SPANNER-SUM OF CORONA GRAPHS AND EXTENDED MESS

Dr.V.Kaladevi[1], G. Sharmila Devi[2]

Abstract: ***For a spanning tree T of a graph G, the spanner – sum is defined by,*** $\xi(T,G)=\sum d_T(u,v)$, ***where*** $(u,v) \in E(G) - E(T)$ ***and*** $d_T(u,v)$ ***denotes the distance between u and v in T. Then the minimum spanner sum of G is defined as*** $\xi(G)=\min\xi(T,G)$, ***where the minimum is taken over all the spanning trees T of G. The spanner-sum problem of a graph G is to find a spanning tree T of G that induces the minimum spanner–sum*** $\xi(G)$. ***In this paper, spanner-sum of Corona graphs and extended mesh are found.***

Keywords : Corona Graphs, Extended mesh, Spanner-sum.

1. INTRODUCTION

A tree spanner of a graph G is a spanning tree T that minimizes the distance between the vertices in the original graph G. For a given graph G (V, E) and a spanning subgraph H of G, $d_G(u,v)$ and $d_H(u,v)$ denotes the shortest distance between the vertices u and v in G and H respectively. A spanning subgraph H of a graph G is a t-spanner of G if $d_H(u,v) = td_G(u,v)$ for every pair of vertices u and v of G[1, 2, 9]. For a spanning tree T of G, the spanner sum is defined by $\xi(T,G)=\sum d_T(u,v)$, where $(u,v) \in E(G) - E(T)$ and $d_T(u,v)$ denotes the distance between u and v in T. Then the minimum spanner sum of G is defined as $\xi(G)=\min\xi(T,G)$, where the minimum is taken over all the spanning trees T of G [4,7,8]. The spanner-sum problem of a graph G is to find a spanning tree T of G that induces the minimum spanner sum $\xi(G)$.

In this paper the spanner – sum of corona graphs and extended mesh is estimated.

2. SPANNER-SUM OF CORONA GRAPHS

Definition : 2.1

The corona graph $G_1 \circledast G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ was defined by Frucht and Harary as the graph G obtained by taking one copy of G_1 and p_1 copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. For example the corona graph $G = C_6 \circledast P_2$ is shown in Fig.1.

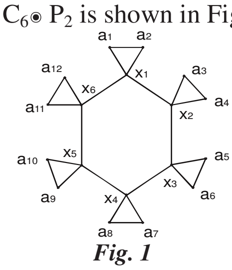

Fig. 1

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Theorem 2.2 : Let G be the corona graph of C_n and P_m. Then the spanner-sum of G is $\xi(G) = 2mn - (n+1)$.

Proof : Let C_n be a cycle with n-vertices and P_m be a path with m vertices. The corona graph $C_n \odot P_m$ is obtained by taking one copy of C_n and n copies of P_m and then joining i^{th} vertex of C_n to every vertex in the i^{th} copy of P_m.

Let $C_n, n \geq 3$ be a cycle with the consecutive vertices $x_1, x_2, x_3, \ldots\ldots, x_n$ in the clockwise direction with x_1 and x_n respectively the start and end vertex.

Let $P_m^1, P_m^2, P_m^3, \ldots\ldots, P_m^n$ be the first, second, third… n^{th} copies of P_m respectively.

Let $V(P_m^1) = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$

$V(P_m^2) = \{a_{m+1}, a_{m+2}, a_{m+3}, \ldots\ldots, a_{2n}\}$ …………

$V(P_m^n) = \{a_{(n-1)m+1}, a_{(n-1)m+2}, a_{(n-1)m+3}, \ldots\ldots, a_{nm}\}$

be the vertex sets $P_m^1, P_m^2, P_m^3, \ldots\ldots, P_m^n$ respectively. By the definition of corona graph the vertex of G is

$V(G) = \{x_i ; 1 \leq i \leq n\} \cup \{a_i ; 1 \leq i \leq mn\}$ and the edge set of G is $E(G) = \{x_n x_1, x_i x_{i+1} ; 1 \leq i \leq n-1\} \cup \{a_i a_{i+1} ; 1 \leq i \leq m-1\} \cup \{a_i a_{i+1} ; m+1 \leq i \leq 2m-1\} \cup \{a_i a_{i+1}; 2m+1 \leq i \leq 3m-1) \cup \ldots\ldots \cup \{a_i a_{i+1}; nm-m+1 \leq i \leq nm-1) \cup \{x_1 a_i ; 1 \leq i \leq m\} \cup \{x_2 a_i ; m+1 \leq i \leq 2m\} \cup \ldots. \cup \{x_n a_i ; (n-1) m+1 \leq i \leq nm\}$

Case (i) : Let $G = C_3 \odot P_2$
The graph G is shown in Fig. 2

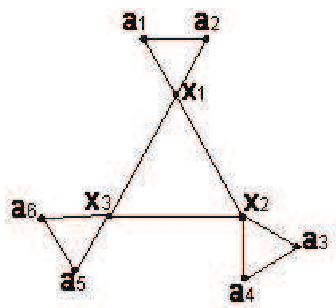

Fig. 2

The graph G consists of 4 number of 3-cycles. The removal edges $a_1 a_2, a_3 a_4, x_1 x_2$ leaves the spanning tree as shown in the Fig. 3

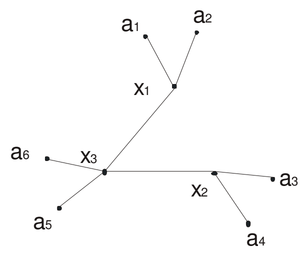

Fig. 3

The minimum spanner sum of G is $\xi(G) = (3-1)+3(2) = 8$

Case (ii) : Let $G = C_4 \circledcirc P_3$
The graph G is shown in Fig. 4

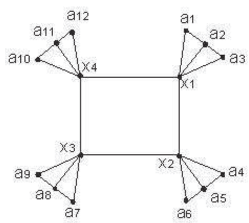

Fig. 4

The graph consists of 1-number of 4-cycle & 8-numbers of 3-cycles. The removal of edges $a_1a_2, a_2a_3, a_4a_5, a_5a_6, a_7a_8, a_8a_9, a_{10}a_{11}, a_{11}a_{12}, x_1x_2$ leaves the spanning tree as shown in Fig. 5

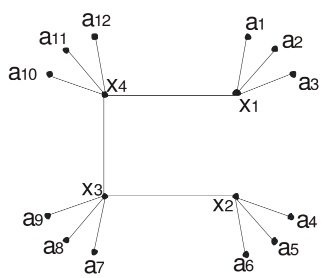

Fig. 5

The minimum spanner sum of G is $\xi(G) = (4-1)+8(2) = 19$

In general, Let $G = C_n \circledcirc P_m$. The graph G consists of 1-number of n-cycle and (m-1)n number of 3-cycles. The removal of edges

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$a_1a_2, a_2a_3, \ldots\ldots, a_{m-1}a_m$ and $a_{m+1}a_{m+2}, a_{m+2}a_{m+3}, \ldots, a_{2m-1}a_{2m}$ and … $a_{(n-1)m+1}a_{(n-1)m+2}, \ldots, a_{nm-1}a_{nm}$ and x_1x_2 gives the spanner tree, with minimum spanner sum.

The Minimum spanner sum of G is
$\xi(G) = (n-1) + (m-1)n(2) = n-1 + 2mn - 2n = 2mn - (n+1)$

Theorem 2.3 : Let G be the corona graph of P_m and C_n. Then the spanner-sum of G is $\xi(G) = 2mn$.

Proof : Let P_m be the path with m-vertices and C_n be a cycle with n-vertices. The corona graph $P_m \odot C_n$ is obtained by taking one copy of P_m and m copies of C_n and then joining the i^{th} vertex of P_m to every vertex in the i^{th} copy of C_n. Let P_m be a path with consecutive vertices $x_1, x_2, x_3, \ldots, x_m$. Let $C_n^1, C_n^2, C_n^3, \ldots, C_n^m$ be the first, second, third ……..m^{th} copy of C_n respectively.

Let $V(C_n^1) = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$ $V(C_n^2) = \{a_{n+1}, a_{n+2}, a_{n+3}, \ldots\ldots, a_{2n}\}$
………..
$V(C_n^m) = \{a_{(m-1)n+1}, a_{(m-1)n+2}, a_{(m-1)n+3}, \ldots\ldots, a_{mn}\}$

be the vertex sets of $C_n^1, C_n^2, C_n^3, \ldots\ldots, C_n^m$ respectively. By the definition of corona graph the vertex set of G is

V (G) $= \{x_i;\ 1 \le i \le m\} \cup \{a_i,\ 1 \le I \le mn\}$ and the edge set of G is
E(G) $= \{x_i\, x_{i+1};\ 1 \le i \le m-1\} \cup \{x_1a_i;\ 1 \le i \le n\}$
$\cup \{x_2a_i;\ 1 \le i \le n\} \cup \ldots\ldots \cup \{x_ma_i;\ 1 \le i \le n\}$
$\cup \{a_na_1,\ a_ia_{i+1};\ 1 \le i \le n-1\}$

Case (i) : Let $G = P_2 \odot C_3$

The Graph G is shown in Fig. 6.

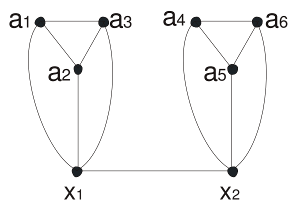

Fig. 6

Now generate the spanning tree of G, by deleting edges in all the copies of C_3 as shown in Fig. 7.

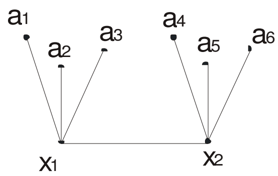

Fig. 7

The spanning tree gives the minimum spanner sum of G.

$\xi(G) = 2(3 \times 2) = 12$

Case (ii) : Let G = $P_3 \odot C_4$

The graph G is shown in Fig. 8.

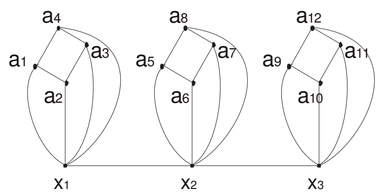

Fig. 8

Now construct the spanning tree of G, by deleting edges all the copies of C_4, is shown in Fig. 9.

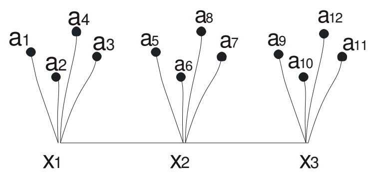

Fig. 9

The above such spanning tree gives the minimum spanner sum of G.

$\xi(G) = 3(4 \text{ x } 2) = 24$

In general, when G = $P_m \odot C_n$, the deletion of all edges in C_n, gives the spanner sum tree of G.

Thus $\xi(G) = m(n \text{ x } 2) = 2mn$.

3. EXTENDED MESH

A cellular neural network (CNN) is a network in which connections are limited to units in local neighbourhoods of individual units with bidirectional signal paths. It

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has a structure similar to a cellular automaton. Units in a CNN are arranged in a d-dimensional array. When d = 2, CNN provides a good match to computations in image processing applications. A two-dimensional cellular neural network can be modeled in graph theory by taking the units as vertices and the connection between the units as edges.

If the two-dimensional CNN has m rows and n-columns then it is nothing but the extended mesh denoted by EX(m,n). The number of vertices in EX(m,n) is mn and the number of edges in EX(m,n) is 4mn-3m-3n+2. We name the vertices of EX(m,n) as $v_{n(j-1)+k}$ for j = 1,2,…..m, k = 1,2,….n. The graph EX(4,4) is show in the Fig. 10.

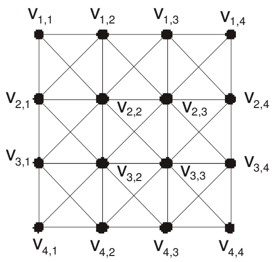

Fig. 10

Theorem 3.1 :

The spanner-sum $\xi(EX(2,2)) = 6$, $\xi(EX(3,3)) = 24$,
$\xi(EX(4,4)) = 68$, $\xi(EX(2,3)) = 12 = \xi(EX(3,2))$,
$\xi(EX(2,4)) = 19 = \xi(EX(4,2))$, $\xi(EX(3,4)) = 38 = \xi(EX(4,3))$.
Proof : Let G = EX(2,2)

The graph G is shown in Fig. 11.

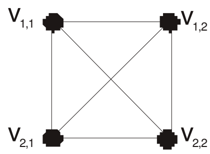

Fig. 11

The removal of edges $v_{12}v_{22}$, $v_{21}v_{22}$, $v_{12}v_{21}$ leaves the spanning tree as star graph with v_{11} as the center vertex as shown in Fig. 12.

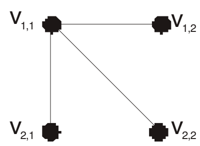

Fig. 12

The minimum spanner-sum of G is $\xi(G) = 6$.
Let G = EX(2,3). The graph G is shown in Fig. 13.

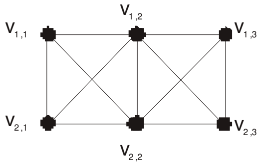

Fig. 13

Now generating the spanning tree as star graph with v_{12} as the center vertex and remaining all the vertices are pendant vertices as shown in Fig. 14.

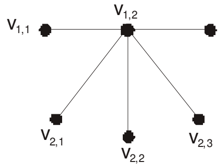

Fig. 14

The minimum spanner-sum of G is $\xi(G) = 12$.
Similarly, the spanner-sum $\xi(EX(3,3)) = 24$,
$\xi(EX(4,4)) = 68$, $\xi(EX(3,2)) = 12$,
$\xi(EX(2,4)) = \xi(EX(4,2)) = 19$, $\xi(EX(3,4)) = 38 = \xi(EX(4,3))$.

Theorem 3.2 :
The spanner- sum $\xi(EX\ (2, n)) = 7(n – 1)$,
$\xi(EX\ (3, n)) = 14(n – 1)$, $\xi(EX\ (4, n)) = 3(9n-13)$,
$\xi(EX\ (5, n)) = 4(10n – 16)$ and $n \geq 5$.

Proof : Let G = Ex (m, n) be the extended mesh.
Let $V(G) = \{v_{ij} ; 1 \leq i \leq m, 1 \leq j \leq n\}$ and
$E(G) = \{(v_{ij},v_{i(j+1)}) ; 1 \leq i \leq m, 1 \leq j \leq n\text{-}1\}$
$\cup \{(v_{ij},v_{j(i+1)}) ; 1 \leq i \leq m\text{-}1, 1 \leq j \leq n\}$
$\cup \{(v_{ij},v_{(i+1)(j+1)}) ; 1 \leq i \leq m\text{-}1, 1 \leq j \leq n\text{-}1\}$
$\cup \{(v_{ij},v_{(i+1)(j-1)}) ; 1 \leq i \leq m\text{-}1, 2 \leq j \leq n+1\}$
be respectively the vertex set and the edge set of G.

Case : 1
Let G = EX (2, n) and $n \geq 5$. There are (n-1) number of 4 cycles and 2(n-1) numbers of 3 cycles in G. Hence $\xi(G) \geq 3(n\text{-}1) + 2 \times 2(n\text{-}1)$

Deleting the edges in the second row and deleting diagonal edges in each square, we get the minimum spanner-sum.
$\xi(G) = 3(n-1) + 2 \times 2(n-1) = 3n-3+4n-4 = 7n-7 = 7(n-1)$

Case : 2
Let G = EX (3, n) and $n \geq 5$.
Following the method of construction of spanning trees as in case (1), the minimum spanner sum $\xi(G) = 14(n - 1)$.

Case : 3

Let G = EX (4, n), $n \geq 5$.

A 4 cycle of G is said to be of type I if there exist an edge e in the 4-cycle such that G – e merges the region bounded by the 4-cycle with the exterior region. If a 4-cycle of G does not possess such an edge e, then the 4-cycle is said to be of type II. In G there are (n – 3) number of 4-cycles of type II. For each cycle of type II, deleting any edge of 4-cycle merges in the region bounded by the 4-cycle with another region of G yielding a region bounded by the 6-cycle. Now, label the vertices of i^{th} row and j^{th} column of an order pair (i,j), $1 \leq i \leq m$, $1 \leq j \leq n$ and construct a spanning T of G as follows. Remove the edges [(i,j), (i,j+1)], i = 1,2,4 and j = 2,3,….n-2 and all the edges in the first and last column of G and all diagonal in each square. Now there are 2n cycles of length 4 and 4n cycles of length 3, (n – 3) cycles of length 6, 2(n-3) cycles of length 5. Therefore, the minimum spanner-sum

$$\xi(G) = 3(2n)+2(4n)+5(n-3)+8(n-3) = 3(9n-13).$$

Case : 4
Let G = EX (5, n) and $n \geq 5$.
There are 8 cycles of length 4, 16 cycles of length 3, 2(n – 3) cycles of length 4, 2 x 2(n – 3) cycles of length 3, 2(n – 3) cycles of length 6 and 2 x 2(n – 3) cycles of length 5. Hence $\xi(G) \geq 8(3) + 16(2) + 6(n - 3) + 8(n - 3) + 10(n - 3) + 16(n - 3)$.

Construct a spanning tree T of G as follows:

(i) Include all the edges in 2 to n – 1.
(ii) Include all the edges in 3^{rd} row.
(iii) Include all the edges ((i,1), (i,2)), $1 \leq i \leq 5$ and the edges ((i, n–1), (i,n)), $1 \leq i \leq 5$. Hence the minimum spanner-sum of G is

$$\xi(G) = 8(3) + 16(2) + 6(n - 3) + 8(n - 3) + 10(n - 3) + 16(n - 3) = 40n - 64 = 4(10n - 16).$$

For the graph EX (6, 5), EX (7, 5) the spanner-sum will be obtained by the above method is the minimum sum and the spanner-sums are 176 and 216 respectively.
For the case n = 6, the spanner-sum of EX (6, 6), EX (7, 6) will be minimum if we construct the spanning tree by the following method.

Generate the spanning tree from any of the center vertex having maximum degree. If this tree is the spanning tree then we stop the process. Otherwise, a tree is generated from the next center vertex of highest degree. If the resulting tree is the minimum spanner-sum tree, then we stop the process. Otherwise the process is repeated until the spanner-sum tree is obtained.

$$\xi(EX\,(6, 6)) = 242$$
$$\xi(EX\,(7, 6)) = 304.$$

4. CONCLUSION

In this paper the spanner-sum of EX(m,n) when $2 \leq m \leq 5$ and $n \geq 2$, EX(m,n) when m = 6 & 7, n = 5 & 6 are derived. For other values of m and n, the spanner sum of EX (m,n) is under study.

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