EFFECT OF HEAT TRANSFERE ON OSCILLATORY FLOW OF A JEFFREY FLUID THROUGH A POROUS MEDIUM IN A CHANNEL

K. Srinivasa Rao[1], P. Koteswara Rao[2], M. V. Subba Reddy[3]

Abstract: In this paper, the effect of heat transfer on oscillatory flow of a Jeffrey fluid through a porous medium in a channel is investigated. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

Keywords: Heat Transfer, Jeffrey fluid, Oscillatory flow, Porous medium.

1. INTRODUCTION

Flow through porous media is very prevalent in nature, and therefore the study of flow through a porous medium has become of principle interest in many and engineering applications. Thermal and solutal transport by fluid flowing through a porous matrix is phenomenon of great interest from the theory and application point of view. Heat transfer in the case of homogenous fluid-saturated porous media has been studied with relation of different applications like dynamic of hot underground springs, terrestrial heat flow through aquifer, hot fluid and ignition front displacements in reservoir engineering, heat exchange between soil and atmosphere and heat exchanges with fluidized beds. Moreover, the oscillatory flows have special relevance in vibrating media with applications in oil-drilling, control of blood flow during surgical operations, chemicals and material processing, isotope separation, irrigation systems, rocket propulsion, filtration mechanism, sweat cooling, cooling of electronic device, heat exchanger and many others. Raptis et al. (1982) have discussed the hydromagnetic free convection flow through a porous medium between two parallel plates. Oscillatory viscous flow in a porous channel with arbitrary wall suction was studied by Jankowski and Majdalani (2002). Makinde and Mhone (2005) have investigated a heat transfer to MHD Oscillatory flow in a channel filled with porous medium. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was studied by Hamza et al. (2011).

Further the non-Newtonian fluids are more appropriate than Newtonian fluids in many practical applications. Examples of such fluids include certain oils, lubricants, mud, shampoo, ketchup, blood at low shear rate, cosmetic products, polymers and many others. Unlike the viscous fluids, all the non-Newtonian fluids (in terms of their diverse characteristics) cannot be described by a single constitutive relationship. Hence, several models of non-Newtonian fluids are proposed in the literature. Al Khatib andWilson (2001) have studied the Poiseuille flow of a yield stress fluid in a channel. Flow of a visco-plastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan (2004). The steady MHD flow of an incompressible electrically conducting viscoelastic fluid through a porous medium between two porous parallel plates under the influence of a transverse magnetic field with heat and mass transfer in porous media was analyzed by Eldabe and Sallam (2005). Ali and Asghar (2011) have analyzed by oscillatory channel flow for non-Newtonian fluid. Rita and Jyoti Das (2012) have studied the effect of heat transfer on MHD oscillatory viscoelastic fluid flow in a channel through a porous medium.

In view of these we studied the effect of heat transfer on oscillatory flow of a Jeffrey fluid through a porous medium in a channel. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

2. MATHEMATICAL FORMULATION:

We consider the flow of a Jeffrey fluid through a porous medium in a channel of width h with radiative heat transfer as depicted in Fig.1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. We choose the Cartesian coordinate system (x, y), where x - is taken along center of the channel and the y - axis is taken normal to the flow direction.

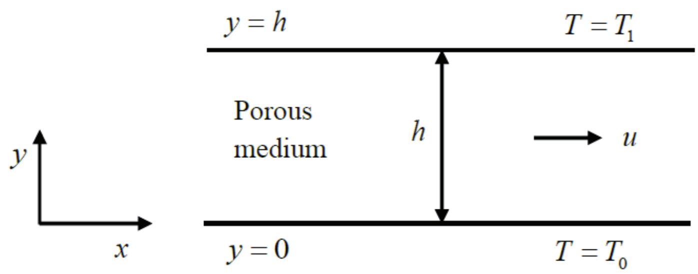

Fig. 1 Geometry of the problem

The constitute equation of S for Jeffrey fluid is

$$S = \frac{\mu}{1+\lambda_1}(\dot{\gamma} + \lambda_2 \ddot{\gamma}) \tag{2.1}$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}u + \rho g \beta \left(T - T_0\right) \tag{2.2}$$

$$\rho \frac{\partial T}{\partial t} = \frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q}{\partial y} \tag{2.3}$$

The boundary conditions are given by

$$u = 0, \quad T = T_0 \quad \text{at} \quad y = 0 \tag{2.4}$$

$$u = 0, \quad T = T_1 \quad \text{at} \quad y = h \tag{2.5}$$

where u is the axial velocity, T is the fluid temperature, p is the pressure, ρ is the fluid density, K is the permeability of the porous medium, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity and q is the radiative heat flux. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 \left(T_0 - T\right) \tag{2.6}$$

here α is the mean radiation absorption coefficient.

Introducing the following non-dimensional variables

$$\bar{x} = \frac{x}{h},\ \bar{y} = \frac{y}{h},\ \bar{u} = \frac{u}{U},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \bar{t} = \frac{tU}{h},\ \bar{p} = \frac{pa}{\mu U},\ Da = \frac{K}{h^2},$$

$$Gr = \frac{\rho g \beta \left(T_1 - T_0\right)}{U\mu},\ \mathrm{Re} = \frac{\rho h U}{\mu},\ Pe = \frac{\rho h U c_p}{k},\ N^2 = \frac{4\alpha^2 h^2}{k}$$

here U is the mean flow velocity, into the equations (2.2) and (2.3), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - \frac{1}{Da}u + Gr\theta \tag{2.7}$$

$$Pe\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + N^2\theta \tag{2.8}$$

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where Re is the Reynolds number, Da is the Darcy number, Gr is the Grashof number, Pe is the Peclet number and N is the radiation parameter.

The corresponding non-dimensional boundary conditions are

$$u=0,\quad \theta=0 \quad \text{at} \quad y=0 \tag{2.9}$$

$$u=0,\quad \theta=1 \quad \text{at} \quad y=1 \tag{2.10}$$

3. SOLUTION:

In order to solve equations (2.7) – (2.10) for purely oscillatory flow, let

$$-\frac{\partial p}{\partial x}=\lambda e^{i\omega t} \tag{3.1}$$

$$u(y,t)=u_0(y)e^{i\omega t} \tag{3.2}$$

$$\theta(y,t)=\theta_0(y)e^{i\omega t} \tag{3.3}$$

where λ is a real constant and ω is the frequency of the oscillation.

Substituting the equations (3.1) - (3.3) in to the equations (2.7) – (2.10), we get

$$\frac{d^2u_0}{dy^2}-m_2^2u_0=-\lambda(1+\lambda_1)-Gr(1+\lambda_1)\theta_0 \tag{3.4}$$

$$\frac{d^2\theta_0}{dy^2}+m_1^2\theta_0=0 \tag{3.5}$$

with the boundary conditions

$$u_0=0,\quad \theta_0=0 \quad \text{at} \quad y=0 \tag{3.6}$$

$$u_0=0,\quad \theta_0=1 \quad \text{at} \quad y=1 \tag{3.7}$$

in which $m_1=\sqrt{N^2-i\omega Pe}$ and $m_2=\sqrt{\frac{1}{Da}+i\omega\mathrm{Re}}$.

Solving equations (3.4) and (3.5) using the boundary conditions (3.6) and (3.7), we obtain

$$u_0(y)=-A\cosh m_2y+C\frac{\sinh m_2y}{\sinh m_2}+A+B\frac{\sin m_1y}{\sin m_1} \tag{3.8}$$

and $$\theta_0(y)=\frac{\sin m_1 y}{\sin m} \tag{3.9}$$

where $A=\frac{\lambda(1+\lambda_1)}{m_2^2}$, $B=\frac{Gr(1+\lambda_1)}{(m_1^2+m_2^2)}$ and $C=(A\cosh m_2-A-B)$.

Therefore, the fluid velocity and temperature are given as

$$u(y,t)=\left(-A\cosh m_2 y+C\frac{\sinh m_2 y}{\sinh m_2}+A+B\frac{\sin m_1 y}{\sin m_1}\right)e^{i\omega t} \tag{3.10}$$

and $$\theta(y,t)=\frac{\sin m_1 y}{\sin m}e^{i\omega t} \tag{3.11}$$

4. DISCUSSION OF THE RESULTS

Fig. 2 shows the effect of material parameter λ_1 on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.5$, $N=1$ and $Da=0.1$. It is observed that, the axial velocity u increases with increasing λ_1. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid $(\lambda_1>0)$ than that of Newtonian fluid $(\lambda_1\to 0)$.

Effect of Darcy number Da on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$ is shown in Fig.3. It is found that, the axial velocity u increases with increasing Da.

Fig. 4 depicts the effect of radiation parameter N on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.5$, $Da=0.1$ and $\lambda_1=0.3$. It is noted that, the axial velocity u increases with an increase in N.

Effect of Grashof number Gr on velocity u for $\mathrm{Re}=1$, $Da=0.1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$ is depicted in Fig.5. It is observed that, the axial velocity u increases with increasing Gr.

Fig. 6 illustrates the effect of Reynolds number Re on velocity u for $Da=0.1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$. It is found that, the axial velocity u decreases with decreasing Re.

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Effect of λ on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $Da=0.1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$ is illustrated in Fig. 7. It is noted that, the axial velocity u increases with an increase in λ.

Fig. 8 shows the effect of ω on velocity u for $\mathrm{Re}=1$, $Gr=1$, $Pe=0.71$, $\lambda=1$, $Da=0.1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$. It is observed that, the axial velocity u decreases with decreasing ω.

Effect of Peclet number Pe on velocity u for $\mathrm{Re}=1$, $Da=0.1$, $Gr=1$, $\lambda=1$, $\omega=1$, $t=0.5$, $N=1$ and $\lambda_1=0.3$ is depicted in Fig. 9. It is noted that, the axial velocity u increases with increasing Pe.

Fig.10 shows the effect of N on temperature θ for $Pe=0.71$, $\omega=1$ and $t=0.5$. It is noted that, the temperature θ increases with an increase in N.

Effect of Pe on temperature θ for $N=1$, $\omega=1$ and $t=0.5$ is shown in Fig. 11. It is observed that, the temperature θ increases with increasing Pe.

Fig. 12 depicts the effect of ω on temperature θ for $N=1$, $Pe=0.71$ and $t=0.5$ Fig. 11. It is found that, the temperature θ decreases with an increase in ω

5. CONCLUSIONS

In this paper, we studied the effect of heat transfer on oscillatory flow of a Jeffrey fluid through a porous medium in a channel. The expressions for the velocity and temperature are obtained analytically. It is found that, the velocity u increases with increasing $\lambda_1, Da, N, Gr, \mathrm{Re}, Pe$ and λ, while it decreases with increasing ω. Also, it is observed that the temperature θ increases with increasing N and Pe, while it decreases with increasing ω. Further, it is found that, the velocity is more for Jeffrey fluid than that of Newtonian fluid.

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[1]Department of Mathematics,Acharya Nagarjuna University, Guntur, India
thrinaina12@gmail.com.
[2]Department of Mathematics, Acharya Nagarjuna University, Guntur, India
[3]Department of CSE, Sri Venkatesa Perumal College of Engineering & Technology, Puttur, India,
drmvsr1979@gmail.com