A TWO SPECIES AMENSALISM MODEL WITH CONSTANT HARVESTING OF THE FIRST SPECIES

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***Abstract:** In this paper a two species Amensalism model is taken up for analytical study. In this model a constant number of first species are harvested. Moreover both the species are provided with limited resources. The series solution of the non-linear system was approximated by the Homotopy analysis method (HAM) and the results are supported by numerical simulations.*

***Keywords:** Amensalism, embedded parameter, linear operator,HAM and zero order deformation ,.*

1. INTRODUCTION

Symbioses are a broad class of interactions among organisms --amensalism involves one organism affecting another negatively without having any positive or negative benefit for itself K.V.L.N.Acharyulu and N.CH.PattabhiRamacharyulu [1] studied the stability of enemy amensal species pair with limited resources and B.Shivaprakash, T.Karunanithi [10] studied the growth rate of both the species were tested for the validity of the logistic model under culture operation in chemostat.. In this paper a two species Amensalism model is taken up for analytical study. In this model a constant number of first species are harvested. Moreover both the species are provided with limited resources. The series solution of the non-linear system was approximated by the Homotopy analysis method (HAM) and the results are supported by numerical simulations.

About HAM: In 1992 Liao employee the basic idea of homotopy in topology to propose a powerful analytical method for nonlinear problems namely Homotopy Analysis Method [2,3].Later on M.Ayub,A.Rasheed,T.Hayat,Fadi &Awawdeh [6,7] successfully applied this technique to solve different types of non-linear problems. The HAM itself provides us with a convenient way to control adjust the convergence region and rate of approximation series. In this paper we propose HAM method to investigate the series solutions of a two species amensalism model with a cover for the first species to protect it from the attacks of the second species.

Basic equations:

The governing equations of the system are as follows

$$\frac{dx}{dt} = a_1 x(t) - \alpha_{11} x^2(t) - \alpha_{12} x(t)\, y(t) - h$$
$$\frac{dy}{dt} = a_2 y(t) - \alpha_{22} y^2(t) \qquad (1.1)$$

With the notation
x(t) ,y(t): populations of the species1 and species2
a_1, a_2: rates of natural growth of species 1 and species 2.
α_{11}, α_{22} : rates of decrease due to insufficient food of the species 1 and species 2.
α_{12} : rate of decrease of the species 1 due to inhibition by the species 2.
h: constant harvesting

2. BASIC IDEAS OF HAM:

In this paper, we apply the homotopy analysis method to the discussed problem. To show the basic idea, let us consider the following differential equation

$$N(u,t) = 0, \tag{2.1}$$

Where N is a nonlinear operator, t denote independent variable, u (t) is an unknown function, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the traditional homotopy method, Liao constructs the so-called zero-order deformation equation

$$(1-p)L[\phi(t;p)-u_0(t)] = p\bar{h}H(t)N[\phi(t;p)], \tag{2.2}$$

Where $p \in [0,1]$ is the embedding parameter , $\bar{h}$ is a nonzero auxiliary, parameter H is an auxiliary ,parameter H is an auxiliary function L is an auxiliary linear operator, $u_0(t)$ is an initial guess of u(t),φ(t,p) is a unknown function, respectively. It is important that one has great freedom to choose auxiliary things in HAM. Obviously,

When P=0 and p=1, it holds $\varphi(t, 0) = u_0(t)$, $\varphi(t, 1) = u(t)$ (2.3)

Respectively. Thus as p increases from 0 to 1, the solution φ (t; p) varies from the initial guesses $u_0(t)$ to the solution u(t). Expanding φ(t; p) in Taylor series with respect to p,one has

$$\phi(t;p) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t) p^m \tag{2.4}$$

Where

$$u_m(t) = \frac{1}{m!} \frac{\partial^m \phi(t;p)}{\partial p^m}\bigg|_{p=0} \tag{2.5}$$

If the initial guess (2.3), the auxiliary linear parameter L_i, the non-zero auxiliary parameter h_i and the auxiliary function H_i are properly choosen,so that the power series (2.4) converges at p=1.

Then we have under these assumptions the solution series

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t) \tag{2.6}$$

Which must be one of solution s of original non linear equation, as proved by Liao [2].As $\bar{h}$ =-1 and H(t)=1,Eq (2.2)becomes

$$(1-p)L[\phi(t;p) - u_0(t)] + pN[\phi(t;p)] = 0, \tag{2.7}$$

which is used mostly in the homotopy perturbation method, whereas the solution obtained directly, without using Taylor series which is explained by H.Jafari,M.Zabihi and M.Saidy [8] and S.J.Liao [2] compare the HAM and HPM. According to the definition, the governing equation can be deduced from the zero-order deformation equation (2.7) .

Define the vector $\vec{u}_k = \{u_{0,} u_1, ...u_k\}$ Differentiating Eq. (2.7) ,m times with respect to embedding parameter p and then setting p=0 and finally dividing them by m! , we have the so-called mth-order deformation equation

$$L[u_m(t) - \chi_m u_{m-1}(t)] = \bar{h} H(t) R_m(\vec{u}_{m-1}), \tag{2.8}$$

$$R_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(t;p)]}{\partial p^{m-1}}\bigg|_{p=0},$$

Where *and*

$$\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1. \end{cases} \tag{2.9}$$

3. APPLICATION

Consider the nonlinear differential equation(1.1) with initial conditions .we assume the solution of the system(1.1) ,x(t),y(t) can be expressed by following set of base functions in the form

$$x(t) = \sum_{m=1}^{+\infty} a_m t^m, \; y(t) = \sum_{m=1}^{+\infty} b_m t^m \tag{3.1}$$

Where a_m, b_m are coefficients to be determined. This provides us the so called rule of solution expression i.e., the solution of (1.1) must be expressed in the same from as (3.1) and the other expressions must be avoided. According to (1.1) and (3.1) we chose the linear operator. To solve the system of Eqs.(1.1),Homotopy analysis method is employed. We consider the following initial approximations

$$x_0(t) = x(t=0) = x_0 \qquad y_0(t) = y(t=0) = y_0 \tag{3.2}$$

The linear and non-linear operators are denoted as follows.

$$L_1[x(t;p)] = \frac{dx(t;p)}{dt}, L_2[y(t;p)] = \frac{dy(t;p)}{dt}, \tag{3.3}$$

$$N_1[x(t,p)] = \frac{dx(t;p)}{dt} - a_1 x(t;p) + \alpha_{11} x^2(t;p) + \alpha_{12} x(t;p) y(t;p) + h \tag{3.4}$$

$$N_2[y(t,p)] = \frac{dy(t;p)}{dt} - a_2 y(t;p) + \alpha_{22} y^2(t;p) \tag{3.5}$$

Using above definition the zero order deformation equation can be constructed

$$\begin{aligned}(1-p)L_1[x(t;p) - x_0(t)] &= ph_1 N_1[x,y],\\ (1-p)L_2[y(t;p) - y_0(t)] &= ph_2 N_2[x,y],\end{aligned} \tag{3.6}$$

When p=0 and p=1, from the zero-deformation equations one has,

$$\begin{aligned}x(t;0) = x_0(t) \quad x(t;1) = x(t)\\ y(t;0) = y_0(t) \quad y(t;1) = y(t)\end{aligned} \tag{3.7}$$

And expanding x(t;p) and y(t;p) in Taylors series, with respect to embedding parameter p. one obtains

$$\begin{aligned}x(t;p) &= x_0(t) + \sum_{m=1}^{+\infty} x_m(t) p^m\\ y(t;p) &= y_0(t) + \sum_{m=1}^{+\infty} y_m(t) p^m\end{aligned} \tag{3.8}$$

$$\begin{aligned}x_m(t) &= \frac{1}{m!} \frac{d^m x(t;p)}{dp^m}\bigg|_{p=0}\\ y_m(t) &= \frac{1}{m!} \frac{d^m y(t;p)}{dp^m}\bigg|_{p=0}\end{aligned} \tag{3.9}$$

$$p = 1 \begin{cases} x_m(t) = x_0(t) + \sum_{m=1}^{+\infty} x_m(t) \\ y_m(t) = y_0(t) + \sum_{m=1}^{+\infty} y_m(t) \end{cases} \tag{3.10}$$

Define the vector

$$\vec{x}_m = [x_0(t), x_1(t), \ldots . x_m(t)]$$
$$\vec{y}_m = [y_0(t), y_1(t), \ldots . y_m(t)] \tag{3.11}$$

And apply the procedure stated before. The following mth-order deformation Eq will be achieved.

$$L_1[x_m(t) - \chi_m x_{m-1}(t)] = \bar{h}_1 H_1(t) R_{1m}(\vec{x}_{m-1}, \vec{y}_{m-1}),$$
$$L_2[y_m(t) - \chi_m y_{m-1}(t)] = \bar{h}_2 H_2(t) R_{2m}(\vec{x}_{m-1}, \vec{y}_{m-1}),$$
(3.12)

Let us consider $H_1(t) = H_2(t) = 1$ and the initial conditions $x_0(t) = x(t=0) = x_0$ $y_0(t) = y(t=0) = y_0$ in above equations

$$R_{1m}(x_{m-1}, y_{m-1}) = \frac{1}{(m-1)!}\frac{d^{m-1}}{dp^{m-1}}N[x(t,p)] = \frac{d}{dt}x_{m-1}(t) - a_1 x_{m-1} + \alpha_{11}\sum_{n=1}^{m} x_n(t)x_{m-n-1}(t) + \alpha_{12}\sum_{n=0}^{m-1} x_n(t)y_{m-n-1}(t) + h$$

$$R_{2m}(x_{m-1}, y_{m-1}) = \frac{1}{(m-1)!}\frac{d^{m-1}}{dp^{m-1}}N[y(t,p)] = \frac{d}{dt}y_{m-1}(t) - a_2 y_{m-1} + \alpha_{22}\sum_{n=1}^{m} y_n(t)y_{m-n-1}(t) \tag{3.13}$$

The following will be obtained successively

$$L_1\left(x_1(t) - \chi_1 x_0(t)\right) = h_1\left[\frac{d}{dt}x_0(t) - a_1 x_0(t) + \alpha_{11}x_0^2(t) + \alpha_{12}x_0(t)y_0(t) + h\right]$$

$$\chi_m(t) = 0, m \le 0$$
$$1, m > 0$$

$$L_1\left(x_1(t)\right) = h_1\left[-a_1 x_0(t) + \alpha_{11}x_0^2(t) + \alpha_{12}x_0(t)y_0(t) + h\right]$$

$$x_1(t) = h_1\left[-a_1 x_0 + \alpha_{11}x_0^2 + \alpha_{12}x_0 y_0 + h\right]t = h_1 k_1 t$$

$$where\, k_1 = -a_1 x_0 + \alpha_{11}x_0^2 + \alpha_{12}x_0 y_0 + h$$

$$L_1\left(y_1(t) - \chi_1 y_0(t)\right) = h_2\left[\frac{d}{dt}y_0(t) - a_2 y_0(t) + \alpha_{22}y_0^2(t)\right]$$

$$L_1\left(y_1(t)\right) = h_2\left[-a_2 y_0(t) + \alpha_{22}y_0^2(t)\right]$$

$$y_1(t) = h_2\left[-a_2 y_0 + \alpha_{22}y_0^2\right]t$$

$$L_1\left(x_2(t)-\chi_2 x_1(t)\right)=h_1\left[\frac{d}{dt}x_1(t)-a_1x_1(t)+\alpha_{11}\sum_{n=0}^{1}x_n(t)x_{1-n}(t)+\alpha_{12}\sum_{n=0}^{m-1}x_n(t)y_{m-n-1}(t)+h\right]$$

$$x_2(t)=(h_1+h_1^2)k_1t+\frac{t^2}{2}\left[-a_1h_1^2k_1+2\alpha_{11}h_1^2k_1x_0+\alpha_{12}h_1^2k_1y_0+\alpha_{12}h_1h_2k_2x_0\right]+hh_1t$$

$$where\, M_1=-a_1h_1^2k_1+2\alpha_{11}h_1^2k_1x_0+\alpha_{12}h_1^2k_1y_0+\alpha_{12}h_1h_2k_2x_0$$

$$k_2=(-a_2y_0+\alpha_{22}y_0^2)$$

$$x_2(t)=h_1[h+(1+h_1)k_1]t+M_1\frac{t^2}{2}$$

$$L_1\left(y_2(t)-\chi_2 y_1(t)\right)=h_2\left[\frac{d}{dt}y_1(t)-a_2y_1(t)+\alpha_{22}\sum_{n=0}^{m-1}y_n(t)y_{m-n-1}(t)\right]$$

$$y_2(t)=(h_2+h_2^2)\left[-a_2y_0+\alpha_{22}y_0^2\right]t+[-a_2h_2^2k_2+2\alpha_{22}h_2^2k_2y_0]\frac{t^2}{2}=(h_2+h_2^2)k_2t+M_2\frac{t^2}{2}$$

$$L_1\left(x_3(t)-\chi_3x_2(t)\right)=h_1\left[\frac{d}{dt}x_2(t)-a_1x_2(t)+\alpha_{11}\sum_{n=0}^{2}x_n(t)x_{1-n}(t)+\alpha_{12}\sum_{n=0}^{2}x_n(t)y_{1-n+m}(t)+h\right]$$

$$x_3(t)=[h+(1+h_1)k_1](h_1+h_1^2)t+hh_1t+[M_1-a_1h_1^2(h+(1+h_1)k_1)+2\alpha_{11}h_1^2x_0(h+(1+h_1)k_1)+\alpha_{12}h_1x_0(h_2+h_2^2)k_2$$

$$+\alpha_{12}h_1^2y_0(h+(1+h_1)k_1)]\frac{t^2}{2}+[\frac{1}{2}h_1M_1+\alpha_{11}h_1M_1x_0+\alpha_{11}h_1^3k_1^2+\alpha_{12}h_1^2h_2k_1k_2x_0+\frac{1}{2}\alpha_{12}h_1^2M_1y_o]\frac{t^3}{3}$$

$$(1+h_1)(h_1+h_1^2)M_1t+\left[(1+h_1)M_2+h_1\alpha_{12}y_0(h_1+h_1^2)M_1+\alpha_{12}h_1x_0(h_2+h_2^2)(-a_2y_0+\alpha_{22}y_0^2)\right]\frac{t^2}{2}$$

$$+\left[-2a_1h_1M_2+2h_1\alpha_{11}M_2x_0+\alpha_{11}h_1^3M_1^2+2h_1\alpha_{12}h_2^2x_0y_0^2(a_2^2-3a_2\alpha_{22}y_0+2\alpha_{22}^2y_0^2)+h_1^2h_2\alpha_{12}M_1(-a_2y_0+\alpha_{22}y_0^2)+2h_1\alpha_{12}M_2y_0\right]\frac{t^3}{3}$$

$$M_2=\left[-a_1h_2^2k_2+2h_2^2\alpha_{22}k_2y_0\right]$$

$$L_1\left(y_3(t)-\chi_3y_2(t)\right)=h_2\left[\frac{d}{dt}y_2(t)-a_2y_2(t)+\alpha_{22}\sum_{n=0}^{2}y_n(t)y_{1-n}(t)\right]$$

$$y_3(t)=(1+h_2)(h_2+h_2^2)k_2t+[(-a_2h_2(h_2+h_2^2)k_2+(1+h_2)M_2+2h_2\alpha_{22}k_2y_0(h_2+h_2^2)]\frac{t^2}{2}+[-\frac{1}{2}a_2h_2M_2+h_2\alpha_{22}M_2y_0+h_2^3\alpha_{22}k_2^2]\frac{t^3}{3}$$

Where

$$k_1=\left[-a_1x_0+\alpha_{11}x_0^2+\alpha_{12}x_0y_0+h\right]$$

$$k_2=\left[\left(-a_2y_0+\alpha_{22}y_0^2\right)\right]$$

The three terms approximation to the solution will be considered as

$$x(t)\approx x_0+x_1(t)+x_2(t)+x_3(t)$$

$$y(t)\approx y_0+y_1(t)+y_2(t)+y_3(t)$$

$$x(t)=\{h_1k_1+h_1[h+(1+h_1)k_1]+([h+(1+h_1)k_1](h_1+h_1^2)+hh_1\}t+\{M_1+[M_1-a_1h_1^2(h+(1+h_1)k_1)+2\alpha_{11}h_1^2x_0(h+(1+h_1)k_1)+\alpha_{12}h_1x_0(h_2+h_2^{\ 2})k_2$$
$$+\alpha_{12}h_1^2y_0(h+(1+h_1)k_1)]\}\frac{t^2}{2}+\{\frac{1}{2}h_1M_1+\alpha_{11}h_1M_1x_0+\alpha_{11}h_1^3k_1^2+\alpha_{12}h_1^2h_2k_1k_2x_0+\frac{1}{2}\alpha_{12}h_1^{\ 2}M_1y_o\}\frac{t^3}{3}$$

$$y(t)=\{h_2\left[-a_2y_0+\alpha_{22}y_0^2\right]+(h_2+h_2^2)k_2+(1+h_2)(h_2+h_2^2)k_2\}t+\{M_2+[(-a_2h_2(h_2+h_2^2)k_2+(1+h_2)M_2+2h_2\alpha_{22}k_2y_0(h_2+h_2^2)]\}\frac{t^2}{2}$$
$$+\{[-\frac{1}{2}a_2h_2M_2+h_2\alpha_{22}M_2y_0+h_2^3\alpha_{22}k_2^{\ 2}]\}\frac{t^3}{3}$$

4.NUMERICAL EXAMPLE :

1.x0=8;y0=10;a1=.5;a2=.09;a11=05;a12=2.6;a22=7;h1=1;

h2=1;h=6;

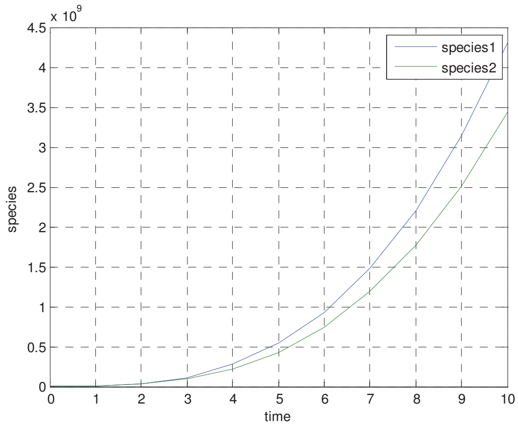

Fig 4.1 .The variation of x(t) and y(t) with respective Time (t) for the system of Eq (1.1)

1. x0=5;y0=10;a1=.5;a2=.09;a11=05;a12=2.6;a22=7;h1=1;h2=1;

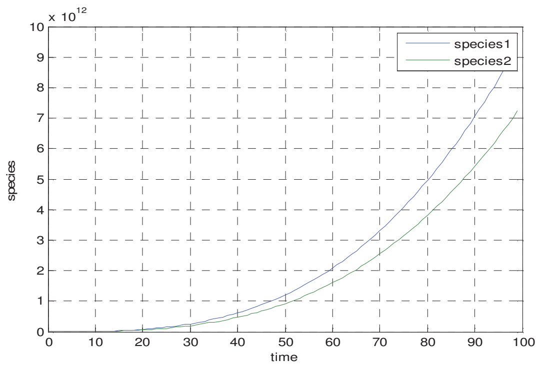

Fig 4.2 .The variation of x(t) and y(t) with respective Time (t) without harvesting

5. CONCLUSION

From the above discussion it is clear that a limited constant harvesting stabilizes system further

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