REMARKS ON NON-RARELY CONTINUITY

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Abstract: The notion of rare continuity was introduced by Popa.

In this paper, we introduce a new class of functions called rarely $\hat{\Omega}$ -continuity, which is independent of rarely continuous function. It is weaker than rarely δ -continuity and stronger than rarely g continuity. Also we explore some concepts associated with semi T1/2 and $_{\omega}T_{\hat{\Omega}}$ spaces.

Keywords: rare set, rarely $\hat{\Omega}$ -continuous, semi T1/2 and $_{\omega}T_{\hat{\Omega}}$ spaces.

1. INTRODUCTION

In 1979, Popa [5] introduced the notion of rarely continuity and then [3] M.Caldas and S.Jafari introduced rarely δ -continuity and rarely g continuity. In this paper we introduce a non rarely continuity namely, rarely $\hat{\Omega}$ continuity which is weaker than δ -continuity and stronger than rarely g continuity. Also we investigate some of it's applications via *semi* $T_{1/2}$ and $_{\omega}T_{\hat{\Omega}}$ spaces.

2. PRELIMINARIES

Throughout this paper (X,τ) or X represent a topological space on which separation axioms are assumed unless explicitly stated. The family of all open (resp. δ -open,regular open,g-open, $\hat{\Omega}$ -open) sets in X are denoted by O(X) (resp. δ O(X),RO(X),GO(X), $\hat{\Omega}$ O(X)).Some of the notations which we use are as follows.

$$O(X,x)=\{U\in X:x\in U\in O(X)\},$$
$$\delta O(X,x)=\{U\in X:x\in U\in \delta O(X)\}$$

$$RO(X,x)=\{U\in X:x\in U\in RO(X)\},$$
$$GO(X,x)=\{U\in X:x\in U\in GO(X)\}$$

$$\hat{\Omega}O(X,x)=\{U\in X:x\in U\in \hat{\Omega}O(X)\}$$

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. A set R in (X,τ) is said to be a rare set if int(R) = Φ.

Definition 2.2. [5] A function $f: X \to Y$ is said to be rarely (resp.[2] rarely δ ,[3] rarely g) continuous if for each $x \in X$ and each $G \in O(Y, f(x))$,there exists a rare set R_G with $G \cap cl(R_G) = \Phi$ and $U \in O(X,x)$ (resp. $U \in \delta O(X,x)$, $U \in GO(X,x)$) such that $f(U) \subseteq G \cup R_G$.

Definition 2.3. [4] A function $f: X \to Y$ is called weakly continuous at $x \in X$ if for each $x \in X$ and each open set $G \in O(Y, f(x))$,there exists $U \in O(X,x)$ such that $f(U) \subseteq cl(G)$.If f is weakly continuous at each $x \in X$,then it is weakly continuous on X.

Definition 2.4. [2] A function $f: X \to Y$ is called weakly $\hat{\Omega}$ -continuous for each $x \in X$ and each open set $G \in O(Y, f(x))$,there exists $U \in \hat{\Omega}O(X,x)$ such that $f(U) \subseteq cl(G)$. If f is weakly continuous at each $x \in X$, then it is weakly $\hat{\Omega}$ continuous on X.

Definition 2.5. [2] A function $f: X \to Y$ is called $\hat{\Omega}$ irresolute if the inverse image of every $\hat{\Omega}$ -open set is $\hat{\Omega}$ -open in X.

Definition 2.6. [2] A topological space (X,τ) is said to be ${}_{\omega}T_{\hat{\Omega}}$ if every ω open set is $\hat{\Omega}$ -open in X.

Lemma 2.7. A topological space (X,τ) is said to be *semi* $T_{1/2}$ if and only if every $\hat{\Omega}$ -open set is open in X.

Lemma 2.8. A topological space (X,τ) is said to be ${}_{\omega}T_{\hat{\Omega}}$ if and only if every open set is $\hat{\Omega}$ -open set in X.

3. RARELY $\hat{\Omega}$ -CONTINUOUS FUNCTION

Definition 3.1. A function $f: X \to Y$ is said to be rarely $\hat{\Omega}$ -continuous if for each $x \in X$ and each $G \in O(Y, f(x))$,there exists a rare set R_G with $G \cap cl(R_G) = \Phi$ and $U \in \hat{\Omega}O(X,x)$ such that $f(U) \subseteq G \cup R_G$.

Example 3.2. Let X = {a,b,c,d}, τ ={ Φ,{a},{bcd},X} , $\hat{\Omega}O(X) = P(X)$, and Y = X, $\sigma = \{\varnothing,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},Y\}$. and rare sets in (Y,σ) are $\{\varnothing,\{d\}\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=c, f(c)=d,$ and $f(d)=a.$ Then f is rarely $\hat{\Omega}$-continuous function.

Theorem 3.3. Every rarely δ-continuous function is rarely $\hat{\Omega}$-continuous function.

Proof- It follows from the fact that [1] every δ- open set is $\hat{\Omega}$ -open set.

Theorem 3.4. Every rarely $\hat{\Omega}$-continuous function is rarely g-continuous function.

Proof- By [1],every $\hat{\Omega}$ - open set is g-open set and hence it follows.

Remark 3.5. It follows from example 3.2 that

- Rarely $\hat{\Omega}$ -continuous function is not always rarely δ-continuous function.
- Rarely $\hat{\Omega}$ -continuous function is not always rarely continuous function.

Remark 3.6. The following example illustrates, "it is not always true that every rarely continuous is rarely $\hat{\Omega}$ -continuous function".

Example 3.7. Let $X = \{a,b,c\}, \tau = \{\varnothing,\{a\},\{a,b\},\{a,c\},X\}$ and $Y = \{a,b,c\}, \sigma = \{\varnothing,\{a\},\{b\},\{a,b\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=b, f(c)=a.$ Then f is rarely continuous but not a rarely $\hat{\Omega}$ -continuous function.

Remark 3.8. From the above discussions,rarely continuous and rarely $\hat{\Omega}$-continuous functions are independent.

Remark 3.9. Rarely g continuous function is not always rarely $\hat{\Omega}$ -continuous function from the following example.

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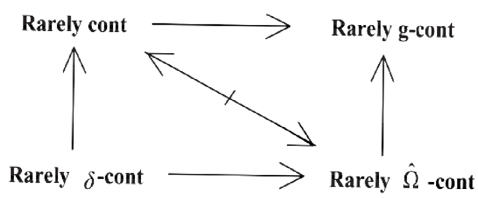
Example 3.10. *Let* $X=\{a,b,c,d\}=Y,\tau=\sigma=\{\varnothing,\{a\},\{b\},\{a,b\},Y\}$. Define $f:(X,\tau)\to(Y,\sigma)$ *as* $f(a)=a, f(b)=b, f(c)=a, f(d)=b.$ Then f is rarely g continuous function but not rarely $\hat{\Omega}$ -continuous function.

Remark 3.11. Pictorial representation of our discussions and the existing results is shown

bellow.In the figure, no reversible implication is possible.



Let us characterize rarely $\hat{\Omega}$ -continuous function in the next theorem.

Theorem 3.12. Let $f:(X,\tau)\to(Y,\sigma)$ be any function.Then the following are equivalent.

(i) f is rarely $\hat{\Omega}$ -continuous at $x\in X$.

(ii) For every $V\in O(Y,f(x))$, there exists $U\in\hat{\Omega}O(X,x)$ such that $int(f(U)\cap(Y\setminus V))=\varnothing.$

(iii) For every $V\in O(Y,f(x))$, there exists $U\in\hat{\Omega}O(X,x)$ such that $int(f(U))\subseteq cl(V).$

(iv) For every $V\in O(Y,f(x))$, there exists a rare set R_V with $cl(V)\cap R_V=\varnothing$ and there exists $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq cl(V)\cup R_V$.

(v) For every $V\in RO(Y,f(x))$, there exists a rare set R_V with $V\cap cl(R_V)=\varnothing$ and there exists $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq V\cup R_V$.

Proof- (i)$\Rightarrow$(ii). Let $V \in O(Y, f(x))$. By hypothesis, there exists a rare set R_V with $V \cap cl(R_V) = \varnothing$ and $U \in \hat{\Omega}O(X, x)$ such that $f(U) \subseteq V \cup R_V$. Therefore,

$R_V \subseteq Y \setminus V$ and hence $f(U) \cap R_V = f(U) \cap (Y \setminus V)$. Also,

$int(f(U) \cap (Y \setminus V)) = int(f(U) \cap R_V) = int(f(U)) \cap int(R_V) = \varnothing$. Thus, $int(f(U) \cap (Y \setminus V)) = \varnothing$.

(ii) $\Rightarrow$(iii). Let $V \in O(Y, f(x))$. By hypothesis, there exists $U \in \hat{\Omega}O(X, x)$ such that $int(f(U) \cap (Y \setminus V)) = \varnothing$. That is, $int(f(U)) \cap (Y \setminus cl(V)) = \varnothing$. Hence $int(f(U)) \subseteq cl(V)$.

(iii) $\Rightarrow$ (iv). Let $V \in O(Y, f(x))$ be arbitrary. By hypothesis, there exists $U \in \hat{\Omega}O(X, x)$ such that $int(f(U)) \subseteq cl(V)$. If $R_1 = f(U) \cap (Y \setminus V)$, then R_1 is a rare set with $V \cap cl(R_1) = \varnothing$ such that $f(U) \subseteq V \cup R_1$. If $R_V = [Y \setminus cl(V)] \cap R_1$, then R_V is a rare set with $cl(V) \cap R_V = \varnothing$ and $f(U) \subseteq cl(V) \cup R_V$.

(iv)$\Rightarrow$(v). Let $V \in RO(Y, f(x))$. By hypothesis, there exists a rare set R_V with $cl(V) \cap R_V = \varnothing$ and there exists $U \in \hat{\Omega}O(X, x)$ such that $f(U) \subseteq cl(V) \cup R_V$. Define $R_1 = R_V \cup (cl(V) \setminus V)$. If $int(R_1) \neq \varnothing$, we can choose a point $z \in int(R_1)$. Then there exists $W \in O(Y, z)$ such that $z \in W \subseteq R_1 = R_V \cup (cl(V) \setminus V)$. If $W \cap [cl(V) \setminus V] \neq \varnothing$, that is $W \cap Fr(V) \neq \varnothing$. Then $W \cap V \neq \varnothing$, a contradiction to $W \subseteq R_1$. Therefore,

$W \subseteq R_V$ and hence $z \in int(R_V)$, a contradiction to R_V is a rare set. Therefore, $int(R_1) = \varnothing$. Thus, R_1 is a rare set with $cl(R_1) \cap V = \varnothing$ and $f(U) \subseteq V \cup R_1$.

(v)$\Rightarrow$(i) Let $x \in X$ and $V \in O(Y, f(x))$. If $V_1 = int(cl(V))$, then V_1 is a regular open set in (X, τ).

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By hypothesis, there exists a rare set R_{V_1} with $V_1 \cap cl(R_{V_1}) = \varnothing$ and there exists $U \in \hat{\Omega}O(X,x)$ such that $f(U) \subseteq V_1 \cup R_{V_1}$. If $R_V = [cl(V) \setminus V] \cup R_{V_1}$, then R_V is a rare set with $cl(R_V) \cap V = \varnothing$ and $f(U) \subseteq V \cup R_V$. Thus, f is rarely $\hat{\Omega}$ -continuous at $x \in X$.

Remark 3.13. The following example shows that composition of two rarely $\hat{\Omega}$ -continuous function is not always rarely $\hat{\Omega}$ -continuous function.

Example 3.14. $X = Y = Z = \{a,b,c,d\}$ and $\tau = \{\varnothing,\{a\},\{a,b\},X\}, \sigma = \{\varnothing,\{a\},\{bcd\},Y\}$, $\eta = \{\varnothing,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},Z\}$. Define $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ by
$f(a) = b, f(b) = c, f(c) = d, f(d) = b,$
$g(a) = a, g(b) = a, g(c) = d, g(d) = b.$

Then f and g are rarely $\hat{\Omega}$ -continuous functions but $g \circ f:(X,\tau) \to (Z,\eta)$ defined by $(g \circ f)(x) = g(f(x))$ for all $x \in X$ is not a rarely $\hat{\Omega}$ -continuous function.

Theorem 3.15. If $f:(X,\tau) \to (Y,\sigma)$ is a rarely $\hat{\Omega}$ -continuous function and $g:(Y,\sigma) \to (Z,\eta)$ is continuous bijective,then $g \circ f:(X,\tau) \to (Z,\eta)$ is rarely $\hat{\Omega}$ -continuous function.

Proof- Let $x \in X$ and $G \in O(Z,(gof)(x))$ be arbitrary.Since g is continuous, $g^{-1}(G)$ is an open set in (Y,σ) containing $f(x)$.Define $G_1 = g^{-1}(G)$. Since f is rarely $\hat{\Omega}$ -continuous, there exists a rare set R_{G_1} with $G_1 \cap cl(R_{G_1}) = \varnothing$ and $U \in \hat{\Omega}O(X,x)$ such that $f(U) \subseteq G_1 \cup R_{G_1}$. By [7], $g(R_{G1})$ is a rare set in Z and $g(R_{G_1}) \subseteq Z \setminus G$ because $R_{G_1} \subseteq Y \setminus G_1$ and g is bijective, and hence $cl(g(R_{G_1})) \cap G = \varnothing,\ (gof)(U) \subseteq G \cup g(R_{G_1})$. Thus, $g \circ f:(X,\tau) \to (Z,\eta)$ is rarely $\hat{\Omega}$ -continuous function.

Theorem 3.16. If $f:(X,\tau)\to(Y,\sigma)$ is $\hat{\Omega}$-irresolute and $g:(Y,\sigma)\to(Z,\eta)$ is rarely $\hat{\Omega}$-continuous function,then $g\circ f:(X,\tau)\to(Z,\eta)$ is rarely $\hat{\Omega}$-continuous function.

Proof- Suppose that $x\in X$ and $V\in O(Z,(gof)(x))$ be arbitrary.Since g is rarely $\hat{\Omega}$-continuous,there exists a rare set R_V in (Z,η) with $V\cap cl(R_V)=\varnothing$ and some $U\in\hat{\Omega}O(Y,f(x))$ such that $g(U)\subseteq V\cup R_V$. Since f is $\hat{\Omega}$- irresolute, $f^{-1}(U)\in\hat{\Omega}O(X,x)$.If $W=f^{-1}(U)$,

then $f(W)\subseteq U$ and hence $(gof)(W)\subseteq V\cup R_V$.

Theorem 3.17. If $f:(X,\tau)\to(Y,\sigma)$ is surjective such that image of every $\hat{\Omega}$-open set in *X* is $\hat{\Omega}$-open in *Y* and $g:(Y,\sigma)\to(Z,\eta)$ is a function such that $g\circ f:(X,\tau)\to(Z,\eta)$ is rarely $\delta\omega$-continuous function, then $g:(Y,\sigma)\to(Z,\eta)$ is rarely $\hat{\Omega}$-continuous function.

Proof- Let $y\in Y$. Since f is surjective there exists $x\in X$ such that $y=f(x)$. Let $V\in O(Z,(gof)(x))$ be arbitrary.Since gof is rarely $\hat{\Omega}$-continuous,there exists a rare set R_V in (Z,η) with $V\cap cl(R_V)=\varnothing$ and some $U\in\hat{\Omega}O(X,x)$ such that $(gof)(U)\subseteq V\cup R_V$. Put $G=f(U)$, then f is strongly open implies that $G\in\hat{\Omega}O(Y,y)$ and $g(G)\subseteq V\cup R_V$. Thus g is rarely $\hat{\Omega}$-continuous function.

Definition 3.18. A function $f:(X,\tau)\to(Y,\sigma)$ is strongly rarely $\hat{\Omega}$-continuous at $x\in X$ if for each $G\in\hat{\Omega}O(Y,f(x))$, there exists $U\in\hat{\Omega}O(X,x)$ such that $int(f(U))\subseteq G$. If f is strongly rarely $\hat{\Omega}$ continuous at each $x\in X$, then f is strongly rarely $\hat{\Omega}$-continuous on X.

Example 3.19. Let $X=Y=\{a,b,c,d\},\tau=\{\varnothing,\{a\},\{bcd\},X\}$,

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σ = {Φ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, Y}. $\delta\omega O(X)$ = P(X). Define $f:(X,\tau)\to(Y,\sigma)$ *as* f(a) = b,f(b) = c,f(c) = d,and f(d) = a. Then f is strongly rarely $\hat{\Omega}$-continuous function.

Remark 3.20. Let $X=Y=\{a,b,c,d\},\tau=\{\varnothing,\{a\},\{bcd\},X\}$ $\sigma=\{\varnothing,\{a\},\{a,b\},Y\}$. $\delta\omega O(X)=\{\varnothing,\{a\},X\}$. Define $f:(X,\tau)\to(Y,\sigma)$ as $f(a)=b, f(b)=c, f(c)=d,$ and $f(d)=a.$ Then f is rarely $\hat{\Omega}$-continuous function but not strongly rarely $\hat{\Omega}$-continuous function.

Theorem 3.21. Let (Y,σ) be a regular space. Then every rarely $\hat{\Omega}$-continuous function $f:(X,\tau)\to(Y,\sigma)$ is strongly rarely $\hat{\Omega}$-continuous function.

Proof- Let $x\in X$ and $G\in O(Y,f(x))$ be an arbitrary.Since Y is a regular space, there exists $G_1\in O(Y,f(x))$ such that $cl(G_1)\subseteq G$. Since f is rarely $\hat{\Omega}$-continuous function,there exists $U\in\hat{\Omega}O(X,x)$ such that $int(f(U))\subseteq cl(G_1)$ and hence $int(f(U))\subseteq G$. Thus, f is strongly rarely $\hat{\Omega}$-continuous function.

Theorem 3.22. If $f:(X,\tau)\to(Y,\sigma)$ is weakly $\hat{\Omega}$-continuous,then f is rarely $\hat{\Omega}$-continuous function.

Proof- Let $x\in X$ and $G\in O(Y,f(x))$ be an arbitrary.By hypothesis, there exists $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq cl(G)$. Then $int(f(U))\subseteq f(U)\subseteq cl(G)$. Thus, f is rarely $\hat{\Omega}$-continuous function.

Theorem 3.23. If $f:(X,\tau)\to(Y,\sigma)$ is rarely $\hat{\Omega}$-continuous function such that image of every $\hat{\Omega}$-open set in X is open in Y, then f is weakly $\hat{\Omega}$-continuous function.

Proof- Let $x\in X$ and $G\in O(Y,f(x))$ be arbitrary.Since f is rarely $\hat{\Omega}$-continuous,there exists a rare set R_G with $G\cap cl(R_G)=\varnothing$ and $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq G\cup R_G$. Then

$f(U)\cap(Y\setminus cl(G))\subseteq R_G$. Since image of every $\hat{\Omega}$ -open set in X is open in $Y, f(U)$ is open in Y. Therefore, $(Y\setminus cl(G))\cap f(U)$ is open in Y. Since R_G is a rare set, $(Y\setminus cl(G))\cap f(U)=\varnothing$. Hence $f(U)\subseteq cl(G)$. Thus, f is weakly $\hat{\Omega}$ -continuous function.

4 APPLICATIONS

In this section,we apply the concept of *semi $T_{1/2}$* and ${}_{\omega}T_{\hat{\Omega}}$ spaces which provide key for obtaining relationship between rarely continuous and rarely $\hat{\Omega}$ -continuous function. Also it helps us to make the graph of a rarely $\hat{\Omega}$ -continuous function into a rarely $\hat{\Omega}$ -continuous function.

Theorem 4.1. Let (X,τ) be a ${}_{\omega}T_{\hat{\Omega}}$ and (Y,σ) be a *semi $T_{1/2}$* spaces. If $f:(X,\tau)\to(Y,\sigma)$ is a continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is rarely $\hat{\Omega}$ -continuous ,then $g\circ f:(X,\tau)\to(Z,\eta)$ is rarely $\hat{\Omega}$ -continuous function.

Proof- Suppose that $x\in X$ and $V\in O(Z,(gof)(x))$ be arbitrary.Since g is rarely $\hat{\Omega}$ -continuous,there exists a rare set R_V in (Z,η) with $V\cap cl(R_V)=\varnothing$ and some $U\in\omega O(Y,f(x))$ such that $g(U)\subseteq V\cup R_V$. Since Y is *semi $T_{1/2}$* , $U\in O(Y,f(x))$ .Since f is continuous,there exists open set $W\in O(X,x)$ such that $f(W)\subseteq U$. Also $(gof)(W)\subseteq V\cup R_V$. By the ${}_{\omega}T_{\hat{\Omega}}$ ness of (X,τ), $W\in\hat{\Omega}O(X,x)$. Thus $g\circ f$ is rarely $\hat{\Omega}$ -continuous function.

Theorem 4.2. Let (X,τ) be a *semi $T_{1/2}$* space and if $f:(X,\tau)\to(Y,\sigma)$ is rarely $\hat{\Omega}$ -continuous then $f:(X,\tau)\to(Y,\sigma)$ is rarely continuous.

Proof- Let $x\in X$ and $G\in O(Y,f(x))$ be arbitrary.By hypothesis, there exists a rare set R_G in Y such that $cl(R_G)\cap G=\varnothing$. with $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq G\cup R_G$. Since (X,τ) is *semi $T_{1/2}$*, U is open in X. Hence f is rarely continuous.

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Theorem 4.3. Let (X,τ) be a ${}_{\omega}T_{\hat{\Omega}}$ space and if $f:(X,\tau)\to(Y,\sigma)$ is rarely continuous then $f:(X,\tau)\to(Y,\sigma)$ is rarely $\hat{\Omega}$-continuous.

Proof- Let $x\in X$ and $G\in O(Y,f(x))$ be arbitrary.By hypothesis, there exists a rare set R_G in Y such that $cl(R_G)\cap G=\varnothing$. with $U\in O(X,x)$ such that $f(U)\subseteq G\cup R_G$. By the ${}_{\omega}T_{\hat{\Omega}}$ ness of (X,τ) , $U\in\hat{\Omega}O(X,x)$ Hence f is rarely $\hat{\Omega}$-continuous.

Theorem 4.4. Let (X,τ) be a ${}_{\omega}T_{\hat{\Omega}}$ space.If $f:(X,\tau)\to(Y,\sigma)$ is rarely $\hat{\Omega}$-continuous function,then the graph function $g:X\to X\times Y$, defined by $g(x)=(x,f(x))$ for every $x\in X$ is rarely $\hat{\Omega}$-continuous function.

Proof- Suppose that $x\in X$ and W is any open set containing $g(x)$. Then there exists $U\in O(X,x)$ and $V\in O(Y,f(x))$ such that $(x,f(x))\in U\times V\subseteq W$. Since f is rarely $\hat{\Omega}$-continuous, there exists a rare set R_V in Y with $cl(R_V)\cap V=\varnothing$ and $U_1\in\hat{\Omega}O(X,x)$ such that $f(U_1)\subseteq V\cup R_V$. Define $G=U\cap U_1$. Since X is ${}_{\omega}T_{\hat{\Omega}}$ space,by lemma 2.9, $U\in\hat{\Omega}O(X,x)$ By [1] theorem 4.12, $G\in\hat{\Omega}O(X,x)$ such that $f(G)\subseteq V\cup R_V$. Also, $G\times R_V$ is a rare set. If $R_W=(G\times R_V)\setminus W=(G\times R_V)\cap(X\times Y)\setminus W$, then R_W is a rare set in $X\times Y$ with $W\cap cl(R_W)=\varnothing$. If $p\in G$, then $f(p)\in V\cup R_V$. Therefore, $g(p)=(p,f(p))\in G\times(V\cup R_V)=(G\times V)\cup(G\times R_V)\subseteq W\bigcup R_W$ .Therefore,$g(G)\subseteq W\bigcup R_W$ .Thus, g is rarely $\hat{\Omega}$-continuous function.

Theorem 4.5. Let (X,τ) and (Y,σ) be any tow topological spaces. If $f:(X,\tau)\to(Y,\sigma)$ is a rarely $\hat{\Omega}$-continuous, then f is rarely $\hat{\Omega}$-continuous function on a subspace (A,τ_A) provied that A is both open and pre-closed subset of X.

Proof-Let $V\in O(Y,f(x))$.BY hypothesis,there exist a rare set R_V with $cl(R_V)\bigcap V=\Phi$ and $U\in\hat{\Omega}O(X,x)$ such that $f(U)\subseteq V\bigcup R_V$ .By [1] theorem 6.8, $A \bigcap U$ is $\hat{\Omega}$-open in the subspace (A, τ_A). Also, $f(U \bigcap A) \subseteq V \bigcup R_V$. Thus, f is rarely $\hat{\Omega}$ - continuous function on (A, τ_A).

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