APPLICATION OF DIFFERENTIAL EQUATIONS IN HEAT GENERATION ON THE ONSET OF MARAGONI CONVECTION OF FERROMAGNETIC FLUIDS

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Abstract: The role of differential equations towards the onset of Marangoni Convection is studied. Internal heat generation is considered. The lower surface is a rigid surface and the upper surface is taken as a non-deformable surface. Using Galerkin's technique, the critical Eigen values are obtained and different parameters are analyzed towards the onset of convection. The internal heating is found to decrease the critical condition. The application of differential equations in the above convection problem is portrayed.

Key Words: Marangoni Convection,Heat generation, ferromagnetic fluids and differential equations.

1. INTRODUCTION

Differential equations play a vital role in all fields of study. In this modern world, investigation of fluids attracted many researchers because of its application in instrumentation, lubrication, vacuum technology, metals recovery, and acoustics. Surface tension driven convection known as Marangoni convection has been a subject of interest mainly because of its importance in many branches of science, engineering and technology.

The present work is focused on the effect of heat transfer on the onset of Marangoni convection in a ferromagnetic fluid.

2. REVIEW OF LITERATURE

Das Gupta investigated the convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis reference number, as in[1]. Gasser and Kazimi studied the onset of convection in a porous medium with internal heat generation by employing a rigid lower surface with a free upper surface and isothermal conditions at the upper and lower surfaces reference number, as in[2]. Hindenburg considered a very coarse porous medium which can be described in terms of the Brinkman model, to justify the existence of a sharp boundary between a liquid saturated porous medium and the external gaseous phase reference number, as in[3]. Nieldet investigated the convection instability of a fluid overlying a porous region saturated with the same fluid subject to a uniform temperature gradient reference number, as in[4].Pearson theoretically analyzed the problem of convection due to temperature dependent surface tension reference number, as in[5]. Sekar] investigated convective instability of a magnetized ferrofluids in a rotating porous medium reference number, as in[6]. Shiva kumara have obtained numerically closed form solution for Darcy-Bernard-Marangoni convection in a sparsely packed porous medium by employing the Brinkman-Forchheimer-Lap wood-extended-Darcy flow

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model with effective viscosity different from fluid viscosity reference number, as in[6]. The effect of dust particles on non-magnetic fluids has been investigated by many authors reference number, as in[8]-[11].In the last decade, stability problems on ferromagnetic fluids have been studied in porous and non-porous medium by many authors reference number, as in[12]-[17]. Wilson investigated the effect of the internal heat generation on Marangoni instability of a horizontal fluid layer when the lower boundary is conducting and insulating to temperature perturbations reference number, as in[18].

3. MATHEMATICAL FORMULATION

Here we consider an infinite horizontal layer of thickness d of an electrically non conducting incompressible ferromagnetic fluid embedded in dust particles in porous medium heated from below. A uniform magnetic field. H_0 acts along the vertical direction which is taken as z-axis. The temperature at top and bottom surfaces $Z = \pm(1/2)d$ are T_0 *and* T_1 respectively, and a uniform temperature gradient $\beta = (|dT/dz|)$ is maintained. Both the boundaries are taken to be free and perfect conductors of heat. The gravitational field $\vec{g} = (0, 0, -g)$ pervades the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ϵ and medium of permeability K_1. The surface tension σ is assumed to vary linearly in the form $\sigma = \sigma_0 - \sigma^*(T - T_0)$ following Pearson [5], where σ_0 is the constant reference value and $-\sigma^*$ is the rate of change of the surface tension with temperature. The Governing equations with Boussinesq approximation are considered.

The continuity equation is

$$\nabla . q = 0 \tag{1}$$

The momentum equation is

$$\rho_0 \left[\frac{\partial}{\partial t} + (q.\nabla) q \right] = -\nabla p + \rho g - \mu q \tag{2}$$

The density of the state is

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_a \right) \right] \tag{3}$$

The temperature equation for an incompressible ferromagnetic fluid is

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$$\rho_0 C_{v,H}\frac{DT}{Dt}+\rho_s C_s\frac{\partial T}{\partial t}+QWT=K_1\nabla^2 T \quad (4)$$

where $\rho, \rho_0, q, \mu, p, t$ are the density, reference density, velocity, dynamic viscosity and pressure of ferromagnetic fluid, time respectively. x=(x, y, z) and

$K = 6\pi\mu\eta,\eta$ and $T_a=\frac{T_0+T_1}{2}$, $C_{v,H}, C_s, C_{pt}, k_1, T and\, \mu_0$ are the specific heat at constant volume and magnetic field, specific heat of solid material, thermal conductivity, temperature and magnetic permeability, respectively.

Allowing a perturbation of the system, the governing equations are modified and normal mode technique is applied.

Using suitable non-dimensional terms, the non-dimensional equations are given below

$$L_1^*\frac{\partial}{\partial t}(D^2-a^2)W^*=0 \quad (5)$$

$$L_1^*(P_r\frac{\partial T}{\partial t})-2ZQW^*=L_1^*(D^2-a^2)T \quad (6)$$

$$L_1^*=\tau\frac{\partial}{\partial t^*}+1, t^*=(v/d^2)t, p_r=\left(\frac{v}{k_1}\right)\rho c \ and \ w^*=(d/v)w$$

The boundary conditions are $W = D^2 + a^2MT = D\varphi = 0 \ at \ z = \pm\frac{1}{2}$

Using Galerkin technique, the expression for Marangoni Convection is derived as

$$M_a=\frac{240r_1}{(5r_2+Q)} \text{ Where } r_1=3\alpha r_2 \text{ and } r_2=6(8+6\alpha) \quad (7)$$

4. CONCLUSION

The application of differential equations in analyzing the stability nature of ferromagnetic fluids with heat generation is studied. Marangoni convection is considered.

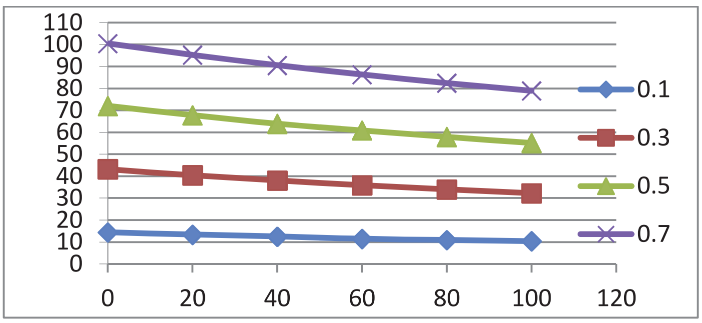
From figure (1), it is clear that as the internal heat generation Q increases the Marangoni number M_a decreases for $\alpha = 0.1, 0.3, 0.5, 0.7$. Thus the internal heating in the fluid layer is found to decrease the critical conditions.

α	Q	M_a
0.1	0	14.4
	20	13.36
	40	12.46
	60	11.46
	80	10.9
	100	10.37
0.3	0	43.2
	20	40.44
	40	38.02
	60	35.87
	80	33.95
	100	32.23
0.5	0	72
	20	67.8
	40	63.9
	60	60.9
	80	57.9
	100	55.2
0.7	0	100.45
	20	95.25
	40	90.5
	60	86.30
	80	82.43
	100	78.90

Figure

Marangoni number M_a at the onset of convection as a function of internal heat generation

M_a



Q

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5. REFERNCES

1. Das Gupta.M,Gupta.A.S,Int.J.Eng.Sci 17 (1979) 271.
2. Gasser. R.D, Kazimi.M.S, Onset of convection in a porous medium with internal heat generation, Journal of Heat Transfer, 76 (1976).49-54.
3. Hennenberg. M, Saghir. M.Z, Rednikov.A, Legros. J.C, Porous media and the Benard-Marangoni problem, Transport in Porous Media, 27 (1997), 327-355.
4. Nield. D.A, Bejan.A, Convection in Porous Media, 3-rd Edition, Springer, New York (2006).
5. Pearson. J.R.A, On convection cell induced by surface tension, J. Fluid Mech., 4 (1958), 489-500.
6. R.Sekar,G.Vaidyanathan, Int.J. Eng.Sci.31(1993) 1139.
7. I.S. Shivakumara, C.E. Nanjundappa, K.B. Chavaraddi, Darcy-Benard-Marangoni convection in porous media, Int. J. Heat Mass transfer, 52 (2009), 2815-2823.
8. R. C. Sharma, Sunil, Polym-Plastics technol. Eng. 33 (3) (1994) 323.
9. R. C. Sharma, Sunil, Y.D. Sharma, R.S. Chandel, Arch. Mech. 54 (4) (2002) 287.
10. Sunil, R. C. Sharma, R. S. Chandel, Int. J. Appl. Mech. Eng. 8 (4) (2003) 693.
11. Sunil, R. C. Sharma, R. S. Chandel, J. Porous Media 7 (1) (2004) 9.
12 Sunil, P.K. Bharti, R.C. Sharma, Arch. Mech. 56 (2004) 87.
13. Sunil, Divya, R.C. Sharma, J. Geophys. Eng. 1 (2004) 116.
14 S. Venkatasubramanian, P.N. Kaloni, Int. J. Eng. Sci 32 (1994) 237.
15. G. Vaidyanathan, R. Sekar, A. Ramanathan, J. Magn. Magn.Mater.149 (1995) 137.
16. G. Vaidyanathan, R. Sekar, A. Ramanathan, J. Magn. Magn.Mater.176 (1997) 321.
17. G. Vaidyanathan, R. Sekar, A. Ramanathan, J. Magn. Magn.Mater.250 (2002) 65.
18. S.K. Wilson, The effect of uniform internal heat generation on the onset of steady Marangoni convection in a horizontal layer fluid, ActaMechanica, 124 (1997), 63-78.

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