FUNDAMENTAL THEOREM OF HOMOMORPHISMS AND FIRST ISOMORPHISM THEOREM ON A*-ALGEBRAS

B. Vijaya Kumar[1]

Abstract: This paper presents Fundamental theorem of homomorphisms on

A*-algebras and First isomorphism theorem on A*-algebras.

1. INTRODUCTION

The concept of A*-algebra is firstly introduced by Koteswara Rao. P in his thesis[1]. For X, $\mathbf{3}^X$ is an A*-algebra, where **3**={0,1,2}. Every A*-algebra can be imbedded in $\mathbf{3}^X$ for some X. In his thesis, Koteswara Rao. P studied equivalences between A*-algebras, Adas, C-algebras and their connections with Stone type representation and introduced the concepts of A*-clones and If-Then-Else structures over A*-algebras and ideals of A*-algebras. [5] and [6] are the study of A*-algebras and introduced subdirect representations of A*-algebras, congruences and prime ideals of A*-algebras, categorical aspects of A*-algebras, products and co-products of A*-algebras, the concept of pre A*-algebras, equivalence of if-then-else algebras over pre A*-algebras and modules over A*-algebras, A*-fields of sets, n-ary classes and prime ideal spaces of A*-algebras. After that in the year 2009, Vijaya Kumar. B firstly presented Fundamental theorem of homomorphisms on A*-algebras and First isomorphism theorem on A*-algebras.

Keywords: A*-algebra, Congruence, Homomorphism, Isomorphism and Sub-A*-algebra.

Definition 1: An algebra $(A, \wedge, *, (-)^{\sim}, (-)_{\pi}, 1)$ is an A*-algebra if it satisfies:

(i) $a_\pi \vee (a_\pi)^{\sim}=1$, $(a_\pi)_\pi = a_\pi$ where $a \vee b = (a^{\sim} \wedge b^{\sim})^{\sim}$

(ii) $a_\pi \vee b_\pi = b_\pi \vee a_\pi$

(iii) $(a_\pi \vee b_\pi) \vee c_\pi = a_\pi \vee (b_\pi \vee c_\pi)$

(iv) $(a_\pi \wedge b_\pi) \vee (a_\pi \wedge (b_\pi)^{\sim}) = a_\pi$

(v) $(a \wedge b)_\pi = a_\pi \wedge b_\pi$, $(a \wedge b)^{\#} = a^{\#} \vee b^{\#}$ where $a^{\#} = (a_\pi \vee a^{\sim}{}_\pi)^{\sim}$

(vi) $a^{\sim}{}_\pi = (a_\pi \vee a^{\#})^{\sim}$, $a^{\sim\#} = a^{\#}$

(vii) $(a * b)_\pi = a_\pi$, $(a * b)^{\#} = (a_\pi)^{\sim} \wedge (b^{\sim}{}_\pi)^{\sim}$

(viii) a = b if and only if $a_\pi = b_\pi$, $a^{\#} = b^{\#}$.

We write 0 for $1^{\sim}$, 2 for $0 * 1$.

Remark: The motivation for the operation * is the "disjointifica-tion" of the familiar rectangular bands of semigroup theory which provide an equational way of the composing a set into a Cartesian product with two factors.

Example: **3**={0, 1, 2} with the operations defined below is an A*- algebra.

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

$*$	0	1	2
0	0	2	2
1	1	1	1
2	0	2	2

x	0	1	2
$x^{\sim}$	1	0	2
x_π	0	1	0
$x^{\#}$	0	0	1

Note: From Definition 1 (i) through (iv) and Huntington's Theorem $B(A)= \{a_\pi \mid a \in A\}$ is a Boolean algebra with $\wedge$, $\vee, (-)^{\sim}$, 0 and a $\in B(A) \Rightarrow a_\pi=a$. Since 1, 0, $(a_\pi)^{\sim} \in B(A)$, we have $1_\pi=1$, $0_\pi=0$, $(a_\pi)^{\sim}{}_\pi=(a_\pi)^{\sim}$ and $a_\pi \wedge a^{\#}=0$, $a * 0=a_\pi$.

Lemma 1: For any x, y, z in an A*- algebra

(i) $x^{\sim\sim} = x$
(ii) $(x \wedge y)^{\sim}{}_\pi=(x^{\sim} \wedge y)_\pi \vee (x \wedge y^{\sim})_\pi \vee (x^{\sim} \wedge y^{\sim})_\pi$
(iii) $(x \vee y)_\pi =(x^{\sim} \wedge y)_\pi \vee (x \wedge y^{\sim})_\pi \vee (x \wedge y)_\pi$
(iv) $x \wedge (y \vee z)= (x \wedge y) \vee (x \wedge z)$.

Lemma 2: For any x,y in A

(i) $(x * y)^{\sim}{}_\pi=(x_\pi)^{\sim} \wedge (y^{\sim})_\pi$
(ii) $x = x_\pi * (x^{\sim})_\pi{}^{\sim}=(x_\pi) * x^{\#}$
(iii) If $x = e * f$, where $e, f \in B(A)$, $e \wedge f = 0$, then $x_\pi = e$, $x^{\#} = f$.

Theorem 1: Every A*-algebra $(A, \wedge, *, (-)_\pi, (-)^{\sim}, 1)$ satisfies the following conditions:

For x, y, z in A

(i) $x \wedge (y \wedge z)= (x \wedge y) \wedge z$
(ii) $x \wedge y = y \wedge x$
(iii) $x \wedge x = x$
(iv) $1 \wedge x = x$
(v) $x^{\sim\sim} = x$
(vi) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ where $x \vee y =(x^{\sim} \wedge y^{\sim})^{\sim}$
(vii) $1_\pi= 1$
(viii) $[(x_\pi)^{\sim}]_\pi = (x_\pi)^{\sim}$
(ix) $(x \wedge y)_\pi= x_\pi \wedge y_\pi$
(x) $(x \wedge x^{\sim})_\pi= 0$ where $1^{\sim}=0$
(xi) $x_\pi \wedge (x_\pi \vee y_\pi)= x_\pi$

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(xii) $(x \wedge y)^{\sim}{}_{\pi} = (x \wedge y^{\sim})_{\pi} \vee (x^{\sim} \wedge y)_{\pi} \vee (x^{\sim} \wedge y^{\sim})_{\pi}$
(xiii) $(x_{\pi})_{\pi} = x_{\pi}$
(xiv) $(x * y)_{\pi} = x_{\pi}$
(xv) $(x * y)^{\sim}{}_{\pi} = (x_{\pi})^{\sim} \wedge (y^{\sim})_{\pi}$
$x = x_{\pi} * (x^{\sim}{}_{\pi})^{\sim}$

E.G. Manes around 1989, in a rough draft of his paper entitled "The Equational Theory of Disjoint Alternatives", the algebra $(A, \wedge, *, (-)_{\pi}, (-)^{\sim}, 1)$ satisfying Th.1(i) through (xvi) called as an ada, which however differs from the definition of an ada.

Theorem 2: An algebra $(A, \wedge, *, (-)^{\sim}, (-)_{\pi}, 1)$ satisfying axioms of the above theorem is an A*-algebra.

Definition 2: Let $(A, \wedge, *, (-)^{\sim}, (-)_{\pi}, 1)$ be an A*-algebra and $A_1 \subseteq A$, A_1 is called a sub A*-algebra of A if A_1 is closed under $\wedge, *, (-)^{\sim}, (-)_{\pi}, 0, 1$.

Definition 3: An A*-algebra of sets(with universal set X) is a subset of T_X , closed under $\wedge, \vee, (-)^{\sim}, (-)_{\pi}, *$.

Definition 4: Let $(A_1, \wedge, \vee, (-)^{\sim}, (-)_{\pi}, *, 1)$ and $(A_2, \wedge, \vee, (-)^{\sim}, (-)_{\pi}, *, 1)$ be A*- algebras. A mapping $f : A_1 \rightarrow A_2$ is called an A*-homomorphism if

(i) $f(a \wedge b) = f(a) \wedge f(b)$ (ii) $f(a * b) = f(a) * f(b)$
(iii) $f(a_{\pi}) = (f(a))_{\pi}$ (iv) $f(a^{\sim}) = (f(a))^{\sim}$
(v) $f(1) = 1$ and (vi) $f(0) = 0$.

If in addition f is bijective, then f is called an A*-isomorphism, and A_1, A_2 are said to be isomorphic, denote in symbols $A_1 \cong A_2$.

Definition 5: A congruence relation $\emptyset$ on an A*-algebra is an equivalence relation on A satisfying
(i) $a \emptyset b \Rightarrow a_{\pi} \emptyset b_{\pi}$, $a^{\#} \emptyset b^{\#}$, $a^{\sim} \emptyset b^{\sim}$
(ii $a \emptyset b, c \emptyset d \Rightarrow (a * c) \emptyset (b * d), (a \wedge c) \emptyset (b \wedge d)$.

Note: Definition 5 is equivalent to
(i) $a \emptyset b \Rightarrow a^{\sim} \emptyset b^{\sim}$
(ii) $a \emptyset b, c \emptyset d \Rightarrow (a \wedge c) \emptyset (b \wedge d)$
(iii) $a \emptyset b, c \emptyset d \Rightarrow (a * c) \emptyset (b * d)$.

Theorem 3: Fundamental theorem of homomorphisms of

A*-algebras:

Let A and B be A*-algebras, f a homomorphism of A into B. Then $\emptyset = f^{-1}f$ is a congruence on A and f(A) is a sub A*-algebra of B. Moreover, we have a unique homomorphism $\bar{f}$ of $A/\emptyset$ into B such that $f = \bar{f}\vartheta$, where ϑ is the homomorphism a $\mapsto \bar{a}$ where $\bar{a}=\emptyset(a)$ of A into A/∅. The homomorphism $\bar{f}$ is injective and ϑ is surjective.

Proof: A, B are A*-algebras. f : A → B is an A*-homomorphism.

Claim: ∅ is a congruence relation on A.

(i) ∅ is reflexive: Let a ∈ A.
Since f(a)= f(a) ⇒(a, a) ∈ $f^{-1}f$ ⇒ (a, a) ∈ ∅.
Therefore ∅ is reflexive.

(ii) ∅ is symmetric: Let (a, b) ∈ ∅.
Then (a, b) ∈ $f^{-1}f$ ⇒ f(a)= f(b) ⇒f(b)= f(a)
⇒ (b, a) ∈ $f^{-1}f$ ⇒ (b, a) ∈ ∅.

Therefore ∅ is symmetric.

(iii) ∅ is transitive: Let (a, b) ∈ ∅, (b, c) ∈ ∅.
⇒ (a, b) ∈ $f^{-1}f$, (b, c) ∈ $f^{-1}f$
⇒ f(a) = f(b), f(b) = f(c) ⇒ f(a) = f(c)
⇒ (a, c) ∈ $f^{-1}f$ ⇒ (a, c) ∈ ∅.
Therefore ∅ is transitive.

Therefore ∅ is an equivalence relation.
Suppose a ∅ b, c ∅ d ⇒ f(a)= f(b), f(c)= f(d).
f(a ∧ c)= f(a) ∧ f(c)= f(b) ∧ f(d)= f(b ∧ d).
Therefore (a ∧ c) ∅ (b ∧ d).
f(a ∗ c)= f(a) ∗ f(c)= f(b) ∗ f(d)= f(b ∗ d).
Therefore (a ∗ c) ∅ (b ∗ d).
$f(a_\pi) = (f(a))_\pi = (f(b))_\pi = f(b_\pi)$.
Therefore a_π ∅ b_π.

$f(a^\sim) = (f(a))^\sim = (f(b))^\sim = f(b^\sim)$

Therefore $a^\sim$ ∅ $b^\sim$.
Therefore ∅ is a congruence relation on A.
(ii) f(A) is a sub A*-algebra:
Since 0, 1, 2 ∈ A ⇒**0, 1, 2** ∈ f(A) (**0**= f(0), **1**= f(1), **2**= f(2)).
Let x, y ∈ f(A). Then ∃ a, b ∈ A ∋ f(a)= x, f(b)= y.
x ∧ y = f(a) ∧ f(b) = f(a ∧ b) ∈ f(A) (since a ∧ b ∈ A)
x ∗ y = f(a) ∗ f(b) = f(a ∗ b) ∈ f(A) (since a ∗ b ∈ A)

$x_\pi = (f(a))_\pi = f(a_\pi) \in f(A)$ (since $a_\pi \in A$)
$x^\sim = (f(a))^\sim = f(a^\sim) \in f(A)$ (since $a^\sim \in A$)

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Therefore f(A) is a sub A*-algebra of B.

(iii) To show there exists unique $\bar{f}$:A/∅ → B such that $\bar{f}\vartheta$= f :

Define $\bar{f}$: A/∅ → B as follows:

Let $\bar{a} \in$ A/∅. $\bar{f}(\bar{a})$ = f(a).

$\bar{a} = \bar{b} \Leftrightarrow$ a ∈ $\bar{b} \Leftrightarrow a \emptyset b \Leftrightarrow f(a) = f(b) \Leftrightarrow \bar{f}(\bar{a}) = \bar{f}(\bar{b})$.

Therefore $\bar{f}$ is well defined and one-one.

Now define ϑ : A → A/∅ by ϑ(a) = $\bar{a}$, ∀ a ∈ A.

Clearly ϑ is surjective.

Claim: $\bar{f}\,\vartheta$ = f.

Clearly f, $\bar{f}\,\vartheta$: A → B.

Let a ∈ A.

$\bar{f}\,\vartheta(a) = \bar{f}(\vartheta(a)) = \bar{f}(\bar{a}) = f(a)$.

Therefore $\bar{f}\,\vartheta$ = f.

Claim: $\bar{f}$: A/∅ → B is a homomorphism.

Let $\bar{a}$, $\bar{b}$ ∈ A/∅.

$\bar{f}(\bar{a} \wedge \bar{b}) = \bar{f}(\overline{a \wedge b})$= f(a ∧ b)= f(a) ∧ f(b)= $\bar{f}(\bar{a}) \wedge \bar{f}(\bar{b})$

$\bar{f}(\bar{a} * \bar{b}) = \bar{f}(\overline{a * b})$= f(a ∗ b)= f(a)∗f(b)= $\bar{f}(\bar{a}) * \bar{f}(\bar{b})$

$\bar{f}(\bar{a}_\pi) = \bar{f}(\overline{a_\pi})$ = f(a_π)= $(f(a))_\pi = (\bar{f}(\bar{a}))_\pi$

$\bar{f}(\bar{a}^\sim) = \bar{f}(\overline{a^\sim}) = f(a^\sim) = (f(a))^\sim = (\bar{f}(\bar{a}))^\sim$

$\bar{f}(\bar{0}) = f(0) = \mathbf{0}$, $\bar{f}(\bar{1}) = f(1) = \mathbf{1}$, $\bar{f}(\bar{2}) = f(2) = \mathbf{2}$.

Therefore $\bar{f}$ is a homomorphism.

Suppose g: A/∅ → B is a homomorphism ∋ gϑ = f.

Claim: $\bar{f}$ = g.

Let $\bar{a} \in$ A/∅.

$\bar{f}(\bar{a}) = \bar{f}(\vartheta(a)) = (\bar{f}\vartheta)(a) = f(a) = (g\vartheta)(a) = g(\vartheta(a)) = g(\bar{a})$.

Therefore $\bar{f} = g$.

Therefore $\bar{f}$ is unique.

Theorem 4: First isomorphism theorem on A*-algebras:

Let ∅ be a congruence on an A*-algebra A, A_1 a sub A*-algebra of A. Let A_1' be the union of the ∅ - equivalence classes that meet A_1. Then A_1' is a subalgebra of A containing A_1,$\emptyset_1$=∅ ∩ (A_1 x A_1) and $\emptyset_1'$ = ∅ ∩ (A_1' x A_1') are congruences on $A_1$and A_1' respectively, and $\bar{a}_{1\emptyset_1} \mapsto \bar{a}_{1\emptyset_1'}$ is an isomorphism of $A_1/\emptyset_1$ onto $A_1'/\emptyset_1'$.

Proof: Clearly $A_1' = \cup_{a \in A_1} \bar{a}$.

Claim: $A_1'$is a sub A*-algebra of A containing A_1.

Clearly $A_1 \subset A_1'$.

Since 0, 1, 2 ∈ A_1 ⇒0, 1, 2 ∈ A_1'(since 0 ∈ $\bar{0}$, 1 ∈ $\bar{1}$, 2 ∈ $\bar{2}$).
Let $a_1, b_1 \in A_1'$. Then $\exists a, b \in A_1 \ni a_1 \in \bar{a},\ b_1 \in \bar{b}$
$\Rightarrow a_1 \emptyset\, a,\ b_1 \emptyset\, b \Rightarrow (a_1 \wedge\ b_1)\emptyset(a \wedge b) \Rightarrow a_1 \wedge b_1 \in \overline{a \wedge b}$
$\Rightarrow a_1 \wedge b_1 \in A_1'$ (since a ∧ b ∈ A_1).
since $a_1 \emptyset\, a,\ b_1 \emptyset\, b \Rightarrow (a_1 * b_1)\ \emptyset(a * b) \Rightarrow a_1 * b_1 \in \overline{a * b}$
$\Rightarrow a_1 * b_1 \in A_1'$ (since a ∗ b ∈ A_1)
$a_1 \emptyset\, a \Rightarrow a_{1_\pi} \emptyset\, a_\pi \Rightarrow a_{1_\pi} \in\ \bar{a}_\pi \Rightarrow a_{1_\pi} \in A_1'$(since $a_\pi \in A_1$)
$a_1 \emptyset\, a \Rightarrow a_1{}^{\sim} \emptyset\, a^{\sim} \Rightarrow a_1{}^{\sim} \in \overline{a^{\sim}} \Rightarrow a_1{}^{\sim} \in A_1'$ (since $a^{\sim} \in A_1$)
Therefore A_1' is a sub A*-algebra of A containing A_1.

<u>Claim</u>: $\emptyset_1 = \emptyset \cap (A_1 \text{ x } A_1)$ is a congruence relation on A_1.
Clearly $\emptyset_1$ is an equivalence relation on A_1.
Let a $\emptyset_1$b, c $\emptyset_1$d. Then a ∅ b, c ∅ d, a, b, c, d ∈ A_1.
a ∅ b, c ∅ d ⇒ (a ∧ c) ∅ (b ∧ d) (since ∅ is a congruence)
⇒ (a ∧ c) $\emptyset_1$(b ∧ d) (since a ∧ c, b ∧ d ∈ A_1).
a ∅ b, c ∅ d ⇒ (a ∗ c) ∅ (b ∗ d) (since ∅ is a congruence)
⇒ (a ∗ c) $\emptyset_1$(b ∗ d) (since a ∗ c, b ∗ d ∈ A_1).
a ∅ b ⇒ $a_\pi \emptyset\, b_\pi$(since ∅ is a congruence)
⇒ $a_\pi \emptyset_1 b_\pi$ (since $a_\pi, b_\pi \in A_1$).
a ∅ b ⇒ $a^{\sim} \emptyset\, b^{\sim}$(since ∅ is a congruence)
⇒ $a^{\sim} \emptyset_1 b^{\sim}$ (since $a^{\sim}, b^{\sim} \in A_1$).
Therefore $\emptyset_1$is a congruence on A_1.
<u>Claim</u>: $\emptyset_1' = \emptyset \cap (A_1' \text{ x } A_1')$ is a congruence relation on A_1'.
Clearly $\emptyset_1'$ is an equivalence relation on A_1'.
Let a $\emptyset_1'$b, c $\emptyset_1'$d. Then a ∅ b, c ∅ d, a, b, c, d ∈ A_1'.
a ∅ b, c ∅ d ⇒ (a ∧ c) ∅ (b ∧ d) (since ∅ is a congruence)
⇒ (a ∧ c) $\emptyset_1'$(b ∧ d) (since a ∧ c, b ∧ d ∈ A_1').
a ∅ b, c ∅ d ⇒ (a ∗ c) ∅ (b ∗ d) (since ∅ is a congruence)
⇒ (a ∗ c) $\emptyset_1'$(b ∗ d) (since a ∗ c, b ∗ d ∈ A_1').
a ∅ b ⇒ $a_\pi \emptyset\, b_\pi$(since ∅ is a congruence)
⇒ $a_\pi \emptyset_1' b_\pi$ (since $a_\pi, b_\pi \in A_1'$).
a ∅ b ⇒ $a^{\sim} \emptyset\, b^{\sim}$(since ∅ is a congruence)
⇒ $a^{\sim} \emptyset_1' b^{\sim}$ (since $a^{\sim}, b^{\sim} \in A_1'$).
Therefore $\emptyset_1'$is a congruence on A_1'.
Define f : $A_1/\ \emptyset_1 \rightarrow A_1'/\ \emptyset_1'$ by f($\bar{a}_{1\emptyset_1}$) = $\bar{a}_{1\emptyset_1'}$ $\forall\ \bar{a}_{1\emptyset_1} \in A_1/\ \emptyset_1$.
$\bar{a}_{1\emptyset_1} = \bar{b}_{1\emptyset_1} \Leftrightarrow a_1\ \emptyset_1\ b_1$, where $a_1, b_1 \in A_1$

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$\Leftrightarrow a_1 \emptyset b_1, a_1, b_1 \in A_1^{'}$

$\Leftrightarrow a_1 \emptyset_1^{'} b_1 \Leftrightarrow \bar{a}_{1\emptyset_1^{'}} = \bar{b}_{1\emptyset_1^{'}} \Leftrightarrow f(\bar{a}_{1\emptyset_1})$=f($\bar{b}_{1\emptyset_1})$

Therefore f is well defined and one-one.

Let $\bar{a}_{1\emptyset_1^{'}} \in A_1^{'}/ \emptyset_1^{'}$.

Since $a_1 \in A_1^{'} \Rightarrow a_1 \in \bigcup_{a\in A_1} \bar{a} \Rightarrow a_1 \in \bar{a}$, for some $a \in A_1$

$\Rightarrow \bar{a}_{1\emptyset} = \bar{a}_{\emptyset}$, $a \in A_1$

$\Rightarrow \bar{a}_{1\emptyset_1^{'}} = \bar{a}_{\emptyset_1^{'}} \Rightarrow f(\bar{a}_{\emptyset_1}) = \bar{a}_{\emptyset_1^{'}} = \bar{a}_{1\emptyset_1^{'}}$.

Therefore f is onto.

<u>Claim</u>: f is a homomorphism.

Let $\bar{a}_{1\emptyset_1}, \bar{b}_{1\emptyset_1} \in A_1/ \emptyset_1$.

$f(\bar{a}_{1\emptyset_1} \wedge \bar{b}_{1\emptyset_1}) = f((\overline{a_1 \wedge b_1})_{\emptyset_1}) = (\overline{a_1 \wedge b_1})_{\emptyset_1^{'}} = \bar{a}_{1\emptyset_1^{'}} \wedge \bar{b}_{1\emptyset_1^{'}}$

$= f(\bar{a}_{1\emptyset_1}) \wedge f(\bar{b}_{1\emptyset_1})$

$f(\bar{a}_{1\emptyset_1} * \bar{b}_{1\emptyset_1}) = f((\overline{a_1 * b_1})_{\emptyset_1}) = (\overline{a_1 * b_1})_{\emptyset_1^{'}} = \bar{a}_{1\emptyset_1^{'}} * \bar{b}_{1\emptyset_1^{'}}$

$= f(\bar{a}_{1\emptyset_1}) * f(\bar{b}_{1\emptyset_1})$

$f((\bar{a}_{1\emptyset_1})_{\pi}) = f((\overline{a_{1\pi}})_{\emptyset_1}) = (\overline{a_{1\pi}})_{\emptyset_1^{'}} = (\bar{a}_{1\emptyset_1^{'}})_{\pi} = f(\bar{a}_{1\emptyset_1})_{\pi}$

$f(\bar{a}_{1\emptyset_1}{}^{\sim}) = f((\overline{\tilde{a_1}})_{\emptyset_1}) = (\overline{\tilde{a_1}})_{\emptyset_1^{'}} = (\overline{a_{1\emptyset_1^{'}}})^{\sim} = f(\bar{a}_{1\emptyset_1})^{\sim}$

Clearly $f(\bar{0}_{\emptyset_1}) = \bar{0}_{\emptyset_1^{'}}$, $f(\bar{1}_{\emptyset_1}) = \bar{1}_{\emptyset_1^{'}}$, $f(\bar{2}_{\emptyset_1}) = \bar{2}_{\emptyset_1^{'}}$.

Therefore f is a homomorphism.

Therefore f is an isomorphism.

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Dr. B. Vijaya Kumar,

Head, Dept. of Mathematics,
Andhra Christian College,
GUNTUR -522001,
Andhra Pradesh, India.
bussavijay@gmail.com