A NEW CLASS OF TOTALLY CONTINUOUS FUNCTIONS

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Abstract: In this paper we introduce a new class of functions called totally $\hat{\omega}$-continuous functions with the aid of $\hat{\omega}$-open sets and $\hat{\omega}$-closed sets.We study the relationships between totally $\hat{\omega}$-continuity and several types of continuity.Moreover we introduce totally $\hat{\omega}$-irresolute and totally $\hat{\omega}$-clopen functions.

Keywords: Totally $\hat{\omega}$-continuous functions, $\hat{\omega}$-clopen sets, $\hat{\omega}$-clopen regular space and $\hat{\omega}$-clopen normal space.

1. INTRODUCTION

Continuity is one of the most basic and important notion of General Topology. Many authors investigated and studied various types of continuities.The concept of totally continuous functions was introduced and studied by Jain[5].Also T.M.Nour [5] introduced the concept of totally semi-continuous functions and several properties of them were obtained. In this paper, we introduce a new class of functions called totally $\hat{\omega}$-continuous functions with the aid of $\hat{\omega}$-open sets and $\hat{\omega}$-closed sets. We study the relationships between totally $\hat{\omega}$-continuity and several types of continuity. Moreover we introduce totally $\hat{\omega}$-irresolute and totally $\hat{\omega}$-clopen functions.

2. PRELIMINARIES

Throughout this paper X,Y and Z (or (X, τ) and (Y, σ) and (Z, η)) represent non-empty topological spaces. For a subset A of (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions which are useful in the sequel.

Definition 2.1 A subset A of of a space (X, τ) is called an a-open set [3] if A $\subseteq$ int(cl(int $_\delta$ (A)).

The complement of the above set is called a-closed set. The a-closure of a subset A of (X, τ) is the intersection of all a-closed sets containing A and is denoted by acl(A)).

Definition 2.2 A subset A of a space (X, τ) is called δ-closed [9] if A=cl_δ(A) where cl_δ(A) ={x $\in$ X:int (cl(U)) $\cap A \neq \phi$, U $\in \tau$ and x $\in$ U}.The complement of δ-closed set is called δ-open set.

Definition 2.3 A subset A of a space (X, τ) is called a

(i) $\hat{g}$ - closed set [8] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

(ii) $\alpha\hat{g}$ -closed set [2] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$ -open in (X, τ).

(iii) $\hat{\omega}$ -closed set [6] if acl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\alpha\hat{g}$ -open in (X, τ).

The complements of $\hat{g}$ -closed , $\alpha\hat{g}$ -closed and $\hat{\omega}$ -closed set are called $\hat{g}$ -open, $\alpha\hat{g}$ -open and $\hat{\omega}$ -open .

Definition 2.4 A function f: (X, τ) $\rightarrow$ (Y, σ) is called

(i) $\hat{\omega}$ -continuous if f^{-1} (V) is $\hat{\omega}$ - closed in (X, τ) for every closed set V of (Y, σ).

(ii) totally continuous function [5] if f^{-1}(V) is clopen in (X, τ) for every open set V of (Y, σ).

(iii) strongly continuous function [4] if f^{-1}(V) is clopen in (X, τ) for every subset V of (Y, σ).

(iv) semi- totally continuous function [1] if f^{-1}(V) is clopen in (X, τ) for every semi-open set V of (Y, σ).

Remark 2.5 Let (X, τ) be a topological space. If X is submaximal , then every pre-open set is open[7].By [6], every $\hat{\omega}$ -open set is pre-open. Hence in a submaximal space , every $\hat{\omega}$ -open set is open.

3. TOTALLY $\hat{\omega}$ - CONTINUOUS FUNCTIONS

In this section we introduce a new class of functions called totally $\hat{\omega}$-continuous functions.

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Definition. 3.1 A function f: (X, τ) $\rightarrow$ (Y, σ) is called totally $\hat{\omega}$-continuous function if f $^{-1}$ (V) is $\hat{\omega}$- clopen ($\hat{\omega}$-open and $\hat{\omega}$-closed) in (X, τ) for every open set V of (Y, σ).

Example 3.2 Let X = Y={a,b,c}, τ ={ ϕ ,{a},{b},{a,b},{a,c},X} and σ ={ ϕ ,{a},{b,c},Y}.Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = a and f(c) = c. Then f is totally $\hat{\omega}$-continuous.

It is evident that totally $\hat{\omega}$-continuous function is $\hat{\omega}$-continuous. But the converse need not be true as shown in the following example.

Example3.3 Let X=Y={a,b,c}, τ ={ ϕ ,{a,b},X} and σ ={ ϕ ,{a} ,{a,c},Y}.Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = c, f(b) = a and f(c) = b.Then f is $\hat{\omega}$-continuous but not totally $\hat{\omega}$-continuous , since for the open set {a} of (Y, σ), f $^{-1}${a}={b} is not $\hat{\omega}$-clopen in (X, τ).

Proposition 3.4 Every clopen set is a $\hat{\omega}$-clopen set.

Proof. Let A be a clopen set in (X, τ).Then A = int(A) and A = cl(A).Hence A = int(cl(A)) and A= cl(int(A)).Consequently A is regular open and regular closed. By proposition 3.2[6] ,A is $\hat{\omega}$-open and $\hat{\omega}$-closed and so A is $\hat{\omega}$-clopen in (X, τ).

The converse of the above theorem need not be true as shown in the following example.

Example 3.5 Let X = {a,b,c}, τ = { ϕ ,{a},{b,c},X}.Then {c} is $\hat{\omega}$-clopen ,but not clopen in (X, τ).

Theorem 3.6 In a submaximal space, every $\hat{\omega}$-clopen set is a clopen set.

Proof. Follows from remark 2.5

Theorem 3.7 Every totally continuous function is totally $\hat{\omega}$-continuous.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be a totally continuous function and V be an open set in (Y, σ). Since f is totally continuous, f $^{-1}$ (V) is clopen in (X, τ).By proposition 3.4,

f $^{-1}$(V) is $\hat{\omega}$- clopen in (X, τ).Thus f is totally $\hat{\omega}$-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.8

Let X= Y= {a,b,c,d}, τ = { ϕ ,{a,b},{c,d},X} and σ = { ϕ ,{a,d}, {a,c,d}, {a,b,d},Y}. Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = c, f(b) = a , f(c) = b and f(d) = d.Then f is totally $\hat{\omega}$-continuous but not totally continuous , since for the open set {a,d} of (Y, σ), f^{-1}({a,d }) = {b,d} is not clopen in (X, τ).

Theorem 3.9 Every totally $\hat{\omega}$-continuous function whose the domain is a submaximal space is totally continuous.

Proof. Follows from theorem 3.6.

Theorem 3.10 Every strongly continuous function is totally $\hat{\omega}$-continuous.

Proof: Since every strongly continuous function is totally continuous, proof follows from theorem 3.7

The converse of the above theorem need not be true as shown in the following example.

Example 3.11 Let X =Y = {a,b,c,d}, τ = { ϕ ,{a}, {b}, {a,b}, {c,d},{a,c,d},{b,c,d},X} and σ = { ϕ ,{a},{a,b},{a,c,d},Y}.Define f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = c, f(b) = d ,f(c) = a and f(d) = b. Then f is totally $\hat{\omega}$-continuous function but not strongly continuous since for the set {b} of (Y, σ), f^{-1}({b})={d} is not clopen in (X, τ).

Theorem 3.12 Every semi-totally continuous function is totally $\hat{\omega}$-continuous.

Proof. Let f: (X, τ) $\rightarrow$ (Y, σ) be a totally continuous function and V be an open set in (Y, σ). Then V is a semi-open set in (Y, σ) .Since f is semi-totally continuous, f^{-1} (V) is clopen in (X, τ).By proposition 3.4, f^{-1}(V) is $\hat{\omega}$- clopen in (X, τ).Thus f is totally $\hat{\omega}$-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.13 Let X ,Y, τ , σ and f be same as in example 3.11.Then f is totally $\hat{\omega}$-continuous but not semi-totally continuous since for the semi-open set {a,d} of (Y, σ),f^{-1}({a,d})={b,c} is not clopen in (X, τ).

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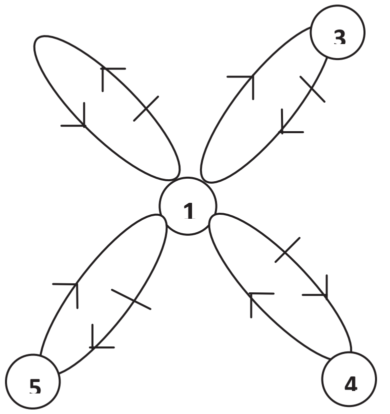
Remark 3.14 From the above discussions we have the following diagram where

A $\longrightarrow$B represents A implies B and A$\longrightarrow$B represents A does not imply B.



1.Totally $\hat{\omega}$-continuous 2.Strongly continuous 3. $\hat{\omega}$- continuous 4.Semi-totally continuous.5. Totally continuous.

4.CHARACTERIZATIONS

Theorem 4.1 A function f: (X, τ) $\rightarrow$ (Y, σ) is totally $\hat{\omega}$-continuous if and only if the inverse image of every closed subset of (Y, σ) is $\hat{\omega}$-clopen in (X, τ).

Proof. Necessity. Let V be any closed subset of (Y, σ).Then Y-V is an open subset of (Y, σ).Since f is totally $\hat{\omega}$-continuous , f^{-1}(Y-V) = X - f^{-1}(V) is $\hat{\omega}$-clopen in (X, τ) and so f^{-1}(V) is $\hat{\omega}$-clopen in (X, τ).

Sufficiency. Let V be any open subset of (Y, σ).Then Y-V is closed in (Y, σ).Hence f^{-1}(Y-V) = X - f^{-1} (V) is $\hat{\omega}$-clopen in (X, τ) and so f^{-1}(V) is $\hat{\omega}$-clopen in (X, τ).Thus f is totally $\hat{\omega}$-continuous.

Recall that a topological space (X, τ) is said to be locally indiscrete, if every open set of X is closed in X.

Theorem 4.2 Let f: (X, τ) $\rightarrow$ (Y, σ) be a function. Then if f is continuous and X is locally indiscrete ,then f is totally $\hat{\omega}$-continuous.

Proof. Let V be an open set in Y .Since f is continuous , $f^{-1}(V)$ is an open set in X. Since X is locally indiscrete, $f^{-1}(V)$ is also closed in X and so $f^{-1}(V)$ is clopen in X. By proposition 3.4, $f^{-1}(V)$ is $\hat{\omega}$-clopen in X. Thus f is totally $\hat{\omega}$-continuous.

Definition 4.3 A topological space X is said to be $\hat{\omega}$-connected, if X cannot be written as the union of two disjoint non-empty $\hat{\omega}$-open sets.

Theorem 4.4 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a totally $\hat{\omega}$-continuous, surjective function from a $\hat{\omega}$-connected space into any space Y, then Y is an indiscrete space.

Proof. Suppose that Y is not an indiscrete space. Let A be a proper non-empty open subset of (Y, σ).Since f is surjective and totally $\hat{\omega}$-continuous , $f^{-1}(V)$ is a proper non-empty $\hat{\omega}$-clopen subset of (X, τ), a contradiction to the fact that (X, τ) is a $\hat{\omega}$-connected space.

Theorem 4.5 A topological space (X, τ) is $\hat{\omega}$-connected if and only if every totally $\hat{\omega}$-continuous function from a space (X, τ) into any T_0-space (Y, σ) is constant.

Proof. Necessity. Suppose there exists a totally $\hat{\omega}$-continuous function f from a space (X, τ) into a T_0-space (Y, σ) which is not constant. Then there exists x ,y $\in$ X ,

$x \neq y$ such that $f(x) \neq f(y)$.Since Y is T_0, there exists an open set V in Y containing f(x) but not f(y). Since f is totally $\hat{\omega}$-continuous, $f^{-1}(V)$ is a proper non-empty $\hat{\omega}$-clopen subset of X, a contradiction to the fact that X is $\hat{\omega}$-connected.

Sufficiency. Suppose X is not $\hat{\omega}$-connected, there exists a proper non-empty $\hat{\omega}$-clopen subset A of X. Let Y={a,b} and $\sigma = \{\phi, \{a\}, \{b\}, Y\}$.Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a function such that f(A)={a} and f(X-A) ={b}.Then f is non-constant and totally $\hat{\omega}$-continuous such that Y is T_0, a contradiction. Hence X is $\hat{\omega}$-connected.

Theorem 4.6 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a totally $\hat{\omega}$ -continuous, surjective function and X is $\hat{\omega}$-connected, then Y is connected.

Proof. Suppose that Y is a disconnected space. Then there exist non-empty disjoint open sets U and V such that $Y = U \cup V$. Since f is totally $\hat{\omega}$-continuous, $f^{-1}(U)$ and

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f^{-1} (V) are $\hat{\omega}$-clopen in X. Moreover, f^{-1} (U) and f^{-1} (V) are disjoint and X = f^{-1} (U) ∪ f^{-1} (V). Since f is surjective, f^{-1} (U) and f^{-1} (V) are non-empty. Therefore, X is not $\hat{\omega}$- connected, a contradiction and hence Y is connected.

Recall that for a function f: (X, τ) → (Y, σ), the subset {(x, f(x)): x ∈ X } ⊂ X x Y is called the graph of f and is denoted by G(f).

Theorem 4.7 A function f: (X, τ) → (Y, σ) is totally $\hat{\omega}$-continuous if its graph function is totally $\hat{\omega}$-continuous.

Proof. Let g: X → X x Y be the graph function and V be an open subset of (Y, σ).Then X x V is open in X x Y. Since g is totally $\hat{\omega}$-continuous , f^{-1} (V) = g^{-1} (X x V) is $\hat{\omega}$-clopen in X and so f is totally $\hat{\omega}$-continuous.

Theorem 4.8 Let {X_λ : λ ∈ Λ} be any family of topological spaces. If f : X → ∏ X_λ is totally $\hat{\omega}$-continuous for each λ ∈ Λ, then $p_\lambda \circ f$:X → X_λ is totally $\hat{\omega}$-continuous for each λ ∈ Λ where p_λ is the projection of ∏ X_λ onto X_λ.

Proof. Suppose U_λ be any open set in X_λ where λ ∈ Λ.Then $p_\lambda^{-1}(X_\lambda)$is open in ∏ X_λ.Since f is totally $\hat{\omega}$-continuous $f^{-1}(p_\lambda^{-1}(U_\lambda)) = (p_\lambda \circ f)^{-1}(U_\lambda)$ is $\hat{\omega}$-clopen in X and so $p_\lambda \circ f$ is totally $\hat{\omega}$-continuous.

Remark 4.9 The composition of two totally $\hat{\omega}$-continuous functions need not be totally $\hat{\omega}$-continuous and this is shown by the following example.

Example 4.10 Let X= {a,b,c} = Y = Z, τ = { ϕ.{a},X} and σ = { ϕ,.Y} and η = { ϕ,{a}, {b,c}, Z}. Define a function f: (X, τ) → (Y, σ) be the identity function and g: (Y, σ) → (Z, η) be defined by g(a) = b, g(b) = c and g(c) = a. Then both f and g are totally $\hat{\omega}$-continuous functions. But g ∘ f: (X, τ) → (Z, η) is not totally $\hat{\omega}$-continuous since

$(g \circ f)^{-1}(\{a\}) = f^{-1}(g^{-1}(\{a\})) = f^{-1}(\{c\}) = \{c\}$ is not $\hat{\omega}$ - clopen in X where {a} is open in Z.

Theorem 4.11 If f: (X, τ) → (Y, σ) is totally $\hat{\omega}$-continuous and g: (Y, σ) → (Z, η) is continuous ,then g ∘ f : (X, τ) → (Y, σ) is totally $\hat{\omega}$-continuous.

Proof: Let V be an open set in (Z, η) .Then g^{-1}(V) is open in (Y, σ) ,Since f is totally $\hat{\omega}$-continuous, $(g \circ f)^{-1}$ (V) =
f^{-1} (g^{-1} (V))= is $\hat{\omega}$-clopen in (X, τ) and so $g \circ f$ is totally $\hat{\omega}$-continuous.

Remark 4.12 From theorem 3.7 , 3.10 and 3.12 we get that, the above theorem 4.12 is true , even if f is totally continuous, strongly continuous or semi-totally continuous.

Theorem 4.13 If f: (X, τ) $\rightarrow$ (Y, σ) is totally $\hat{\omega}$-continuous and g: (Y, σ) $\rightarrow$ (Z, η) is totally $\hat{\omega}$-continuous where (Y, σ) is submaximal, then $g \circ f$: (X, τ) $\rightarrow$ (Y, σ) is totally $\hat{\omega}$-continuous.

Proof: Let V be an open set in (Z, η) .Then g^{-1}(V) is $\hat{\omega}$-clopen in (Y, σ) , By theorem 3.6, g^{-1}(V) is clopen in (Y, σ) and so g^{-1}(V) is open in (Y, σ) .Since f is totally
$\hat{\omega}$-continuous, $(g \circ f)^{-1}$ (V) = f^{-1} (g^{-1} (V))= is $\hat{\omega}$-clopen in (X, τ) and so $g \circ f$ is totally $\hat{\omega}$-continuous.

Definition 4.14 A function f: (X, τ) $\rightarrow$ (Y, σ) is said to be totally $\hat{\omega}$-clopen if the image of every open set in X is $\hat{\omega}$-clopen in Y.

Theorem 4.15 A bijective function f: (X, τ) $\rightarrow$ (Y, σ) is totally $\hat{\omega}$-clopen if and only if the image of every closed set in X is $\hat{\omega}$-clopen in Y.

Proof. Necessity. Let U be closed in X. Then X - U is open in X. Since f is totally $\hat{\omega}$-clopen, f(X-U) = Y-f(U) is $\hat{\omega}$-clopen in Y and so f(U) is $\hat{\omega}$- clopen in Y.

Sufficiency. Let U be open in X. Then X - U is closed in X.,Hence f(X-U) = Y - f(U) is $\hat{\omega}$-clopen in Y and so f(U) is $\hat{\omega}$-clopen in Y. Thus f is totally $\hat{\omega}$-clopen.

Theorem 4.16 A surjective function f: (X, τ) $\rightarrow$ (Y, σ) is totally $\hat{\omega}$-clopen if and only if for every subset F of Y and for each open subset U of X containing f^{-1}(F) ,there is an
$\hat{\omega}$-clopen set V of Y such that F $\subset$ V and f^{-1}(V) $\subset$ U.

Proof. Suppose f is a surjective ,totally $\hat{\omega}$-clopen function .Let F $\subset$ Y and U be an open subset of X such that f^{-1}(F) $\subset$ U. Then X-U closed in (X, τ)and f (X-U) is $\hat{\omega}$-clopen in Y .Take V = Y-f (X-U).then V is an $\hat{\omega}$-clopen subset of (Y, σ) such that F $\subset$ V and f^{-1}(V) $\subset$ U.

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Conversely, let G be closed in X. Then $f^{-1}(Y-f(G)) \subset X-G$ and X- G is open in X. By hypothesis , there exists an $\hat{\omega}$ -clopen set V of Y such that $Y-f(G) \subset V$ and $f^{-1}(V) \subset X - G$. Hence $G \subset X - f^{-1}(V)$ and so $Y-V \subset f(G) \subset f(X - f^{-1}(V)) \subset Y-V$.Thus f(G) =Y-V is $\hat{\omega}$ -clopen in Y .Hence f is totally $\hat{\omega}$ -clopen.

Theorem 4.17 Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a bijective function. Then the following are equivalent.

(i) f^{-1} is totally $\hat{\omega}$-continuous (ii) f is totally $\hat{\omega}$-clopen.

Proof. (i) $\Rightarrow$ (ii) Let V be an open subset of Y. Since f^{-1} is totally $\hat{\omega}$-continuous,$(f^{-1})^{-1}(V) = f(V)$ is $\hat{\omega}$-clopen in Y. Thus f is totally $\hat{\omega}$-clopen .

(ii) $\Rightarrow$ (i) Let G be an open subset of X. Since f is totally $\hat{\omega}$-clopen $f(G) =(f^{-1})^{-1}(G)$ is $\hat{\omega}$-clopen in Y.Thus f^{-1} is totally $\hat{\omega}$-continuous.

Remark 4.18 The composition of two totally $\hat{\omega}$-clopen functions need not be totally $\hat{\omega}$-clopen and this is shown by the following example.

Example 4.19 Let X= {a,b,c} = Y = Z, τ ={ ϕ ,{a}, {b,c}, X} and σ = { ϕ ,Y} and η={ ϕ ,{a}, {a,b}, Z}.Define a function f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = c, f(b) = a and f(c) = b. and g: $(Y, \sigma) \rightarrow (Z, \eta)$ be defined by g(a)= a, g(b) = c and g(c) = b.Then both f and g are totally $\hat{\omega}$-clopen functions. But $g \circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is not totally $\hat{\omega}$-clopen since $(g \circ f)$ ({a})= g(f({a}))= g({c})={b} is not $\hat{\omega}$ - clopen in Z where (a} is open in X.

Theorem 4.20 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is open and g: $(Y, \sigma) \rightarrow (Z, \eta)$ is totally $\hat{\omega}$-clopen, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is totally $\hat{\omega}$-clopen .

Proof: Let V be an open set in X. Then f(V) is open in (Y, σ) .Since g is totally $\hat{\omega}$-clopen, $(g \circ f)(V)= g(f(V))$is $\hat{\omega}$-clopen in (Z, η) and so $g \circ f$ is totally $\hat{\omega}$-clopen.

Theorem 4.21 If f: $(X, \tau) \rightarrow (Y, \sigma)$ is totally $\hat{\omega}$-clopen. and g: $(Y, \sigma) \rightarrow (Z, \eta)$ is totally $\hat{\omega}$-clopen where (Y, σ) is submaximal , then $g \circ f$: $(X, \tau) \rightarrow (Z, \eta)$ is totally $\hat{\omega}$-clopen.

Proof: Let V be an open set in X. Then f(V) is $\hat{\omega}$-clopen in (Y, σ) .Since (Y, σ) is submaximal, by theorem 3.6 f(V) is clopen in (Y, σ) and so f(V) is open in (Y, σ) Since g is totally $\hat{\omega}$-clopen, (g $\circ$ f)(V)= g(f(V))is $\hat{\omega}$-clopen in (Z, η) and so g $\circ$ f is totally $\hat{\omega}$-clopen.

Definition 4.22 A function f: (X, τ) $\rightarrow$ (Y, σ) is said to be

(i) totally $\hat{\omega}$-irresolute if the pre image of every $\hat{\omega}$-clopen subset of Y is an $\hat{\omega}$-clopen subset of X.
(ii) totally $\hat{\omega}$*- clopen if the image of every $\hat{\omega}$-clopen subset of X is an $\hat{\omega}$-clopen subset of Y.

Theorem 4.23 Let f: (X, τ) $\rightarrow$ (Y, σ) and g: (Y, σ) $\rightarrow$ (Z, η) be functions such that g $\circ$ f : (X, τ) $\rightarrow$ (Z, η) is totally $\hat{\omega}$-clopen. Then

(i) if f is continuous and surjective ,then g is totally $\hat{\omega}$-clopen.
(ii) if g is totally $\hat{\omega}$-irresolute and injective, then f is totally $\hat{\omega}$-clopen.

Proof.(i) Let V be an open set in Y. Since f is continuous,

f^{-1}(V) is open in X. Since g $\circ$ f is totally $\hat{\omega}$-clopen,
(g $\circ$ f)(f^{-1} (V))=g(V)) is $\hat{\omega}$-clopen in Z and so g is totally $\hat{\omega}$-clopen.
(ii) Let W be an open set in X. Since g $\circ$ f is totally $\hat{\omega}$-clopen, (g $\circ$ f)(W) is $\hat{\omega}$-clopen in Z. Since g is totally $\hat{\omega}$-irresolute and injective , g^{-1} ((g $\circ$ f)(W) = f(W) is $\hat{\omega}$-clopen in Y and so f is totally $\hat{\omega}$-clopen.

Theorem 4.24 Let f: (X, τ) $\rightarrow$ (Y, σ) be a surjective , totally $\hat{\omega}$-irresolute and totally $\hat{\omega}$*-clopen and g: (Y, σ) $\rightarrow$ (Z, η) be any function. Then g $\circ$ f : (X, τ) $\rightarrow$ (Z, η) is totally $\hat{\omega}$-continuous if and only if g is totally $\hat{\omega}$-continuous.

Proof. Necessity. Let V be an open set in Z. Then $(g \circ f)^{-1}$ (V)= f^{-1} (g^{-1} (V)) is $\hat{\omega}$-clopen in X. Since f is surjective and totally $\hat{\omega}$*-clopen , f(f^{-1}(g^{-1} (V))= g^{-1} (V) is $\hat{\omega}$-clopen in X Hence g is totally $\hat{\omega}$-continuous.
Sufficiency . Let W be an open set in Z. Since g is totally $\hat{\omega}$-continuous, g^{-1}(W) is $\hat{\omega}$-clopen in Y. Since f is totally $\hat{\omega}$-irresolute, f^{-1}(g^{-1}(W)) = $(g \circ f)^{-1}$(W) is $\hat{\omega}$-clopen in X. Thus g $\circ$ f is totally $\hat{\omega}$-continuous.

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5. APPLICATIONS

In this section, using the properties of totally $\hat{\omega}$-continuous functions, some preservation theorems of such functions are investigated.

Definition 5.1 A topological space (X, τ) is said to be

(i) $\hat{\omega}$-clopen T_1 if for each pair of distinct points x and y of X , there exist $\hat{\omega}$-clopen sets U and V containing x and y respectively such that y $\notin$ U and x $\notin$ V.

(ii) $\hat{\omega}$-clopen T_2 if for each pair of distinct points x and y of X, there exist disjoint $\hat{\omega}$- clopen sets U and V such that x $\in$ U and y $\in$ V.

Theorem 5.2 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous injective function and Y is T_1, then X is $\hat{\omega}$-clopen T_1.

Proof. Let x and y be any pair of distinct points of X. Since f is injective, f(x) $\neq$ f(y). Since Y is T_1,there exist disjoint open sets V and W in Y such that f(x) $\in$ V , f(y) $\notin$ V , f(y) $\in$ W and f(x) $\notin$ W. Since f is totally $\hat{\omega}$-continuous , f^{-1}(V) and f^{-1}(W) are $\hat{\omega}$-clopen subsets of X such that x $\in f^{-1}$(V), y $\notin f^{-1}$(V),y $\in f^{-1}$(W) and x $\notin f^{-1}$(W). Hence X is $\hat{\omega}$-clopen T_1.

Theorem 5.3 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous injective function and Y is T_2, then X is $\hat{\omega}$- clopen T_2.

Proof. Let x and y be any pair of distinct points of X. Since f is injective, f(x) $\neq$ f(y). Since Y is T_2, there exist disjoint open sets V and W in Y such that f(x) $\in$ V and f(y) $\in$W Since f is totally $\hat{\omega}$-continuous , f^{-1}(V)and f^{-1}(W) are $\hat{\omega}$-clopen subsets of X such that x $\in f^{-1}$(V) and y $\in f^{-1}$(W) .Also $f^{-1}(V) \cap f^{-1}(W) = \Phi$. Hence X is $\hat{\omega}$-clopen T_2.

Theorem 5.4 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous injective function and Y is T_0, then X is $\hat{\omega}$-clopen T_2.

Proof. Let x and y be any pair of distinct points of X. Since f is injective, f(x) $\neq$ f(y). Since Y is T_0, there exist an open set V containing f(x) but not f(y). Since f is totally $\hat{\omega}$-continuous , f^{-1}(V) is a $\hat{\omega}$-clopen subset of X .Also x $\in f^{-1}$(V)and y $\in$ X - f^{-1}(V) . Hence X is $\hat{\omega}$-clopen T_2.

Definition 5.5 .A topological space (X, τ) is said to be

(i) $\hat{\omega}$-clopen regular if for each $\hat{\omega}$-clopen set F and each point x $\notin$ F, there exists disjoint open sets U and V such that F $\subset$ U and x $\in$ V.

(ii) $\hat{\omega}$-clopen normal if for each pair of disjoint $\hat{\omega}$-clopen sets U and V of X , there exists disjoint open sets G and H such that U $\subset$ G and V $\subset$ H.

(iii) $\hat{\omega}$-Ultra regular if for each closed set F and each point x $\notin$ F, there exists disjoint $\hat{\omega}$-clopen sets U and V such that F $\subset$ U and x $\in$ V.

(iv) $\hat{\omega}$-Ultra normal if for each pair of disjoint closed sets U and V of X , there exists disjoint $\hat{\omega}$-clopen sets G and H such that U $\subset$ G and V$\subset$ H.

Theorem 5.6 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous, injective open function from a $\hat{\omega}$-clopen regular space X onto a space Y ,then Y is regular.

Proof. Let F be a closed set in Y and y $\notin$ F. Let y = f(x). Since f is totally $\hat{\omega}$-continuous, f^{-1}(F) is a $\hat{\omega}$-clopen subset of X .Let G = f^{-1}(F).We have x $\notin$ G. Since X is $\hat{\omega}$-clopen regular, there exist disjoint open sets U and V such that G $\subset$ U and x $\in$ V. Hence F = f(G) $\subset$ f(U) and y $\in$ f(V) .Also f(U) and f(V) are disjoint open sets .Thus Y is regular.

Theorem 5.7 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous, injective open function from a $\hat{\omega}$-clopen normal space X onto a space Y, then Y is normal.

Proof. Let U and V be disjoint closed sets in Y . Since f is totally $\hat{\omega}$-continuous, f^{-1}(U) and f^{-1}(V)are $\hat{\omega}$-clopen subsets of X .Let G = f^{-1} (U) and H = f^{-1}(V).Then G $\cap$H =

f^{-1}(U)$\cap$ f^{-1}(V)= f^{-1}(U $\cap$ V)= Φ.Since X is $\hat{\omega}$-clopen normal, there exist disjoint open sets A and B such that G$\subset$ A and H $\subset$ B. Hence U = f(G) $\subset$ f(A) and V = f(H) $\subset$f(B).Since f is injective and open, f(A) and f(B) are disjoint open sets and so Y is normal.

Theorem 5.8 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous, closed function from a space X into a regular space Y, then X is $\hat{\omega}$-Ultra regular .

Proof. Let F be a closed set in X and x $\notin$ F. Since f is closed , f(F) is a closed set in Y not containing f(x). Since Y is regular, there exists disjoint open sets G and H such that f(x)$\in$ G and f(F) $\subset$ H. Hence x $\in$ f^{-1} (G) and F $\subset$

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f^{-1} (H). Since f is totally $\hat{\omega}$-continuous , f^{-1}(G) and f^{-1}(H) are disjoint $\hat{\omega}$-clopen subsets of X .Thus X is $\hat{\omega}$-Ultra regular.

Theorem 5. 9 If f: (X, τ) $\rightarrow$ (Y, σ) is a totally $\hat{\omega}$-continuous, closed injective function from a space X into a normal space Y, then X is $\hat{\omega}$-Ultra normal.

Proof. Let U and V be disjoint closed sets in X . Since f is closed and injective, f (U) and f (V) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets G and H such that f(U) $\subset$ G and f(V) $\subset$ H. Hence U $\subset$ f^{-1} (G) and V $\subset$ f^{-1} (H). Since f is totally $\hat{\omega}$-continuous , f^{-1} (G) and f^{-1}(H) are disjoint $\hat{\omega}$-clopen subsets of X are and so X is $\hat{\omega}$-Ultra normal.

REFERENCES

[1] S.S.Benchalli and Umadevi I.Neeli,"Semi-totally continuous functions in topological spaces", International Mathematical Forum,Volume 6(2011), no.10,479-492.

[2] E.El–Monsef, S.Rose Mary and M.Lellis Thivagar ,"On $\alpha\hat{g}$ -closed sets in topological spaces", Assiut University Journal of Mathematics and Computer Science, Vol.36 (2007),43-51.

[3] Erdal Ekici,"On a-openSets, A^*-Sets and decompositions of continuity and super-continuity", Annales Univ. Sci. Budapest 51(2008),39-51.

[4] N.Levine," Strong continuity in topological spaces" Amer. Math. Monthly, 1960, 67(3), 269-275.

[5] T.M.Nour,"Totally semi-continuous functions", Pacific J. Pure. Appln. Math., 26(7), (1995), 675-678

[6] H.Maki, M.Lellis Thivagar, C.Santhini and M.Anbuchelvi " New sort of generalized closed sets"-Unpublished

[7] I.L.Reily and M.K. Vamanamurthy," On some questions concerning preopen sets", Kyungpook Math.J. , 30(1990), no.1, 87-93.

[8] M.K.R.S.Veerakumar," $\hat{g}$ -Closed Sets in Topological spaces", Bull. Allah. Math. Soci. 18(2003),99-112.

[9] N.V.Velicko ,"H - closed sets in Topological spaces", Amer. Math. Soc. Transl., 78(1968), 103-118.

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