SOLUTION OF THE COUPLED DRINFELD'S SOKOVWILSON (DSW) SYSTEM BY RDT METHOD

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***Abstract:** In this paper, this method is used for solving the coupled Drinfeld's-Sokolov-Wilson (DSW) system with two different given initial conditions containing arbitrary constants. The results are compared with the known exact solutions by fixing the arbitrary constants.*

***Keywords:** Reduced differential transform method (RDTM), the coupled Drinfeld's-Sokolov-Wilson (DSW) system, numerical solution.*

1. INTRODUCTION

There are various methods for solving the nonlinear PDEs. In this context, the auxiliary equation method [1], the complex hyperbolic-function method [2], the F-expansion method [3], the exp-function method [4] are worth mentioning. The Reduced differential transform method has reduced the complexity of DTM. This method is very effective and easy to execute, Keskin and Oturanç [5]. In this paper, we have applied the Reduced Differential Transform Method (RDTM) [5-6] to solve the coupled Drinfeld's-Sokolov-Wilson (DSW) system [7-11] with two different given initial conditions containing arbitrary constants for showing the efficiency of RDTM. The main advantage of the method is the fact that it provides its user with an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms. The structure of this paper is organized as follows: In section 2, we begin with some basic definitions and explain the reduced differential transformation method. In section 3, we apply this method to solve the above system with two different given initial conditions.

2. The RDTM method: The basic definitions in the reduced differential transform method [2-3] are as follows:

Definition 2.1: Let function $u(x, t)$ be analytic and k-times continuously differentiable with respect to time t and space variable x in the domain of interest, and let

$$U_k(x) = (\frac{1}{k!})(\frac{\partial^k u(x,t)}{\partial t^k})_{t=0}, \tag{1}$$

where the function $U_k(x)$ is the reduced differential transformation of the function $u(x,t)$.

For solving the Partial Differential equation by this method,

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we write the given partial differential equation in the standard form:

$$L(u)+R(u)+N(u)=0, \tag{2}$$

with initial condition $u(x,0)=f(x),$ (3)

where $L(u)=u_t(x,t), R(u)$ is a linear operator which has mixed partial derivatives and $N(u)$ is a nonlinear term. According to the RDTM, the iteration formula is

$$(k+1)U_{k+1}(x)=-R(U_k(x))-N(U_k(x)),\ k=0,1,2.. \tag{4}$$

where $R(U_k(x))$ and $N(U_k(x))$ are the reduced differential transformations of the functions $R(u(x,t))$ and $N(u(x,t)),$ respectively.

Obviously, initial condition implies $U_0(x)=u(x,t).$ (5)

Substituting $U_0(x)$ in the iteration formula (4), we obtain the values of $U_k(x)$. Then the differential inverse transformation of the set of values $[U_k(x)]_{k=0}^{n}$ gives nth order approximation solution as $\overline{u_n}(x,t)=\sum_{k=0}^{n}\frac{1}{k!}(\frac{\partial^k u(x,t)}{\partial t^k})_{t=0}t^k.$ (6)

Therefore, the differential inverse transform of $U_k(x)$ is given by

$$u(x,t)=\lim_{n\to\infty}\overline{u_n}(x,t). \tag{7}$$

3. APPLICATIONS

3.1. The Coupled Drinfeld's-Sokolov-Wilson (DSW) system: We consider the coupled Drinfeld's-Sokolov-Wilson (DSW) system:

$$u_t+\alpha vv_x=0, \tag{8}$$

$$v_t+\beta v_{xxx}+\gamma uv_x+\varepsilon u_x v=0, \tag{9}$$

where $\alpha,\beta,\gamma,\varepsilon$ are nonzero parameters. The system was first proposed by Drinfeld and Sokolov [7, 8] and Wilson [9] when $\alpha=3,\beta=\gamma=2 and \varepsilon=1.$ The above system has the following initial conditions [10]:

$$u(x,0)=\frac{6c}{(\gamma+2\varepsilon)}(\sec h[\sqrt{\frac{c}{\beta}}(x)])^2, \tag{10}$$

$$v(x,0)=2\sqrt{3}\sqrt{\frac{c^2}{\alpha(\gamma+2\varepsilon)}}\sec h[\sqrt{\frac{c}{\beta}}(x)].$$

Here, $u=u(x,t), v=v(x,t)$ are the solutions of (8) and (9). The true solutions for the equations (8) and (9) obtained by the Variational Approach [24] are given by

$$u(x,t)=\frac{6c}{(\gamma+2\varepsilon)}(\sec h[\sqrt{\frac{c}{\beta}}(x-ct)])^2, \tag{11}$$

$$v(x,t)=2\sqrt{3}\sqrt{\frac{c^2}{\alpha(\gamma+2\varepsilon)}}\sec h[\sqrt{\frac{c}{\beta}}(x-ct)]. \tag{12}$$

We solve the system (8) and (9) by the RDTM. Taking the Reduced Differential Transformation of both sides of equations (8) and (9), we obtain the iterative scheme as follows: $(k+1)U_{k+1}(x)=-\alpha N_k(U_k(x)),\ k=0,1,2..$ (13)

and $(k+1)V_{k+1}(x)=-\beta\frac{\partial^3}{\partial x^3}(V_k(x))+\gamma G_k(U_k(x),V_k(x))+$
$\varepsilon M_k(U_k(x),V_k(x)),\ k=0,1,2...,$ (14)

where $N_k(U_k(x))$, $G_k(U_k(x),V_k(x))$ and $M_k(V_k(x),U_k(x))$ are the Reduced Differential Transformations of $vv_x, -uv_x$ and $-u_xv$; respectively. Using the initial conditions (10), we obtain

$$U_0=u(x,0)=\frac{6c}{(\gamma+2\varepsilon)}(\sec h[\sqrt{\frac{c}{\beta}}(x)])^2, \tag{15}$$

$$V_0(x)=v(x,0)=2\sqrt{3}\sqrt{\frac{c^2}{\alpha(\gamma+2\varepsilon)}}\sec h[\sqrt{\frac{c}{\beta}}(x)]. \tag{16}$$

Now, substituting $k=0,1,2,3\ldots$ in equations (13) and (14) and using (15) and (16), we obtain the various terms of the RDTM.

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For brevity, we have not written the other computed terms. However we have obtained the second order approximation solution. Therefore, the differential inverse transformation of the set of values $[U_k(x)]_{k=0}^{2}$ and $[V_k(x)]_{k=0}^{2}$ gives the second order approximation solution for $u(x,t)$ and $v(x,t)$ as

$$u_2(x,t) = \sum_{k=0}^{2} U_k(x)t^k = U_0(x) + U_1(x)t + U_2t^2,$$

$$v_2(x,t) = \sum_{k=0}^{2} V_k(x)t^k = V_0(x) + V_1(x)t + V_2t^2.$$

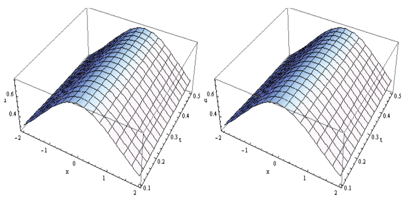
The comparisons of the computed solution with the exact solutions are made in the following tables:

t	x	u_{RDTM}	u_{exact}	Absolute error
0.3	-1	0.54775757	0.54796225	0.00020469
	0	0.74578125	0.74579702	0.00001577
	1	0.62952954	0.62931846	0.00021108
0.5	-1	0.51838072	0.51931436	0.00093364
	0	0.73828125	0.73840225	0.00012099
	1	0.65466734	0.65368498	0.00098236

Table1. Comparison of the RDTM approximate solution $u(x,t)$ with the exact solution (11) of the coupled DSW system for $\alpha = 3, \beta = \gamma = 2, \varepsilon = 1 and c = 0.5.$

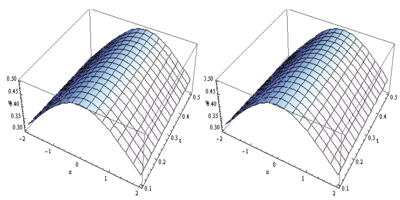
t	x	v_{RDTM}	v_{exact}	Absolute error
0.3	-1	0.42732696	0.42738049	0.00005353
	0	0.49859375	0.49859704	0.00000329
	1	0.45806302	0.45800963	0.00005339
0.5	-1	0.41581146	0.41605863	0.00024717
	0	0.49609374	0.49611902	0.00002528
	1	0.46703824	0.46679224	0.00024600

Table2. Comparison of the RDTM approximate solution $v(x,t)$ with the exact solution (12) of the coupled Drinfeld's-Sokolov-Wilson (DSW) system for $\alpha = 3, \beta = \gamma = 2, \varepsilon = 1 and c = 0.5.$



(i) (ii)

Fig.1. The graphs for $u(x,t)$ ***(given in (i)) in comparison with the exact analytical solutions*** $u(x,t)$ ***(given in (ii)).***



(i) (ii)

Fig.2. The graphs for $v(x,t)$ ***(given in (i)) in comparison with the exact analytical solutions*** $v(x,t)$ ***(given in (ii)).***

3.2. The coupled DSW system with different I.C.

Again, Consider coupled Drinfeld's-Sokolov-Wilson (DSW) system (8)-(9) with initial conditions [11] of the form:

$$U_0(x) = u(x,0) = \frac{(c-4k)}{2} + 3k^2(\sec h[kx])^2, \tag{17}$$

$$V_0(x) = v(x,0) = 2k\sqrt{\frac{c}{2}}\sec h[kx], \tag{18}$$

where c and k are the non-zero arbitrary constants. The exact solutions of the coupled DSW are given by

$$u(x,t) = \frac{(c-4k)}{2} + 3k^2(\sec h[k(x-ct)])^2, \tag{19}$$

and $$v(x,t) = 2k\sqrt{\frac{c}{2}}\sec h[k(x-ct)]. \tag{20}$$

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For brevity, we have not written the other computed terms. However we have obtained the second order approximation solution. Therefore, the differential inverse transformation of the set of values $[U_k(x)]_{k=0}^{2}$ and $[V_k(x)]_{k=0}^{2}$ gives the second order approximation solution for $u(x,t)$ as $u_2(x,t)=\sum_{k=0}^{2}U_k(x)t^k$ $=U_0(x)+U_1(x)t+U_2t^2$.

$$v_2(x,t)=\sum_{k=0}^{2}V_k(x)t^k \;=V_0(x)+V_1(x)t+V_2t^2.$$

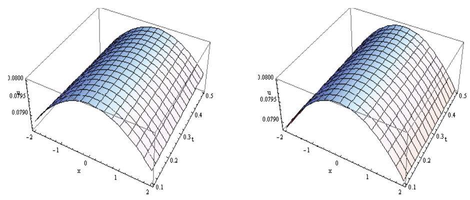
The comparisons of the computed solutions by RDTM with the exact solution are made in the following tables:

t	x	u_{RDTM}	u_{exact}	Absolute error
0.3	-1	0.07961162	0.07969304	0.00008142
	0	0.07999838	0.07999993	0.00000155
	1	0.07978924	0.07971080	0.00007844
0.5	-1	0.07954964	0.07968700	0.00013736
	0	0.07999540	0.07999981	0.00000441
	1	0.07984568	0.07971661	0.00012907

Table3. Comparison of the RDTM approximate solution $u(x,t)$ with the exact solution (19) of the coupled Drinfeld's-Sokolov-Wilson (DSW) system for $\alpha=3,\beta=\gamma=2,\varepsilon=1,c=0.5. and k=0.1.$

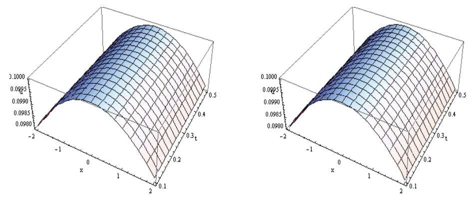
t	x	v_{RDTM}	v_{exact}	Absolute error
0.3	-1	0.09946478	0.09948708	0.00002230
	0	0.09999832	0.09999988	0.00000156
	1	0.09953618	0.09951684	0.00001934
0.5	-1	0.09943814	0.09947698	0.00003884
	0	0.09999535	0.09999969	0.00000434
	1	0.09955715	0.09952656	0.00003059

Table4. Comparison of the RDTM approximate solution $v(x,t)$ with the exact solution (20) of the coupled Drinfeld's-Sokolov-Wilson (DSW) system for $\alpha=3,\beta=\gamma=2,\varepsilon=1,c=0.5. and k=0.1.$



(i) (ii)

Fig.3. The numerical results for $u(x,t)$ (given in (**i**)) in comparison with the exact analytical solutions $u(x,t)$ (given in (**ii**)).



(i) (ii)

Fig.4. The numerical results for $v(x,t)$ ***(given in (i)) in comparison with the exact analytical solutions*** $v(x,t)$ ***(given in (ii)).***

4. CONCLUSION

The main aim of this article is to construct approximate solutions of the coupled Drinfeld's-Sokolov-Wilson (DSW) system. As the method is usually tedious to apply by hand, we have used the software package "MATHEMATICA" to calculate few terms of the series obtained from the RDTM. The numerical results are compared with the exact solutions in Tables 1, 2, 3 and 4. The approximate and exact solutions are also compared in Figures 1, 2, 3 and 4.

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