COMPARISON OF VARIOUS STRATEGIES IN TWO TAILED SECRETARY PROBLEM

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Abstract: In the two tailed secretary problem more emphasis is given on the probability of best selection of items. Instead of considering probability of best selection of item, if we consider cost aspect of selection of items then certainly view of selection of item changes. In this paper we consider comparison of various strategies in the variations in the secretary problem.

Keywords : Two tailed secretary problem, Modified optimality criterion.

1. INTRODUCTION

Secretary problem is a sequential decision procedure in which one has to select an item out of N items presented one by one in random order before an observer. The item which is presented must be either accepted or rejected immediately after an inspection. If the item is accepted the process terminates with the selection of that item. If the item is rejected the next item is called for inspection. If the observer reaches the last item then it must be accepted.

In the original secretary problem we generally follow the procedure as "Inspect first r units without selecting any. Thereafter we are selecting a unit which is best of the first r units. If we fail to select from the first N-1 units then we have to select the last unit i.e. the N^{th} unit."

In the analysis of secretary problem we consider two random variables X & Y where y is the number of units at which the selection is made. And X is the real rank of the selected unit. We discuss explicitly the expression of the probability that we stop after examining y units & the selected unit has real rank x when there are in all N units available & the selection is not made from first r units. This probability is denoted by P(x,y/r,N) where X = 1,2,3,…,N & Y = r+1,r+2,r+3,….,N.

2. ANALYSIS OF SECRETARY PROBLEM :

The joint distribution of X & Y is given by

$$P(x,y/r,N) = \begin{cases} \frac{r\,(N-y)!(x-1)!}{(y-1)N!(x-y)!} & r+1 \le y \le N-1,\ y \le x \le N \\ \frac{r}{N\,(N-1)} & y = N \\ 0 & \text{otherwise} \end{cases}$$

From this joint distribution function we can obtained the marginal distribution of random variable X & Y which is given as under.

The marginal distribution of random variable X is given by

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P_X(x/r,N) = $\frac{r}{N(N-1)}$ $\qquad$ $1 \le x \le r$

$= \frac{r}{N(N-1)} [\, 1 + \sum_{y=r+1}^{x} \frac{\binom{x-1}{y-1}}{\binom{N-2}{y-2}}]$ $\qquad$ $r+1 \le x \le N-1$

$= \frac{r}{N} [\frac{1}{r} + \frac{1}{r+1} + \cdots + \frac{1}{N-1}]$ $\qquad$ x = N

The marginal distribution of random variable Y is given by

P_Y(y/r,N) = 0 $\qquad$ $1 \le y \le r$

$= \frac{r}{y(y-1)}$ $\qquad$ y = r+1, r+2,…., N-1

$= \frac{r}{N-1}$ $\qquad$ y = N

After developing a computer program in BASIC & after executing the program for few values of N & r. It is being found that P_x(x/r,N) is non decreasing function of x.

If the program is executed for a given values of N taking all the values of r ranging from 1 to N-1 it can be observed that probability of selecting the best unit first increases reaches a maximum value and then decreases. The optimum value of r is denoted by r_0

The following table shows the maximum probability for given values of r & N.

Table I: Table of probabilities $P_x(N/r_0,N)$, r_0, and N

N	r_0	$P_x(N/r_0,N)$
10	3	0.3986905
20	7	0.3842089
30	11	0.3786514
40	15	0.3757428
50	18	0.374275
60	22	0.3732099

3. EXPECTED VALUE OF Y

E(Y/r,N) = $\sum_{y=r+1}^{N} y\, P_Y$ (y/r,N)

$= r[\frac{1}{r} + \frac{1}{r+1} + \cdots + \frac{1}{N-2} + \frac{N}{N-1}]$

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4. TWO TAILED SECRETARY PROBLEM:

Let there be large number of items to be observed by an observer. The observer adopts 'Secretary problem procedure' for the selection of best item. If for some reason the observer can not afford to all the available N items and wants to terminate the process quite early under any circumstances, he will have to stop at the most r_2 number of items ($r_2 < N$) irrespective of the rank of the r_2^{th} unit. We introduce here some modifications of the original secretary problem

"Inspect first r_1 units without selecting any. Select the unit thereafter which is better than the best of the first r_1 units. If none of the first r_2-1 units is selected then select the r_2^{th} unit."

5. ANALYSIS OF THE TWO TAILED SECRETARY PROBLEM:

We note that the procedure leads to two random variables X & Y where Y is the number of units at which the selection is made. Let the probability that we stop after observing y units and the selected unit has real rank x be denoted by the probability $P(x,y/r_1.r_2,N)$

The joint distribution x & Y is given by

The probability that (X = x,Y=y) is given by

$$P(x,y/r_1,r_2,N) = \frac{r_1(N-y)!(x-1)!}{(y-1)N!(x-y)!} \qquad r_1+1 \le y \le r_2-1, y \le x \le N$$

$$= \frac{r_1(N-r_2)!}{N!}\sum_{i=0}^{N-r_2}\frac{(N-2-i)!}{(N-r_2-i)!} \quad y = r_2,\ \ 1 \le x \le N$$

$$= \ 0 \qquad\qquad \text{Otherwise}$$

6. MARGINAL DISTRIBUTION OF X:

The joint distribution of X & Y is of vital importance as the marginal distribution of X as well as of Y can be obtained which becomes a complete analysis of the problem

The marginal distribution of random variable X is given by

$$P_x(x/r_1.r_2,N) = \frac{r_1(N-r_2)!}{N!}\sum_{i=0}^{N-r_2}\frac{(N-2-i)!}{(N-r_2-i)!} \qquad x = 1,2,\ldots, r_1$$

$$= \frac{r_1}{N(N-1)}\sum_{y=r_1+1}^{x}\frac{\binom{x-1}{y-1}}{\binom{N-2}{y-2}} + \frac{r_1(N-r_2)!}{N!}\sum_{i=0}^{N-r_2}\frac{(N-2-i)!}{(N-r_2-i)!}$$

$x = r_1+1, r_1+2, \ldots.\ r_2-1$

$$= \frac{r_1}{N(N-1)}\sum_{y=r_1+1}^{r_2-1}\frac{\binom{x-1}{y-1}}{\binom{N-2}{y-2}} + \frac{r_1(N-r_2)!}{N!}\sum_{i=0}^{N-r_2}\frac{(N-2-i)!}{(N-r_2-i)!}$$

$X = r_2, r_2+1, \ldots, N$

We have prepared computer program for calculating $P_x(x/r_1,r_2,N)$. Some outputs after running this program shows that $P_x(x/r_1,r_2,N)$ is nondecreasing function of x.

7. MARGINAL DISTRIBUTION OF Y:

The marginal distribution of random variable Y is given by

$$P_y(y/r_1,r_2,N) = 0 \qquad 1 \leq y \leq r_1$$

$$= \frac{r_1}{y(y-1)} \qquad y = r_1+1, r_1+2, \ldots, r_2-1$$

$$= \frac{r_1(N-r_2)!}{N!} \sum_{i=0}^{N-r_2} \frac{(N-2-i)!}{(N-r_2-i)!} \qquad y = r_2$$

The following table shows the probabilities for various values of r_1,r_2,N.

Table 1.2

N	*r_1*	*r_2*	*$P(x/r_1,r_2,N)$*
10	*2*	*8*	*0.3185714*
15	*5*	*13*	*0.3399591*
20	*7*	*13*	*0.2286237*

8. MODIFIED OPTIMALITY CRITERION :

In case of original secretary problem we try to develop a stopping rule which maximizes probability of best selection. But we can modify our aim from a statistician point of view. Here rather than developing a rule which focuses on maximizing probability of best selection, if we say that the probability of selecting a unit with maximum expected value of X. we denote the value of r for which expectation is maximum as r^*.

9. COMPARISON OF STRATEGIES FROM PROBABILISTIC VIEW

By strategy we mean a triplet (A:B:C) where
A = Selection procedure
B = Value of r_1
C = Value of r_2

Thus we have seen the following major strategies.

1] (original procedure:r_0,N)
2] (Original procedure:r_1,r_2, $r_2 < N$)
3] (Modified criteria: r_1^*,r_2) Where r_1^* is the value where we have maximum expected real rank.

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Thus we notice the following

P[Best selection using 1] > P[Best selection using 2] > P[Best selection using 3]

If maximization of probability of best selection is the only criteria under consideration then original secretary problem is better than two tailed secretary problem and Modified two tailed secretary problem also two tailed secretary problem is better than modified two tailed secretary problem. This is evidence from the following table.

Table III
Comparison of strategies with respect to probability of best selection

Strategy I $r_0 = r_1 = 7$ $R_2 = 20$ N = 20	Strategy II $r_0 = r_1 = 7$ $r_2 = 15$ N = 20	Strategy III $R_1^* = r_1 = 3$ $R_2 = 15$ N = 20
Probability of best selection = 0.3842089	Probability of best selection = 0.2805468	Probability of best selection = 0.2627

9. COMPARISON OF STRATEGIES FROM COST POINT OF VIEW:

If the cost aspect is considered for comparison of strategies then the criteria changes.

The expression of $E(y/r_1,r_{2,N})$ can be used for estimating the cost required in the selection process.

Table IV
Comparison of strategies with respect to cost criterion

Original Secretary Problem $r_1 = 7$ $r_2 = 20$ N = 20	Modified two tailed secretary problem $r_1 = r_1^* = 3$ $r_2 = 15$ N = 20	Two tailed secretary problem $r_1 = 7$ $r_2 = 15$ N = 20
E(Y) = 14.68	E(Y) = 8.2546	E(Y) = 12.61094

If the cost aspect is considered for comparison of strategies then we note that modified two tailed secretary problem is better than two tailed secretary problem which is better than original secretary problem.

10. REFERENCES

1. Freeman P.R., "Secretary problem and its extension: A Review" International statistical review, 51, 1983,189-206.
2. Ferguson T.S, "Who solved the secretary problem" Statistical science, vol. 4 ,1989, 282-296.
3. Ubale P.V. & Kane S.P, "A two tailed secretary problem" Bulletin of pure and applied sciences, vol.19E No. 1, 2000,75-80.
4. Ubale P.V. & Kane S.P, "A modified stopping criteria in TTSP" Int. J. Agricultural & statistical Science, Vol. 6, No. 1,2010, 99-104.

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