SOLVING TRANSPORTATION PROBLEM WITH THE HELP OF REVISED SIMPLEX METHOD

Gaurav Sharma[1], S. H. Abbas[2], Vijay Kumar Gupta[3]

Abstract: The purpose of this paper is establishing usefulness of newly revised simplex method in a limited supply production planning problem with continuous variables. In this respect parameters of problem are modeled in from of linear programming. This paper begins with introduction and method of revised simplex method of revised simplex method for numerical real life example of production planning problem is presented.

Keywords: Optimal Solution, Revised Simplex Method, Transportation Problem

1. INTRODUCTION

The Transportation Problem model consider minimum-cost planning problems for shipping a product from some origins to other destinations such as from factories to warehouses, or from warehouses to supermarkets, with the shipping cost from one location to another being a linear function of the number of units shipped. The Transportation problem model is a special case of the linear programming models and obviously, it can be solved by the regular simplex method. Due to its special structure of the model the stepping – stone method Charnes and Cooper[1] was developed for the efficiency reason while the simplex method is not suitable for the transportation problem, especially for those large- scale transportation problems. Other research results can be found from Ford and Fulkerson[8], Balinski and Gomory[10], Muller- Merbach[6], Grigoriadis and Walker[9], Glover et al[4], Shafaat and Goyal[2], and Arsham and Khan[5]. A brif review on this aera was presented by Gass[11].

2. REVISED SIMPLEX METHOD:

The revised simplex method is another efficient method developed by G B Dantzing for solving LP problems. Its is efficient in the sense that at each iteration, we need not recomputed values of all the variable, namely: y_j, $c_j – z_j$, x_B and Z while moving one iteration to next in search of an improved solution of the LP problem. Here the world revised refers to the procedure of the changing or updating the simplex table. Earlier you have noted that each iteration it was necessary to calculate $c_j – z_j$ corresponding to non basic variable columns to decide whether the current solution is optimal or not. If not then in order to select the non basic variable to enter into the basic metrics B, first we need to know $y_j = B^{-1}a_j$, where y_j refer to updated column a_j in the simplex table being examined. If $y_j \leq 0$, then optimal solution is unbounded. Otherwise, apply minimum ratio rule to decide to which basic variable should leave the basic. Update, basic matrix B by replacing an outgoing vector with an incoming vector.

In revised simplex method we need to recompute value of only B^{-1}, x_B, $c_B B^{-1}$ and Z. value of all these new variable can be computed directly from their definition provided B^{-1} is know. At each iterartion, B-1 is calculated its from its previous values when only one y_j is changed a_i each iteration of revised simplex method are:

(i) Coefficient of non basic variables in the objective function

(ii) Coefficient of the variable to be entered into the basic in the set of constraints.

2.1 Steps of the Algorithms

The revised simplex algorithm can be summarized in the following steps

Step 1: Express the given problem in standard from

Express the given problem in the revised simplex from by consideration the objective function as one the constraints, and adding the slack and surplus variables, if needed, to inequalities to convert them into equalities.

Step 2: Obtain the basic feasible solution

Start with initial basic matrix $B = I_m$ and find B_1^{-1} and B_1^{-1} to from the initial revised simplex table.

Step 3: Select a variable to enter into the basic (key column)

For each non – basic variable, calculate $c_j - z_j$ by using the formula $c_j - z_j = c_j - cB$ B_1^{-1} $a_j^{(1)}$ where B_1^{-1} $a_j^{(1)}$ represented the product of the first row of B_1^{-1} and successive columns of A not in B_1^{-1}

(i) If all $c_j - z_j \leq 0$, then the current basic solution is optimal. Other go to step 4

(ii) If one or more $c_j - z_j$ are positive, then variable to enter into the basic may be the $c_j - z_j = \max\{c_j - z_j : c_j - z_j > 0\}$

Step 4: Select a variable to leave the basic (Key row)

Calculate $y_k^{(1)} = B_1^{-1}$ $a_k^{(1)} = a_k^{(1)}$; where $a_k^{(1)} = [\, - c_k - a_k]$ if all $y_{ik} \leq 0$, then variable to be removed from the basic is determined by calculating the ratio

$$\frac{x_{Br}}{y_{rk}} = Min_i \left\{ \frac{x_{Bi}}{y_{ik}};\ y_{ik} > 0 \right\}$$

That is, the vector $\beta_r^{(1)}$ is selected to leave the basic and go to step 5.

If the minimum ratio is not unique, i.e. the ratio is same for more than one row, then resulting basic feasible solution will be degenerate. To avoid cycling to occur, the usual method of resolving the degeneracy is applied.

Step 5: Update the current solution

Update the initial table by introducing a non- basic variable x_k ($=a_k^{(1)}$) into basic and removing basic variable x_r ($=\beta_r^{(1)}$) from the basic.

Repeat Steps 3 to 5 until an optimal solution is obtained or there is an indication for an unbounded solution.

3. NUMERICAL PROBLEM

To solve the following transportation problem of minimal cost with the initial fuzzy basic feasible solution obtained by Modified Revised Simplex Method whose cost and requirement table is given below.

TABLE-1

	D1	**D2**	**D3**	**D4**	**SUPPLY**
S1	6	3	5	4	22
S2	5	9	2	7	15
S3	5	7	8	6	8
DEMAND	**7**	**12**	**17**	**9**	**35**

An equivalent formulation to derive an optimal solution of the transportation scheme for the above example, apply the Modified Revised Simplex method and convert the inequality constraints into equality constraints.

Min $z = 6x_{11}+3x_{12}+5x_{13}+4x_{14}+5x_{21}+9x_{22}+2x_{23}+7x_{24}+5x_{31}+7x_{32}+8x_{33}+6x_{34}$

$x_{11}+x_{12}+x_{13}+x_{14}+s_1= 22$; $x_{11}+x_{21}+x_{31}+s_4= 07$

$x_{21}+x_{22}+x_{23}+x_{24}+s_2= 15$; $x_{12}+x_{22}+x_{32}+s_5= 12$

$x_{31}+x_{32}+x_{33}+x_{34}+s_3=08$; $x_{14}+x_{24}+x_{34}+s_7=09$

$x_{13}+x_{23}+x_{33}+s_6=17$

The initial basic feasible solution is Table 2

Variable In Basic	Solution values	Basic Inverse B_1^{-1}								$y_k^{(1)}$	Additional Table											
		$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$	$\beta_4^{(1)}$	$\beta_5^{(1)}$	$\beta_6^{(1)}$	$\beta_7^{(1)}$		$a_{11}^{(1)}$	$a_{12}^{(1)}$	$a_{13}^{(1)}$	$a_{14}^{(1)}$	$a_{21}^{(1)}$	$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{31}^{(1)}$	$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$
B	**b(= $x_B^{(1)}$)**	**Z**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**S_7**	**C_k - Z_k**	**x_{11}**	**x_{12}**	**x_{13}**	**x_{14}**	**x_{21}**	**x_{22}**	**x_{23}**	**x_{24}**	**x_{31}**	**x_{32}**	**x_{33}**	**x_{34}**
Z	0	1	0	0	0	0	0	0	0		-6	-3	-5	-4	-5	-9	-2	-7	-5	-7	-8	-6
S_1	22	0	1	0	0	0	0	0	0		1	1	1	1	0	0	0	0	0	0	0	0
S_2	15	0	0	1	0	0	0	0	0		0	0	0	0	1	1	1	1	0	0	0	0
S_3	08	0	0	0	1	0	0	0	0		0	0	0	0	0	0	0	0	1	1	1	1
S_4	07	0	0	0	0	1	0	0	0		1	0	0	0	1	0	0	0	1	0	0	0
S_5	17	0	0	0	0	0	1	0	0		0	1	0	0	0	1	0	0	0	1	0	0
S_6	12	0	0	0	0	0	0	1	0		0	0	1	0	0	0	1	0	0	0	1	0
S_7	09	0	0	0	0	0	0	0	1		0	0	0	1	0	0	0	1	0	0	0	1

To select the vector corresponding to a non basic variable to enter into the basis, we compute

C_k - Z_k = Max { (Cj – Zj) > 0} = Max { -(First row of B_1^{-1})(column $a_j^{(1)}$ not in basis, B_1)}

= Max {-(-5, -3, -5, -4, -5, -9, -2, -7, -5, -7, -8, -6)} = 9

Thus vector $a_{22}^{(1)}$ (= x_{22}) is selected to enter into the basic.

To select a basic variable to leave the basic variable to leave the basic given the entering non-basic variable $x_{22,}$ we compute $y_k^{(1)}$

$$y_1^{(1)} = B_1^{-1}\ a_1^{(1)} = a_1^{(1)} = \begin{bmatrix} -9 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } x_B^{(1)} = B_1^{-1}\text{b} = \text{b} = \begin{bmatrix} 0 \\ 22 \\ 15 \\ 8 \\ 7 \\ 17 \\ 12 \\ 9 \end{bmatrix}$$

After having selected the non basic variable X_{22} to enter into the basis, we shall calculate the minimum ratio to select the basic variable to leave the basis

$$\frac{x_{Br}}{y_{rk}} = Min_i \left\{\frac{x_{Bi}}{y_{ik}};\ y_{ik} > 0\right\} = \text{Min } \{\tfrac{22}{0}, \tfrac{15}{1}, \tfrac{8}{0}, \tfrac{7}{0}, \tfrac{17}{1}, \tfrac{12}{0}, \tfrac{09}{0}\} = 15$$

Thus vector $\boldsymbol{\beta_2^{(1)}}$ **(=s_2)** is selected to leave the basic. Table 1 again reproduced with the new entries in the column $y_1^{(1)}$ and minimum ratio as shown in Table 3

Varia ble In Basic	Solutio n Values	Basic Inverse B_1^{-1}								$y_k^{(1)}$	Minim un Ratio
		$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$	$\beta_4^{(1)}$	$\beta_5^{(1)}$	$\beta_6^{(1)}$	$\beta_7^{(1)}$		
B	**b(=$x_B^{(1)}$)**	**Z**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**S_7**	C_k - Z_k	
Z	0	1	0	0	0	0	0	0	0	-9	-
S_1	22	0	1	0	0	0	0	0	0	0	0
S_2	15	0	0	1	0	0	0	0	0	1	15
S_3	08	0	0	0	1	0	0	0	0	0	0
S_4	07	0	0	0	0	1	0	0	0	0	0
S_5	17	0	0	0	0	0	1	0	0	1	17
S_6	12	0	0	0	0	0	0	1	0	0	0
S_7	09	0	0	0	0	0	0	0	1	0	0

The initial basic feasible solution shown in table 3 is now updated by replacing variable s_2 with the variable x_{22} in the basic. For this we apply the following now operation in the same way as in the simplex method. The improved solution is shown in Table 4

Variable In Basic	Solution values	Basic Inverse B_1^{-1}								$y_k^{(1)}$	Additional Table											
		$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$	$\beta_4^{(1)}$	$\beta_5^{(1)}$	$\beta_6^{(1)}$	$\beta_7^{(1)}$		$a_{11}^{(1)}$	$a_{12}^{(1)}$	$a_{13}^{(1)}$	$a_{14}^{(1)}$	$a_{21}^{(1)}$	$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{31}^{(1)}$	$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$
B	b(= $x_B^{(1)}$)	Z	S_1	x_{22}	S_3	S_4	S_5	S_6	S_7	C_k - Z_k	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	S_2	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
Z	135	1	0	15	0	0	0	0	0		-6	-3	-5	-4	-5	0	-2	-7	-5	-7	-8	-6
S_1	22	0	1	0	0	0	0	0	0		1	1	1	1	0	0	0	0	0	0	0	0
x_{22}	15	0	0	1	0	0	0	0	0		0	0	0	0	1	1	1	1	0	0	0	0
S_3	08	0	0	0	1	0	0	0	0		0	0	0	0	0	0	0	0	1	1	1	1
S_4	07	0	0	0	0	1	0	0	0		1	0	0	0	1	0	0	0	1	0	0	0
S_5	2	0	0	-1	0	0	1	0	0		0	1	0	0	0	1	0	0	0	1	0	0
S_6	12	0	0	0	0	0	0	1	0		0	0	1	0	0	0	1	0	0	0	1	0
S_7	09	0	0	0	0	0	0	0	1		0	0	0	1	0	0	0	1	0	0	0	1

We are applying continuously revised simplex algorithm in above table for getting optimum solution. We are applying this algorithm until we are not getting

Optimal Solution

Table 5

Variable In Basic	Solution values	Basic Inverse B_1^{-1}								$y_k^{(1)}$	Additional Table											
		$\beta_0^{(1)}$	$\beta_1^{(1)}$	$\beta_2^{(1)}$	$\beta_3^{(1)}$	$\beta_4^{(1)}$	$\beta_5^{(1)}$	$\beta_6^{(1)}$	$\beta_7^{(1)}$		$a_{11}^{(1)}$	$a_{12}^{(1)}$	$a_{13}^{(1)}$	$a_{14}^{(1)}$	$a_{21}^{(1)}$	$a_{22}^{(1)}$	$a_{23}^{(1)}$	$a_{24}^{(1)}$	$a_{31}^{(1)}$	$a_{32}^{(1)}$	$a_{33}^{(1)}$	$a_{34}^{(1)}$
B	B= $x_B^{(1)}$	Z	S_1	x_{22}	x_{33}	x_{11}	x_{12}	x_{13}	x_{14}	C_k - Z_k	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	S_2	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
Z	302	1	0	12	8	6	3	5	4		0	0	0	0	-5	0	-2	-7	-5	-7	0	-6
S_1	0	0	1	1	1	-1	-1	-1	-1		0	0	0	0	0	0	0	0	0	0	0	0
x_{22}	15	0	0	1	0	0	0	0	0		0	0	0	0	1	1	1	1	0	0	0	0
x_{33}	08	0	0	0	1	0	0	0	0		0	0	0	0	0	0	0	0	1	1	1	1
x_{11}	07	0	0	0	0	1	0	0	0		1	0	0	0	1	0	0	0	1	0	0	0
x_{12}	02	0	0	-1	0	0	1	0	0		0	1	0	0	0	1	0	0	0	1	0	0
x_{13}	04	0	0	0	-1	0	0	1	0		0	0	1	0	0	0	1	0	0	0	1	0
x_{14}	09	0	0	0	0	0	0	0	1		0	0	0	1	0	0	0	1	0	0	0	1

C_k - Z_k = Max { (Cj – Zj) > 0} = Max { -(First row of B_1^{-1})(column $a_j^{(1)}$ not in basis, B_1)} = Max {-(6, 3, 5, 4, 13, 15, 15, 9, 9, 4, 13, 6)}

$C_k - Z_k < 0$

The current solution is optimal solution

$\mathbf{x_{22} = 15 \quad x_{33} = 08 \quad x_{11} = 07 \; x_{12} = 02 \; x_{13} = 04 \; x_{14} = 09}$

$Z = 6x_{11}+3x_{12}+5x_{13}+4x_{14}+5x_{21}+9x_{22}+2x_{23}+7x_{24}+5x_{31}+7x_{32}+8x_{33}+6x_{34}$

Min Z = 302

4. CONCLUSION

As main conclusions we can remark the following:

1. Proposed technique makes possible to solve parametric problems that have not been solved until now.

2. The algorithm has showed a good performance in finding parametric solutions in comparison with nonparametric solutions obtained with other nonlinear solution methods.

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[1]Author 1: IES Institute of Technology and Management, Bhopal (M.P.)
gaurav.sharma.vds.2010@gmail.com
[2]Author 2: Saifia Science College, Bhopal (M.P.)
[3]Author 3: UIT, RGPV Bhopal (M.P.)