Sigma\, \delta\alpha_i / \alpha_i \in \Gamma^*\} = N$

Proof: Let M=(Q, Γ, B, δ) be a homomorphic group semi turing machine. Clearly $N \subseteq N(M)$. Conversely it suffices to show that N is a near ring, since obviously N contains all δa ($a \in \Gamma$) and $id_Q = \delta$. In fact, we show that N is a sub near-ring of map(Q).

Take $\delta_1 = \Sigma\, \delta\alpha_i \in N$ and $\delta_2 = \Sigma\, \delta\beta_j \in N$

Clearly $\delta_1 + \delta_2 \in N$

Consider $\delta_1\delta_2$,

$$\delta_1\, \delta_2 = (\Sigma\, \delta\alpha_i)(\Sigma\, \delta\beta_j)$$

$= \Sigma(\Sigma\delta\alpha_i) \delta\beta_j$

Let $\beta_j = b_1b_2\ldots\ldots\ldots.b_n \in \Gamma^*$ then

$(\Sigma \delta\alpha_i) \delta\beta_j = (\Sigma \delta\alpha_i)\delta b_1,\delta b_2\ldots\ldots\ldots\ldots\ldots..\delta b_n$

put $b_1 = b$

$(\Sigma \delta\alpha_i) \delta b = (\Sigma \delta\alpha_i) \delta B +\delta_{1b}$

$= (\Sigma \delta\alpha_i \delta\beta)+\delta^1 b$

$= (\Sigma \delta\alpha_i B) +\delta^1 b$

$= (\Sigma \delta\alpha_i B) - \delta B +\delta b \in N$

$\therefore$ we get $\gamma_k \in \Gamma^*$ with

$(\Sigma \delta\alpha_i)\delta b_1 \delta b_2 \ldots\ldots..\delta b_n = (\Sigma \delta\alpha_i) \delta b_1) \delta b_2 \ldots\ldots..\delta b_n$

$= (\Sigma \delta\gamma_k) \delta b_2 \ldots\ldots..\delta b_n$

$= (\Sigma \delta\gamma_k) \in N$

$\therefore \quad N(M) \subseteq N$

$\therefore N(M) =\{\Sigma \delta\alpha_i / \alpha_i\in\Gamma^*\} = N$

Corollary 3.12 Let M=(Q, Γ, B, δ) be a group semi turing machine. If M is homomorphic then by the above theorem N(M) is a generalized distributively generated near ring with identity.

Note 3.13 The zero-symmetric part $N_B(M) = (N(M))_B$

Proposition 3.14 Let M=(Q, Γ, B, δ) be homomorphic then $N_B(M)$ consists of all finite sum of elements of the form $c \pm \delta - c$ with $\delta \in \{ id, \delta B,\delta^2 B,\delta^3 B,\ldots\ldots\ldots.\}$ and $c \in \{ \Sigma \delta^1\alpha_i / \alpha_i \in \Gamma^* \}$.

Proof: Let M=(Q, Γ, B, δ) be homomorphic.

Clearly all elements $c \pm \delta - c \in N_B(M)$, the zero-symmetric part.

Conversely take $\delta_1 = \Sigma \delta\alpha_i \in N_B(M)$

Thus $B = B\delta_1 = B (\Sigma \delta\alpha_i) = \Sigma B \delta\alpha_i = \Sigma \delta^1\alpha_i$

By group theory we can arrange $\delta_1= \Sigma\delta\alpha_i =\Sigma (\delta^n iB + \delta^1\alpha_i)$ of the form $c \pm \delta^n i - c$ where $c \in \{ \Sigma \delta^1\alpha_i / \alpha_i \in \Gamma^* \}$.

Theorem 3.15 For every near ring N with identity there is some group semi turing machine M with $N \cong N(M)$.

Proof: By a known theorem we can find a group Q such that N is isomorphic to a sub near-ring N^1 of map(Q).

Let Γ be an index set for N^1 i.e. $N^1 = \{ \delta a / a \in \Gamma \}$

Let δ(q,a)=qδa then $N \cong N^1 = N(M)$ with M=(Q, Γ, B, δ).

Therefore for every near ring N with identity there is some group semi turing machine M with $N \cong N(M)$.

Corollary 3.16 Since every near ring can be embedded in a near ring with identity we get every near ring can be embedded in the near ring of some group semi turing machine.

Result 3.17 For a near ring N there exists a linear group semi turing machine M with $N \cong N(M)$ iff (N, +) is abelian. N has an identity. There is some $d \in N_d$ such that N_B is generated by {1,d}.

REFERENCES

1. Adaricheva, K.V., and J.B.Nation. "Reflections on lower bounded lattices", *Algebra Universalis* 53(2005).307-330.
2. Beauquier.D. and Pin. J.E . "*Factors of words*". Volume 372 of Lecture notes in computer Science, pages 63-79, Berlin,1989. Spring Verlag.
3. Bernstein E,Vazrani, "*Quantum complexity theory*". SIAMJ comp,1997.
4. Campeanu.C. and Ho. Nu.W.H.,Culik.K. "State complexity of basic operations on finite languages". *In O. Boldt and H.Jurgensen, Proceedings of the 4th International workshop on implementing. Automata*, number 2214 in LNCS,
5. Campeanu.C. and Ho. W.H. "The maximum state complexity for finite language" .*Journal of Automata*, Languages and Combinatorics,9(2-3) 189-202,september2004.
6. Castro JL, Delgado M, Mantas C T. "*A new approach for the execution and adjustment of a fuzzy algorithm"* Fuzzy set system 2001.
7. Champarnadh.J.M. and Pin. J.E ."*A maximum problem on finite automata*". Discrete applied mathematics.
8. Crespi areghizzi.S., Pradella.M.,Pietro.P. "Associative definitions of programming languages". *Computer languages*,26:105-123,2000.
9. Eilenberg.S. "*Automata, Languages, and machines*", Vol. A. Acd Press,1974.
10. Feiner.l. *"Hierarchies of Boolean algebras".*
11. Freese,R.,Jezek.J. amd Nation.J.B."Free Lattices", *Mathematical Surveys and Monographs*, vol. 42, Amer.Math.soc., Providence, RI, 1995.
12. Gile C,Omlin C,Thornber K K. *"Equivalence in knowledge representation: Automata, Recurrent neural network and dynamical fuzzy systems."*
13. Gratzer,G.,"*General Lattice Theory*", SecEdn,Birkhauser Verlag, Basel, 1998
14. Gunter Pilz."*Near-rings*"
15. Hanf.W. "Model-theoretic methods in the study of elementary logic", *Theory of models (Proc.1963 internet. Sympos., Berkeley)*, North-Holland,Amsteerdam,1965,pp.132-145.MR 35#1457.
16. Holcombe."*Near-rings associated with automata*", San B163-166.
17. Holzer.M. and M.Kutrib. *"State complexity of basic operations on nondeterministic finite automata"*
18. Hopcroft J.E,Ullman *"Introduction to Automata theory. Language and Computation."*

19. Hopcroft.J.E. and Ullman.J.D. "*Introduction to automata theory, Languages, and computation"*,addision-Weslwy,1979.
20. Joshi.A.K., Levy.L.S., and Yueh.K. *"Local constraints in the syntax and semantics of programming languages".*
21. Lee E T,Zadeh L.A "A note on fuzzy languages" *inf Sci*, 1969.
22. Li.Y M,Pedrycz W.F. *"Fuzzy finite automata and fuzzy regular expressions with membership values in lattice -ordered monoids."*
23. Li.Y.M., Z.K.Shi, *"Finite Automata with values in lattice-monoid and their languages."*
24. Mordeson J N,Malik *"Fuzzy Automata and Languages: Theory and application."*
25. Nicaud.C "*Average state complexity of operations on unary automata*."
26. Omaratzki. M.,Kisman.D. "On the number of distinct languages accepted by finite automata with n states". *Journal of automata, languages*
27. Pedrycz W.Gomide "*An introduction to Fuzzy sets*": Cambridge mit press.
28. Repnitskii, V.B., "On finite lattices embedded in sub semi group lattices", *Semi group forum* 46(1993),388-397.
29. Salomaa. A., "*Theory of Automata*". Pergamum Press,Oxford,1969.
30. Schols.H.M.J.L. "The maximum number of states after projection", *Computing Science notes* No.87/06, Eindhoven University of technology, Dept. of. Math. And Comp. Sci., The Netherlands, April 1987.
31. Semenova. M.V., "On lattices embeddable in to suborder lattices". *Algebra and Logic* 44(2005),270-285.
32. Sivak, B., "Representation of finite lattices by orders on finite sets", *Math. Siovaca* 28(1978),203-215.

****

----

[1]*P.Vijaya Vani*
Visiting Professor, Dept of IBS
Acharya Nagarajuna University , Guntur, Andhra Pradesh, India.