CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this paper, the authors introduce a new class of sets called generalized b* - closed sets in topological spaces (briefly gb* - closed set). Also we study some of its basic properties and investigate the relations between the associated topology.

Keywords: gb* - closed set, gb-closed set.

1. INTRODUCTION

N.Levine[4] introduced the notion of generalized closed (briefly g-Closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality etc are all g-closed hereditary. J.Dontchev[3], H.Maki, R.Devi and K.Balachandran[6] , A.S.Mashhour, Abd.EI-Monsef.M.E and EI-Deeb.S.N.[7], D.Andrijevic[1] and N.Nagaveni[8] introduced and investigated the concept of generalized semi-pre closed sets , generalized α-closed sets, pre closed sets, semi-pre closed sets and weakly generalized closed sets respectively. D .Andrijevic[2], introduced a class of generalized open sets in a topological space called b-open sets. A.A.Omari and M.S.M.Noorani[10] introduced and studied the concept of generalized b-closed sets (gb-closed) in topological spaces.

In this paper, we introduce a new class of sets called gb^* closed set which is between the class of b-closed sets and the class of gb-closed sets.

Throughout this paper (X, τ) and (Y,σ) (or simply X and Y) represents the non-empty topological spaces on which no separation axioms are assumed, Unless otherwise mentioned. For a subset A of X, cl(A) and int(A) represent the closure of A and interior of A respectively.

2 PRELIMINARIES

In this section let us recall some definitions and results which are used in this section

Definition 2.1 [7] : Let A subset A of a topological space (X, τ) is called a pre open set if A $\subseteq$ int(cl(A)) and preclosed set if cl(int(A)) $\subseteq$ A.

Definition 2.2 [5]: Let A subset A of a topological space (X, τ) is called a semi open set if A $\subseteq$ cl(int(A)) and semiclosed set if int(cl(A)) $\subseteq$ A.

Definition 2.3 [9]: Let A subset A of a topological space (X, τ) is called an α open set if A $\subseteq$ int(cl(int(A))) and α closed set if cl(int(cl(A))) $\subseteq$ A.

Definiotion 2.4 [1]: Let A subset A of a topological space (X, τ) is called a semi-pre open set (β open set) if A ⊆ cl(int(cl(A))) and semi preclosed set (β closed) if int(cl(int(A))) ⊆ A.

Definiotion 2.5 [2]: Let A subset A of a topological space (X, τ) is called a b -open set if A ⊆ cl(int(A)) U int(cl(A)) and b- closed set if cl(int(A)) U int(cl(A)) ⊆ A.

Definiotion 2.6 [4]: Let A subset A of a topological space (X, τ) is called a generalized closed set (g-closed) if cl(A) ⊆ U , whenever A ⊆ U and U is open in X.

Definiotion 2.7 [6]: Let A subset A of a topological space (X, τ) is called a generalized α closed (gα-closed) if αcl(A) ⊆ U , whenever A ⊆ U and U is open in X.

Definiotion 2.8 [10]: Let A subset A of a topological space (X, τ) is called a generalized b- closed (gb-closed) if bcl(A) ⊆ U , whenever A ⊆ U and U is open in X.

Definiotion 2.9 [8]: Let A subset A of a topological space (X, τ) is called a weakly generalized closed (wg-closed) if cl(int(A)) ⊆ U , whenever A ⊆ U and U is open in X.

Definiotion 2.10 [3]: Let A subset A of a topological space (X, τ) is called a generalized semi preclosed set(gsp -closed) if spcl(A) ⊆ U , whenever A ⊆ U and U is open in X.

Definiotion 2.11 [11]: Let A subset A of a topological space (X, τ) is called a generalized * closed set (g*-closed) if cl(A) ⊆ U , whenever A ⊆ U and U is g-open in X.

3. gb^* CLOSED SETS IN TOPOLOGICAL SPACE

In this section we introduce the concept of gb^* closed sets in topological space and we investigate the group of structure of the set of all gb^* closed sets.

Definition 3.1: A subset A of X is called a gb^* closed set if bcl(A) ⊆ U whenever A ⊆ U, U is gb open .

The class of all gb^* closed sets in a topological space (X , τ) is denoted by gb^*-c(X,τ).

Remark 3.2: The complement of gb^* closed set is gb^* open set.

Theorem 3.3: Every closed set in X is gb^* closed set in X but not conversely.

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Proof: Let A be a closed set in X. Let U be a gb open set such that A $\subseteq$ U. Since A is closed, that is cl(A)=A, cl(A) $\subseteq$ U. But bcl(A) $\subseteq$ cl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is gb^* closed in X.

The converse of the above theorem need not be true as seen from the following example

Example 3.4: Consider the topological space X= {a,b,c} with the topology τ = { Φ.X, {c}}. The sets {a} and {b} are gb^* closed sets but not closed

Theorem 3.5: Every b-closed set is gb^* closed set.

Proof: Let A be a b closed set in X such that A $\subseteq$ U, where Uis gb-open. Since A is b-closed, bcl(A) = A. Therefore bcl(A) $\subseteq$ U. Hence A is a gb^* closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example3.6: Let X = {a,b,c} with the topology τ = { Φ, X, {a},{a,c}}. Then the set {a,b} is a gb^* closed set but not a b-closed set of (X, τ).

Theorem 3.7: Every gb^* closed set is gb closed but not conversely

Proof : Let A be gb^* closed set in X .Let U be an open set such that A $\subseteq$ U. Since every open set is gb-open and A is gb^* closed, we have bcl(A) $\subseteq$ U. Therefore *A is* gb closed in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.8: Consider the topological space X = {a,b,c} with the topology τ= { Φ,X, {a}}.Then the sets {a,c} is gb closed but not gb^* closed set.

Theorem 3.9: Every α closed set is gb^* closed set.

Proof: Let A be a α closed set in X such that A $\subseteq$ U, where U is gb open. Since A is α closed, bcl(A) $\subseteq$ αcl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is gb^* closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10: Let X = {a,b,c} with the topology τ = { Φ,X, {a},{a,b}}. Then the set {a,c} is a gb^* closed set but not a α closed set.

Theorem 3.11: Every semi closed set is gb^* closed set.

Proof: Let A be a semi closed set in X such that A $\subseteq$ U, where U is gb open. Since A is semi closed, bcl(A) $\subseteq$ scl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is a gb^* closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let X = {a,b,c} with the topology τ = { Φ,X,{a,b}}. Then the set {a} is a gb^* closed set but not a semi closed set.

Theorem 3.13: Every preclosed set is a gb^* closed set.

Proof: Let A be a preclosed set in X such that A $\subseteq$ U, where U is gb open. Since A is preclosed, bcl(A) $\subseteq$ pcl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is a gb^* closed set in X.

The converse of the above theorem need not be true as seen from the following example

Example 3.14: Let X = {a,b,c} with the topology τ = { Φ,X{a},{b},{a,b}}. Then the sets {a},{b} are gb^* closed set but not a preclosed set.

Theorem 3.15: Every g^* closed set is a gb^* closed set

Proof: Let A be a g^* closed set set in X such that A $\subseteq$ U, where U is gb open. Since every g-open set is gb-open. Then bcl(A) $\subseteq$ cl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is a gb^* closed set in X.

The converse of the above theorem need not be true as seen from the following example

Example 3.16: Let X = {a,b,c} with the topology τ = { Φ,X,{a}}. Then the set {c} is a gb^* closed set but not a g^* closed set.

Theorem 3.17: Every gα closed set is gb^* closed set.

Proof: Let A be a gα closed set in X such that A $\subseteq$ U, where U is gb open. Since A is gα closed, bcl(A) $\subseteq$ αcl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is gb^* closed set.

The converse of the above theorem need not be true as seen from the following example

Example 3.18: Let X = {a,b,c} with the topology τ = { Φ,X,{a}, {a,c}}. Then the set {c} is a gb^* closed set but not a gα closed set.

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Theorem 3.18: Every g closed set is gb* closed set.

Proof: Let A be a g closed set in X such that A $\subseteq$ U, where U is gb open. Since A is g closed, bcl(A) $\subseteq$ cl(A) $\subseteq$ U. Therefore bcl(A) $\subseteq$ U. Hence A is gb^* closed set.

The converse of the above theorem need not be true as seen from the following example

Example 3.19: Let X = {a,b,c} with the topology τ = { Φ,X,{a}, {a,c}}. Then the set {c} is a gb^* closed set but not a g closed set.

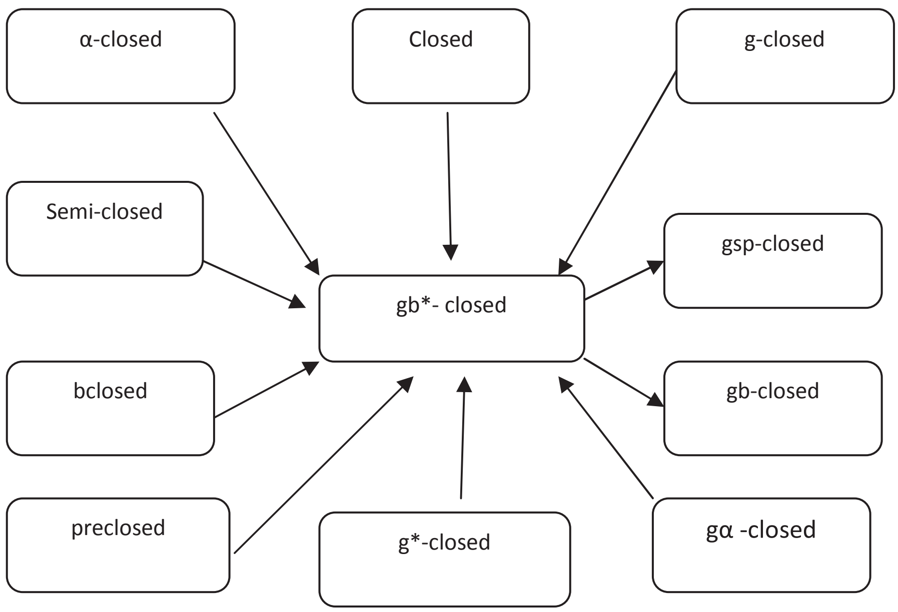
Theorem 3.20: Every gb^* closed set is a gsp closed set.

Proof: Let A be a gb^* closed set in such that A $\subseteq$ U, where U is open. Since every open set is g open and gb^* closed , spcl(A) $\subseteq$ bcl(A) $\subseteq$ U. Hence A is gsp closed.

The converse of the above theorem need not be true as seen from the following example

Example 3.21: Let X = {a,b,c} with the topology τ = { Φ,X,{a}, {a,c}}. Then the set {a,c} is not a gb^* closed set but a gsp closed set.

Remark: We have the following implications but none of this implications are reversible.



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