A TRAVELLING SALESMAN PROBLEM, A CASE OF KRUSKAL ALGORITHM AND SOME DISTRIBUTION CENTRE'S OF ARZIKI TABLE WATER IN ALIERO'S METROPLIS

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Abstract: Kruskal algorithm has been used to determine the best route that can be used to deliver goods in the various distribution centers of Arziki Table water in Aliero's metropolis. It has been discovered that in this case of traveling salesman problem, the total distances traveled in the course of distributing goods of the company to its trading units could be significantly reduced by using some graph theoretic concepts, such as the Kruskal algorithm to construct a minimum spanning tree which ensures least distances and avoids repetition of routes.

Keywords; Traveling Salesman Problem, Kruskal Algorithm, Arziki Table Water and Minimum Spanning Tree

1. INTRODUCTION

A traveling salesman hopes to visit a number of locations and then return to the starting point. Considering the time duration of traveling and costs between such locations, he desires to plan his itinerary so that he visits each location exactly once and travels in all within the shortest possible time possible. In graphical terms, the aim is to find a minimum-weight Hamilton cycle in a weighted complete graph. Such a cycle is called an optimal cycle as in accordance with Ibrahim and Audu[2]. In contrast to the shortest path problem no efficient algorithms exists for finding this optimal cycle (Ibrahim and Audu) [2]).

In fact, according to Ibrahim [4], the traveling salesman problem is NP-complete as also reported by Claude and Dinneen [1]. Efforts are therefore geared towards finding approximation algorithms and heuristics to try and obtain some gains.

On the other hand, the shortest path problem is an optimization problem which consists of finding a minimum route between two specified towns in a network (Ibrahim4]). The shortest path can be conveniently constructed using simple graphs. It is therefore realistic to obtain an optimal and efficient algorithm that calculates the exact weight of these types of routes.

2. METHOD OF COMPUTATION

There are a variety of ways for computing the traveling salesman problem; at least as many as the number of-greedy algorithms which are normally used in the compotation exercise. However, the intent of this report is focused on the use of Kruskal's algorithm.

Kruskal's Algorithm

There are varied methods of solving the traveling salesman problem. One method was suggested by Ibrahim andAudu[2] and includes obtaining a Hamilton cycle C and then searching for another of smaller weight by suitably modifying the cycle.

One possible modification was identified thus:

Let C = $v_1 v_2 \ldots v_v v_1$ (1)

Then , for all / and j such that $1 < i + 1 < j < v$ we can obtain a new Hamilton cycle
$C_{ij} = v_1 v_2 \ldots v_{i+1} v_{j+1} v_{j+2} \ldots v_v v_1$ (2)

by deleting the edges $v_i v_{i+1}$ and $v_j v_{j+1}$ while adding the edges $v_i v_{i+1}$ and $v_j v_{j+1}$ as shown in Fig. 1

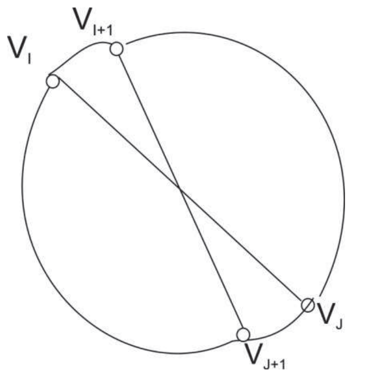

Fig.l: Formation of Hamilton cycles

If for some i and j, (2) holds then, the cycle C_{ij} is an improvement of the original cycle C.

The process of improving upon the existing routine systems is the backbone of minimization algorithms in traveling salesman problems.

In what follows, the methodology of Kruskal's algorithm is outlined:

1. $Chose\ a\ limk\ e_1.$
2. $If\ edges\ e_1, e_2, \ldots e_i\ have\ been\ chosen, then\ choose\ an\ edge\ e_{i+1}.$
$from\ E\ \{e_1, e_2, \ldots e_i\}\ in\ such\ a\ way\ that$
(i) $[\{e_1, e_2, \ldots e_i\}]$ *is acyclic;*

(ii) $w(e_{i+1})$ *is as small as possible subject to (i)*

3. ***Stop when step 2 cannot he implemented further.***

Using the methodology of in Kruskal algorithm as expressed above one can attempt finding a minimum spanning tree from given routes expressed in this case, as a connected graph.

As an illustration, consider the graph in Fig. 2

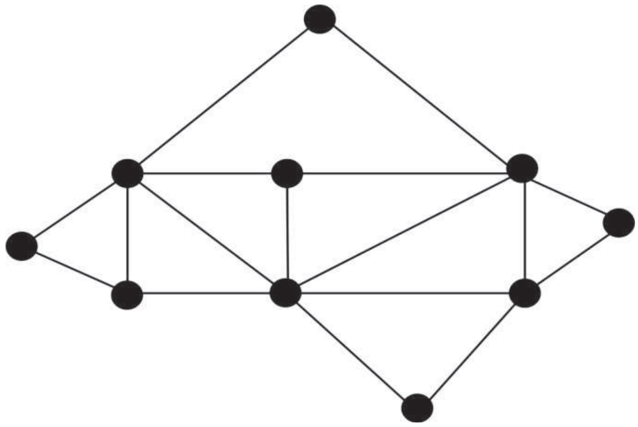

Fig. 2

As highlighted in the Kruskal Algorithm, the corresponding spanning tree of the graph in Fig. 2 can be obtained by choosing edges in such a way that the resulting edge set $\{e_1, e_2, \ldots e_m\}$,say, for some $m \geq 0$ is acyclic. Without loss of generality, it follows that $m = 9$ and the following graph (Fig. 3) is the desired spanning tree under the given conditions.

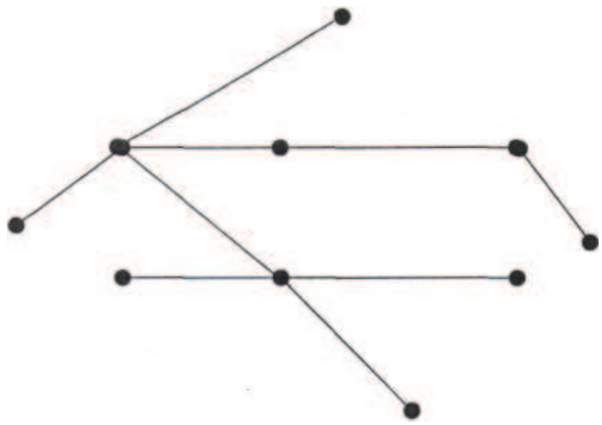

Fig. 3

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3. APPLICATION

In what follows, a distribution network is studied in respect of a manufacturing enterprise. The resulting graph as constructed using the geographical map Sloane [5] is provided in Fig. 4. This graph is then used to contruct a corresponding complete graph (see Fig. 5) thus ensuring all the possible connectivities of the different distribution centres and together with the distances between nodes, taken as edge weights, an optimal route is established (see Fig. 6). This is done using Kruskal algorithm to remove all un wanted routes, ones that result in to cycles, and thus avoiding repitative routing in the iterneracy af the sales man.

We proceed as follows:

Distribution Network

We are mainly concerned with the Area of major concentration of the distribution network of the manufacturing enterprise under study. For operational security, the detailed graph of all the distribution centres is withheld.

Deleting the vertices which do not fall within the major concentration of distribution centres, we obtain the graph of the major concentration of the distribution centres (See Fig. 4 following).

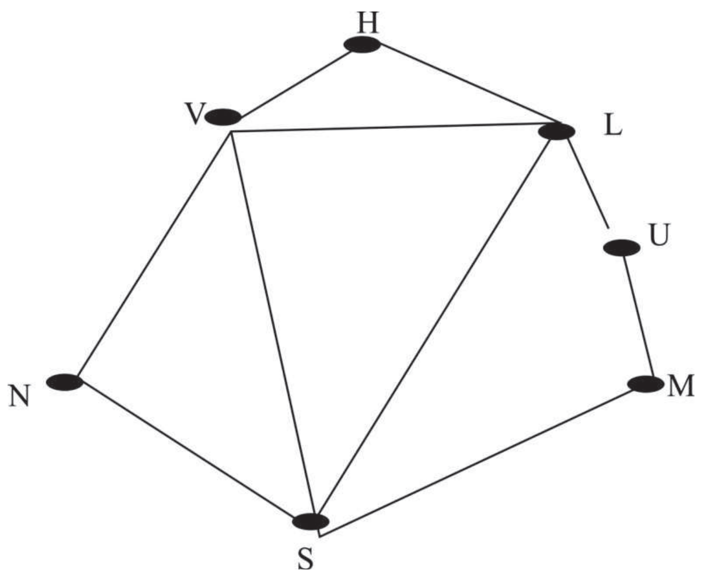

Fig.

Fig4: Major distribution centres

The graphical interpretation in Fig. 4 is that dots represent distribution centres while the lines connecting them represent distances.

Next, we make the graph of Fig. 4 a complete graph, (see Fig. 5) with edge weights (cost). These edge weights were obtained using the actual geographical distances between the different distribution centres ,Drager and Fettweis [3].as considered.

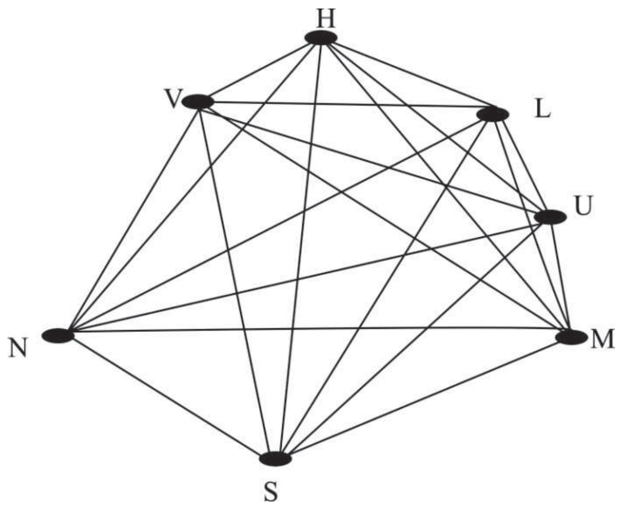

Fig.5: Resulting completegraph

Depicting the edge weight (in kilometers) and sorting in ascending order we have the following values:

MU = 0.900	UV = 3.100	MV = 3.750
UL = 1.010	NV = 2.600	HS = 4.000
LH = 1.340	MH = 3.120	LN = 4.150
HV = 1.350	SM = 3.260	UN = 4.350
NS = 1.950	SV = 3.290	MN = 4.480.120
LM = 2.000	US = 3.500	
HU = 2.300	UN = 3.680	
LV = 2.350	LS = 3.730	

The total weight of **Fig. 3** is 59.3km.

Computation of minimum spanning tree using Kruskal algorithm

Using the graph of **Fig.3** we compute the Minimum Spanning Tree, using Kruskal's alaorithm thus:

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M to U	-	Cost is 0.900 - add to tree
U to L	-	Cost is 1.010- add to tree
L to H	-	Cost is 1.340 - add to tree
H to V	-	Cost is 1.350 - add to tree
N to S	-	Cost is 1.950-add to tree
L to M	-	Cost is 2.000 - reject because it forms a circle
H to U	-	Cost is 2.300 - reject because it forms a circle
Lto V	-	Cost is 2.350 - reject because it forms a circle
N to V	-	Cost is 2.600 - Add to tree
U to V	-	Cost is 3.100 - reject because it forms a circle
H to M	-	Cost is 3.120 - reject because it forms a circle
S to M	-	Cost is 3.260 - reject because it forms a circle
U to S	-	Cost is 3.500 - reject because it forms a circle
H to N	-	Cost is 3.680 - reject because it forms a circle
L to S	-	Cost is 3.730 - reject because it forms a circle
M to V	-	Cost is 3.750 - reject because it forms a circle
H to S	-	Cost is 4.000 - reject because it forms a circle
L to N	-	Cost is 4.150- reject because it forms a circle
U to N	-	Cost is 4.350 - reject because it forms a circle
M to N	-	Cost is 4.480 - reject because it forms a circle

We stop here, because n-1 edges have been added. Hence, the graph of the minimum spanning tree (MST) has a total weight of 9.150km. This graph is obtained by deleting all nodes that produce cycles in the routing scheme and is depicted in **Fig. 6** thus:

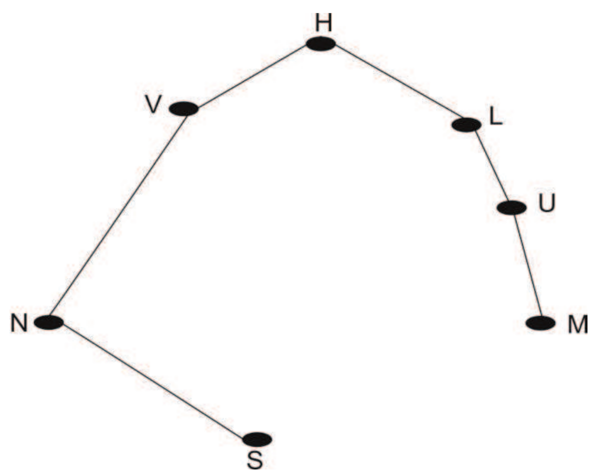

Fig.6: Minimum spanning tree

4. CONCLUSION

It is by now clear the importance of graph theoretical framework in attending to the growing procedures of industrialization the centaral emphasis of which concerns the concepts of maximizing efficiency at minimal cost. The results obtained indicate that a lot of time, cost and energy could be saved using these theoretical models.

5. REFERENCES

1. Claude C.S. and Dinneen M.J (1998) Group-Theoretic Methods for Designing Networks, Discrete Mathematics and Theoretical Computer Science, Research Report.
2. Ibrahim A.A (2007) An Enumeration Scheme and Algebraic properties of a Special (132)-avoiding Class of permutation patterns. Trends in Applied sciences Research Academic Journals Inc. USA. 2(4):334-340
3. Drager T. and Fettweis G.P. (2002) Energy Savings with Appropriate Interconnection Networks in Parallel DSP. 3 Lolloquim des Schwerpunklprogramms de Dculschen Forscungsgescllschaft VIVA: 35-42 , Chemnitz, Germany
4. Ibrahim A.A. (2007) A Counting Scheme And Some Algebraic Properties of A Class ofSpecial Permutation Patterns Journal of Discrete Mathematical Sciences a cryptography Taru Publishers, Vol. 10 No. 4:537-546, New Delhi, India
5. N. J. A. Sloane,(1972) Table of n, a(n) for n = 0..30000

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