A DETERMINISTIC INVENTORY CONTROL MODEL WITH EXPONTIAL INCREASE IN DEMAND RATE

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Abstract: In this paper, a deterministic inventory control model with exponential increase in demand rate with time has been proposed. The demand rate increases by a constant percentage during each time interval. The proposed model will be constrained binomial geometric programming model. This implies that for any exponentially growing quantity, the larger the increase in quantity, the faster it grows. A numerical analysis of the proposed model has been presented. Production rate is considered as finite and approximation procedure is used to solve the model.

Keywords: Deterministic, Inventory Control, Demand Rate, Population, Exponential Growth.

1. INTRODUCTION

A lot of work has been done for determining the inventory level of deteriorating items which allows and does not allow shortage by different researchers over last three decades Maximum physical goods undergo decay or deterioration over time. Fruits, vegetables and food items suffer from depletion by direct spoilage while stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. Deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factor into consideration. A number of researchers have developed models in the area of deteriorating inventory. An exponentially decaying inventory model was developed by Ghare and Schrader (1963). Shah and Jaiswal (1976) presented an order level inventory model for deteriorating items with a constant rate of deterioration. Goyal et al. (1988) have developed an integrated production inventory marketing model for determining the economic production quantity and economic order quantity for raw materials in a multistage production system

Mandal and Phaujdar (1989), Goswami and Choudhary (1991), Bose et al. (1995) and Mak (1982) assumed either instantaneous or finite production rate with different rates of deterioration. Many researchers have extended the EOQ model to accommodate time varying demand pattern.

Goswami and Choudhary (1991), Bhunia and Maitee (1997), Urban (1995) assumed a linear trend in demand. Hong et al. (1993) considered an inventory model with time proportional demand instantaneous replenishment and no shortage. Yang, H.L., J.T. Teng and M.S. Chern, 2001.Deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. Balkhi, Z.T., 2004. On the Optimality of Inventory Models with Deteriorating Items for Demand and On-Hand Inventory Dependent Production Rate. Hou, K.L., 2006. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Lo, S.T., H.M. Wee and W.C. Huang, 2007. An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration under inflation.

In the present model, demand is exponentially increasing, date of deterioration is linear and production rate is finite and depends upon demand as well inventory on hand.

2. ASSUMPTIONS AND NOTATIONS

The proposed deterministic inventory model for deteriorating items with finite production rate is developed under the following assumptions and notations:

(1) Demand rate D(t) is known and increases exponentially, that is at time t, $t \geq 0, D(t) = Ae^{\lambda t}$is the initial demand, λ is a constant which govern the increasing rate of demand.

(2) Shortage is not allowed.

(3) Lead time is zero.

(4) Deterioration occurs when the items are in stock. Deterioration at any time given by $\theta(t) = \alpha t,\quad 0 < \alpha < 1$

(5) No replacement or repair of deteriorated units is during the given cycle.

(6) Production P(t) depends on the on hand inventory level q(t) and the demand rate of the system at any time such that

$$P(t) = K - \beta q(t) + \gamma D(t)$$
$$where\ K > 0$$
$$0 \leq \beta < 1$$
$$0 \leq \gamma < 1.$$

Also, production rate is more than demand rate R (t) $> D(t)$.

(7) q (t) is the inventory level at any time t.

(8) C_1 denotes the holding cost per unit in stock per unit time.

(9) C_2 denotes the cost of deterioration per unit per unit time.

(10) C_3 denotes the set up cost.

(11) C is the total average cost of the system.

(12) S denotes the stock level at time $t = t_1$

3. MATHEMATICAL FORMULATION AND ANALYSIS OF THE MODEL

Initially, the stock is zero. Production starts just after t=0 and then stock level reaches S after t_1 time unit. Then the production is stopped. Just after t_1, the inventory level gradually declines, primarily due to the demand and deterioration. By this process, the stock reaches zero level at t=t_2 this cycle repeats itself.

The differential equation governing the stock status during the time interval $(0, t_2)$ are given by

$$\frac{dq(t)}{dt} = P(t) - D(t) - \theta(t)q(t) \qquad 0 \le t \le t_1 \dots\dots\dots (1)$$

and $$\frac{dq(t)}{dt} = -D(t) - \theta(t)q(t) \qquad 0 \le t \le t_2 \dots\dots\dots \quad (2)$$

Now substituting the value of R(t), D(t) and $\theta(t)$ in equation (1) one can get

$$\frac{dq(t)}{dt} + (\alpha t + \beta)q(t) = K + (\gamma - 1)Ae^{\lambda t}$$

The above equation is linear in q(t)

Its integrating factor= $e^{\int(\alpha t+\beta)dt}$

$$= e^{\frac{\alpha t^2}{2}+\beta t}$$

Hence solution of equation (1) is given by

$$q(t) = e^{-(\frac{\alpha t^2}{2}+\beta t)}\left[\int_0^t e^{\frac{\alpha t^2}{2}+\beta t}(K + (\gamma - 1)Ae^{\lambda t})dt\right]$$

$$q(t) = e^{-\left(\frac{\alpha t^2}{2}+\beta t\right)}\left[K\int_0^t e^{\frac{\alpha t^2}{2}+\beta t}\,dt + (\gamma - 1)A\int_0^t e^{\frac{\alpha t^2}{2}+\beta t}\,e^{\lambda t}dt\right]$$

Applying the series approximation and neglecting the higher power terms in α, β, γ since it these parameter are extremely small we get

$$q(t) = \left[(K + (\gamma - 1)A)\left(t - \frac{\alpha t^3}{3} - \frac{\beta t^2}{2}\right) + \frac{(\gamma-1)A\lambda t^2}{2}\right] \qquad \dots.. \qquad (4)$$

$0 \le t \le t_1$

Similarly, equation (2) on substitution of D (t) and $\theta(t)$ reduces to

$$\frac{dq(t)}{dt} = -Ae^{\lambda t} - \alpha t q(t), \qquad t_1 \le t \le t_2$$

Or $\frac{dq(t)}{dt} + \alpha t q(t) = -Ae^{\lambda t}$ ……………… (5)

The above equation is linear differential equation.

Its integrating factor= $e^{\int \alpha t\, dt} = e^{\frac{\alpha t^2}{2}}$

Therefore, solution of equation (5) is given by

$$q(t).e^{\frac{\alpha t^2}{2}} = A\int_0^t e^{\frac{\alpha t^2}{2}} e^{\lambda t} dt$$

Or

$$q(t) = -Ae^{\frac{-\alpha t^2}{2}} \int_0^t e^{\frac{\alpha t^2}{2} + \lambda t} dt$$

$$q(t) = -A\left(1 - \frac{\alpha t^2}{2}\right)\int_0^t \left(1 + \lambda t + \frac{\alpha t^2}{2}\right) dt$$

$$q(t) = -A\left(1 - \frac{\alpha t^2}{2}\right)\left(t + \frac{\lambda t^2}{2} + \frac{\alpha t^3}{6}\right) + C\left(1 - \frac{\alpha t^2}{2}\right)$$

Solution after adjusting the constant of integration is given by

$$q(t) = \left(1 - \frac{\alpha t^2}{2}\right)\left[q(t_1)\left(1 + \frac{\alpha t_1^2}{2}\right) - A\left(t + \frac{\lambda t^2}{2} + \frac{\alpha t^3}{6} - t_1 - \frac{\lambda t_1^2}{2} - \frac{\alpha t_1^3}{6}\right)\right] \ldots . \qquad (6)$$

$t_1 \le t \le t_2$ Also

$$q(t) = q(t_1)\left(1 + \frac{\alpha t_1^2}{2} - \frac{\alpha t^2}{2}\right) - A\left(t + \frac{\lambda t^2}{2} - \frac{\alpha t^3}{3} - t_1 - \frac{\lambda t_1^2}{2} - \frac{\alpha t_1^3}{6} + \frac{\alpha t_1 t^2}{2}\right) \ldots . (7)$$

Other term are neglecting or cancelled

Where $q(t_1)$ is the value of q (t) at $t = t_1$

Using the boundary condition q(t) =0 at $t = t_2$ in equation (6) we get

$$0 = \left(1 - \frac{\alpha t_2^2}{2}\right)\left[q(t_1)\left(1 + \frac{\alpha t_1^2}{2}\right) - A\left(t_2 + \frac{\lambda t_2^2}{2} + \frac{\alpha t_2^3}{6} - t_1 - \frac{\lambda t_1^2}{2} - \frac{\alpha t_1^3}{6}\right)\right]$$

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$$0 = \left[q(t_1)\left(1+\frac{\alpha t_1^2}{2}\right)\left(1-\frac{\alpha t_2^2}{2}\right) - A\left(1-\frac{\alpha t_2^2}{2}\right)\left(t_2+\frac{\lambda t_2^2}{2}+\frac{\alpha t_2^3}{6}-t_1-\frac{\lambda t_1^2}{2}-\frac{\alpha t_1^3}{6}\right)\right]$$

Therefore

$$q(t_1) = A\left(1-\frac{\alpha t_1^2}{2}\right)\left(t_2+\frac{\lambda t_2^2}{2}+\frac{\alpha t_2^3}{6}-t_1-\frac{\lambda t_1^2}{2}-\frac{\alpha t_1^3}{6}\right) = S \ldots\ldots\ldots \quad (8)$$

Putting the value of $q(t_1)$ in equation (7) we get

$$q(t) = A\left(t_2+\frac{\lambda t_2^2}{2}+\frac{\alpha t_2^3}{6}-t+\frac{\lambda t^2}{2}+\frac{\alpha t^3}{6}-\frac{\alpha t^2 t_2}{2}\right)\ldots\ldots\ldots \quad (9)$$

$t_1 \le t \le t_2$

Putting $t = t_1$ in equation (4) and comparing with equation (8), t_1 and t_2 are related as

$$\left[(K+(\gamma-1)A)\left(t_1-\frac{\alpha t_1^3}{3}-\frac{\beta t_1^2}{2}\right)+\frac{(\gamma-1)A\lambda t_1^2}{2}\right] = A\left(t_2+\frac{\lambda t_2^2}{2}+\frac{\alpha t_2^3}{6}-t+\frac{\lambda t^2}{2}+\frac{\alpha t^3}{6}-\frac{\alpha t^2 t_2}{2}\right)\ldots\ldots\ldots\ldots\ldots \quad (10)$$

The inventory holding cost over the period $(0, t_2)$ is given by

$$= C_1\left(\int_0^{t_1} q(t)\,dt + \int_{t_1}^{t_2} q(t)\,dt\right)$$

Substituting the values we get

$$= C_1\left[\int_0^{t_1}(K+(\gamma-1)A)\left(t-\frac{\alpha t^3}{3}-\frac{\beta t^2}{2}\right)+\frac{(\gamma-1)A\lambda t^2}{2}\right]dt + A\int_{t_1}^{t_2}\left[t_2+\frac{\lambda t_2^2}{2}+\frac{\alpha t_2^3}{6}-t-\frac{\lambda t^2}{2}+\frac{\alpha t^3}{6}-\frac{\alpha t^2 t_2}{2}\right]dt$$

$$= C_1\left[\left\{(K+(\gamma-1)A)\left(\frac{t^2}{2}-\frac{\alpha t^4}{12}-\frac{\beta t^3}{6}\right)+\frac{(\gamma-1)A\lambda t^3}{6}\right\}_0^{t_1} + A\left\{t_2 t+\frac{t\lambda t_2^2}{2}-\frac{t\alpha t_2^3}{6}-\frac{t^2}{2}-\frac{\lambda t^3}{6}+\frac{\alpha t^4}{12}-\frac{\alpha t^3 t_2}{6}\right\}_{t_1}^{t_2}\right]\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

The cost of the deterioration for the period $(0, t_2)$ is given by

$$= C_2\left(\int_0^{t_1} R(t)dt - \int_0^{t_2} D(t)\, dt\right)$$

On substitution the values of S, R (t) and D (t) and integration this cost reduces to

$$= C_2\left[Kt_1 + A\gamma\left(t_1 + \frac{\lambda t_1^2}{2}\right) - A\left(t_2 + \frac{\lambda t_2^2}{2}\right)\right] - \beta\int_0^{t_1} q(t)dt \ldots\ldots\ldots\ldots (12)$$

Now the total cost of the system is given by

$$C = \frac{1}{t_2}\left[(C_1 - \beta C_2)\left\{\left((K + A(\gamma-1)\left(\frac{t_1^2}{2} - \frac{\alpha t_1^4}{12} + \frac{\beta t_1^3}{6}\right) + \frac{(\gamma-1)\lambda A t_1^3}{6}\right)\right\}\right.$$
$$+ AC_1\left\{\frac{t_2^2}{2} + \frac{\lambda t_2^3}{3} + \frac{\alpha t_2^4}{12} - t_1 t_2 - \frac{\lambda t_1 t_2^3}{2} - \frac{\alpha t_1 t_2^3}{6} + \frac{t_1^2}{6} + \frac{\lambda t_1^3}{6}\right.$$
$$\left. - \frac{\alpha t_1^4}{12} + \frac{\alpha t_1^3 t_2}{6}\right\} + C_2\left\{Kt_1 + A\gamma\left(t_1 + \frac{\lambda t_1^2}{2}\right) - A\left(t_2 + \frac{\lambda t_2^2}{2}\right)\right\}$$
$$\left. + C_3\right] \ldots.. \qquad (13)$$

The total cost equation (13) depends upon two variable t_1 and t_2. These variables however are not independent. They are related by equation (10). To obtain the optimal values of t_1 and t_2 which minimize the total cost of the system one has to solve the equations.

$\frac{\partial C}{\partial t_1} = 0$ And $\qquad \frac{\partial C}{\partial t_2} = 0.$

Under the condition

$$\frac{\partial^2 C}{\partial t_1^2} > 0, \qquad \frac{\partial^2 C}{\partial t_2^2} > 0$$

And $\qquad \frac{\partial^2 C}{\partial t_1^2}\cdot\frac{\partial^2 C}{\partial t_2^2} + \left(\frac{\partial^2 C}{\partial t_1 t_2}\right)^2 > 0$

Applying this one can get

$$(C_1 - \beta C_2)\left\{(K + (\gamma-1)A)\left(t_1 - \frac{\alpha t_1^3}{3} - \frac{\beta t_1^2}{2}\right) + \frac{(\gamma-1)A\lambda t_1^2}{2}\right\}$$
$$+ AC_1\left(-t_2 - \frac{\lambda t_2^2}{2} - \frac{\alpha t_2^3}{3} + t_1 - \frac{\lambda t_1^2}{2} - \frac{\alpha t_1^3}{3} + \frac{\alpha t_1^2 t_2}{2}\right)$$
$$+ C_2(K + A\gamma(1 + \lambda t_1)) = 0 \ldots\ldots.. \qquad (14)$$

also

$$(C_1-\beta C_2)\left\{(K+(\gamma-1)A)\left(\frac{t_1^2}{2}-\frac{\alpha t_1^4}{12}-\frac{\beta t_1^3}{6}\right)+\frac{(\gamma-1)A\lambda t_1^3}{6}\right\}$$
$$+AC_1\left\{\left(\frac{t_1^2}{2}+\frac{\lambda t_2^3}{6}+\frac{\alpha t_1^4}{12}+\right)+C_2\left(Kt_1+A\gamma(t_1+\frac{\lambda t_1^2}{2})\right)\right\}+C_3$$
$$+\frac{C_2A\lambda t_2^2}{2}-C_1A\left(\frac{t_2^2}{2}+\frac{2\lambda t_2^3}{3}+\frac{\alpha t_2^4}{4}-\frac{\lambda t_1t_2^2}{2}-\frac{\alpha t_2^3t_1}{3}\right)$$
$$=0\ldots\ldots \qquad (15)$$

Equation (14) and (15) are non linear equations. They can be solved by using computer and suitable software. In this way one can get the optimal values of t_1 (say t_1^*) and t_2 (say t_2^*) and using values t_1^* and t_2^* in equation (10), (11) and (12) one get optimum values of holding cost, deterioration cost and total cost respectively.

4. CONCLUSION

In this paper, an attempt is made to develop an inventory model by assuming that production rate is finite and it depends on demand and on hand inventory simultaneously. Deterioration rate is assumed to be a linear function of time and the model is formulated and solved by taking the exponentially increasing rate for demand. The approximate expression for holding cost, deterioration cost and total cost of the system is obtained. Cost minimization technique is used to get the solution.

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