$\tau_1\tau_2$- G*S LOCALLY CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract: In this paper, we introduce $\tau_1\tau_2$- g*s locally closed sets in bitopological spaces and study some of their basic properties. Further, we introduce and study $\tau_1\tau_2$- g*s*locally closed sets and $\tau_1\tau_2$- g*s**locally closed sets in bitopological spaces.

Keywords: $\tau_1\tau_2$- g*s locally closed sets, $\tau_1\tau_2$- g*s* locally closed sets, $\tau_1\tau_2$- g*s** locally closed sets

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1. INTRODUCTION

The study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology.A triple (X,τ_1, τ_2) when X is non- empty set and τ_1, τ_2 are two topologies on X is called a bitopological space. Kelly [6] initialed the study of these spaces in 1963. Fukutake[4] introduced the concept of g-closed sets in bitopological spaces in 1985. Arya and Nour.T [1] defined gs-open using semi-open sets.Ganster,Arockiarani and Balachandran [5] introduced regular generalized locally closed sets and RGLC continuous functions and discussed some of their properties. K.Chandrasekhara Rao and K.Kannan introduced [3] the concepts of semi star generalized locally closed sets and S*g-submaximal spaces in unital topological spaces and bitopological spaces [2]. A.Pushplatha and K.Anitha [7] introduced the concept of g*s-closed sets in topological spaces.N.Selvanayagi [9] introduced On (i,j)-g*s-closed sets in bitopological spaces. A.Pushpalatha and K.Anitha [8] introduced g*s, g*s* and g*s** -locally closed sets in topological spaces.

In this paper, we introduce $\tau_1\tau_2$-g*s, $\tau_1\tau_2$- g*s*, $\tau_1\tau_2$- g*s**-locally closed sets in bitopological spaces.

2. Preliminaries

In this section, we recollect some notations and definitions which are used in this paper.

Definition:2.1 A subset of a topological space (x,τ) is called a g*s-closed set [7] if scl (A) ⊆U whenever A⊆ U, U is gs-open in X.

Definition: 2.2 [8] A subset s of X is called a

(a) Locally closed if S=P∩Q where P is open and Q is closed in X.

(b) Generalised locally closed if S= P∩Q where P is g-open and Q is g-closed in X.

(c) gs-locally closed if S=P∩Q where P is gs-open and Q is gs-closed in x.

(d) sg-locally closed if S=P∩Q where P is sg-open and Q is sg-closed.

Definition:2.3 A subset S of X is called g*s-locally closed set (g*sl-set)[8] if S=A∩B where A is g*s-open in X and B is gs-losed in X.

Definition:2.4 A subset S of a topological space X is called g*s*-locally closed set[8] if S= P∩Q where P is g*s-open in X and Q is called in X.

Definition:2.5 A subset S of a topological space X is called g*s**-locally closed set[8] if S= P∩Q where P is open in X and Q is g*s-closed in X.

3. $\tau_1\tau_2$-G*S LOCALLY CLOSED SETS IN BITOPOLOGICAL SPACES

In this chapter, we introduce the concept of $\tau_1\tau_2$-g*s local closed sets, $\tau_1\tau_2$- g*s* locally closed sets, and $\tau_1\tau_2$- g*s** locally closed sets and study some of their properties.

Definition:3.1 A subset A of a bitopological spaces (X,τ_1, τ_2) is said to be

(a) $\tau_1\tau_2$-g*s locally closed if S=A∩B where A is a τ_1-g*s open set in X and B is a τ_2-g*s closed set in X.

(b)$\tau_1\tau_2$- g*s* locally closed if S=A∩B where A is a τ_1-g*s open set in X and B is a τ_2- closed set in X.

(c)$\tau_1\tau_2$- g*s** locally closed if S=A∩B where A is a τ_1- open set in X and B is a τ_2- g*s closed set in X.

Remark:3.2 The class of all $\tau_1\tau_2$- g*s locally closed sets, ,$\tau_1\tau_2$- g*s* locally closed sets and $\tau_1\tau_2$- g*s** locally closed sets in (X,τ_1, τ_2) are denoted by $\tau_1\tau_2$-g*sLC(X,τ_1, τ_2), $\tau_1\tau_2$-g*s*LC(X,τ_1, τ_2),and $\tau_1\tau_2$-g*s**LC(X,τ_1, τ_2) respectively.

Remark:3.3 Since every $\tau_1\tau_2$- g*s locally closed set is the intersection of τ_1- g*s open set andτ_2-g*s closed set, we can conclude the following.

Theorem:3.4 A subset A of (X,τ_1, τ_2) is $\tau_1\tau_2$-g*s locally closed set if and only if A^c is the union of a τ_2-g*s open set and τ_1-g*s closed set.

Remark:3.5 Every τ_1-open set {resp. τ_2-closed set} is ,$\tau_1\tau_2$- g*s open {resp. $\tau_1\tau_2$- g*s closed}. According we conclude the following.

Theorem:3.6 In bitopological space(X, τ_1, τ_2),

(a) Every τ_1-open set is $\tau_1\tau_2$-g*s locally closed

(b) Every τ_2-closed set is $\tau_1\tau_2$-g*s locally closed.

(c) Every $\tau_1\tau_2$- locally closed set is $\tau_1\tau_2$- g*s locally closed .

(d) Every $\tau_1\tau_2$- locally closed set is $\tau_1\tau_2$-g*s* locally closed.

(e) Every $\tau_1\tau_2$- locally closed set is $\tau_1\tau_2$- g*s** locally closed.

Remark:3.7 The converse of the above theorem need not be true in general as can be seen in the following examples.

Examples:3.8 (a) Let X={a,b,c}, τ_1={∅,X,{a}},τ_2={∅,X,{c},{a,b}}. Then {a,b} is a $\tau_1\tau_2$- g*s locally closed set, but not τ_1-open in (X, τ_1,τ_2).

(b) Let X={a,b,c}, τ_1={∅,X,{c},{a,b}} τ_2={∅,X,{b,c},{b}}. Then {C} is a $\tau_1\tau_2$-g*s locally closed set but not τ_2-closed in (X, τ_1,τ_2).

(c) Let X={a,b,c}, τ_1={∅,X,{a}}, τ_2={∅,X,{a},{a,b}}. Then {a,b} is a $\tau_1\tau_2$-g*s* locally closed set, but not $\tau_1\tau_2$-locally closed set in (X, τ_1,τ_2).

(d) Let X={a,b,c}, τ_1={∅,X,{a},{b},{a,b}},τ_2={∅,X,{b},{c},{b,c}}.Then {a,c} is a $\tau_1\tau_2$- g*s * locally closed set but not $\tau_1\tau_2$- locally closed set in (X, τ_1,τ_2).

(e) Let X={a,b,c}, τ_1={∅,X,{b,c}}, τ_2={∅,X,{a,c}}. Then {b,c} is $\tau_1\tau_2$- g*s** locally closed set but not $\tau_1\tau_2$- locally closed set in (X, τ_1,τ_2).

Theorem:3.9 In any bitopological space(X, τ_1,τ_2),

(a)S∈ $\tau_1\tau_2$-g*s*LC(X,τ_1, τ_2)⟹ S∈ $\tau_1\tau_2$-g*sLC(X,τ_1, τ_2).

(b) S∈ $\tau_1\tau_2$-g*s**LC(X,τ_1, τ_2)⟹ S∈ $\tau_1\tau_2$-g*sLC(X,τ_1, τ_2).

Proof: (a) Since S is $\tau_1\tau_2$- g*s* locally closed subset in (X, τ_1,τ_2), we have S=A∩B where A is τ_1-g*s open set and B is τ_2-closed in X. Since every τ_2-closed sets are τ_2-g*s closed in (X, τ_1,τ_2), S=A∩B where A is τ_1-g*s open and B is τ_2-g*s closed in (X, τ_1,τ_2). Therefore S∈ $\tau_1\tau_2$-g*sLC(X,τ_1, τ_2).

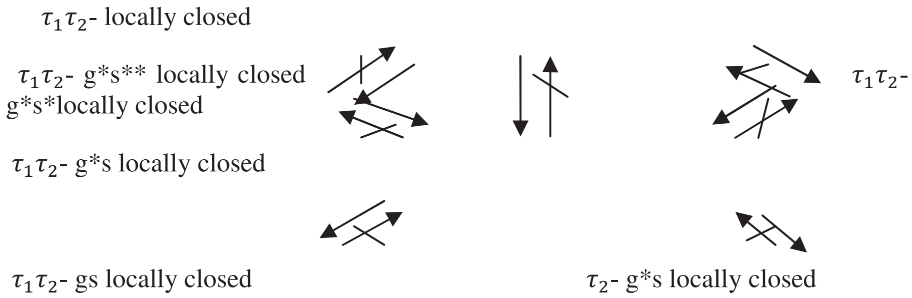
(b) Since S is $\tau_1\tau_2$-g*s** locally closed subset in (X, τ_1,τ_2), we have S=A∩B where A is τ_1- open and B is τ_2-g*s closed in (X, τ_1,τ_2). Since every τ_1- open sets are τ_1-g*s open in (X, τ_1,τ_2), S=A∩B where A is τ_1- g*s open and B is τ_2-g*s closed in (X, τ_1,τ_2). Therefore S∈ $\tau_1\tau_2$-g*sLC(X,τ_1, τ_2).

The converse of (a) and (b) are not true as seen from the following examples.

Examples:3.10 (a) Let X={a,b,c}, τ_1={∅,X,{c},{b,c}},τ_2={∅,X,{b,c},{b}}. Then {c} is $\tau_1\tau_2$-g*s locally closed but not $\tau_1\tau_2$-g*s* locally closed set in (X, τ_1,τ_2).

(b) Let X={a,b,c}, τ_1={∅,X,{c},{b,c}},τ_2={∅,X,{b,c},{b}}. Then {c} is $\tau_1\tau_2$-g*s locally closed but not $\tau_1\tau_2$-g*s**locally closed set in (X, τ_1,τ_2).

From the above results,we conclude the following:



Theorem:3.11 If (X, τ_1,τ_2) is pairwise door space, then every subset of X is both $\tau_1\tau_2$-g*s locally closed and τ_2 τ_1-g*s locally closed.

Proof: Since (X, τ_1,τ_2) is pairwise door space, every subset of (X, τ_1,τ_2) is either τ_1-open (or) τ_2-closed and τ_2-open (or) τ_1-closed. Since every τ_1-open (resp. τ_2-closed) subset of (X, τ_1,τ_2) is τ_1-g*s open (resp. τ_2-closed), we have every subset of (X, τ_1,τ_2) is either τ_1-g*s open (or) τ_2-g*s closed. Since every τ_1-g*s open and τ_2-g*s closed subset of (X, τ_1,τ_2) is $\tau_1\tau_2$- g*s locally closed, we have every subset of X is $\tau_1\tau_2$- g*s locally closed. Similarly we can prove that every subset of X is $\tau_2\tau_1$- g*s locally closed set.

Theorem:3.12. For a subset A of bitopological space (X, τ_1,τ_2), the following are equivalent.

a. A∈ $\tau_1\tau_2$-g*s*LC(X,τ_1, τ_2).

b. A=G∩[τ_2-cl(A)] for some τ_1-g*s open set G.

c. A∪{X- [τ_2-cl(A)]} is τ_1-g*s open.

d. [τ_2-cl(A)]-A is τ_1-g*s closed.

Proof: (a)⟹(b):

Since A is $\tau_1\tau_2$-g*s*locally closed set in (X, τ_1,τ_2), we have A =G∩F where G is τ_1-g*s open set and F is τ_2-closed in X. Since A⊑ τ_2-cl(A) and A⊑G,

we have A⊑ G∩[τ_2-cl(A) ⟶ (1)

Since A$\sqsubseteq$ F and F is τ_2- closed in X, we have τ_2-cl(G) $\sqsubseteq$ F. Therefore G$\cap$[τ_2-cl(A)] $\sqsubseteq$ G$\cap$F=A. Hence G$\cap$ [$\tau_2 -$ cl(A)] $\sqsubseteq$ A$\longrightarrow$ (2)

From (1) a and (2) a we have A=G$\cap$[[τ_2-cl(A)] for some τ_1-g*s open set G in (X, τ_1,τ_2). Since τ_2-cl(A) is τ_2-closed in (X, τ_1,τ_2) and G is τ_1-g*s closed in (X, τ_1,τ_2) , we have A$\in$ $\tau_1\tau_2$-g*s*LC(X,τ_1, τ_2).

(b)⟹(c):

Suppose A=G$\cap$[[τ_2-cl(A)] for some τ_1-g*s open set G in (X, τ_1,τ_2), we have

A$\cup$ {X- [τ_2-cl(A)]}= {$G \cap$ [τ_2-cl(A)]}$\sqcup$ {X − [$\tau_2 -$ cl(A)]}=G. Therefore

A $\cup${ X- [τ_2-cl(A)]} is τ_1-g*s open.

(c)⟹(b):

Suppose that A$\cup$ {X- [τ_2-cl(A)]}is τ_1-g*s open in (X, τ_1,τ_2).Let

G= A$\cup$ {X- [τ_2-cl(A)}.Then G is τ_1-g*s open set in (X, τ_1,τ_2).

Now, $G \cap$ [τ_2-cl(A)]= [A$\cup$ {X- [τ_2-cl(A)]}] $\cap$ [τ_2-cl(A)]

={ [A$\cup$ [τ_2-cl(A)]^c}$\cap$ [τ_2-cl(A)]

={ A$\cap$ [τ_2-cl(A)]} $\cup${[[τ_2-cl(A)]$^c\cap$ [τ_2-cl(A)]}

=A$\cup$ ϕ

=A

Therefore A= $G \cap$ [τ_2-cl(A)] for some τ_1-g*s open in (X, τ_1,τ_2). Let G=A $\cup${ X- τ_2-cl(A)]}. Since G is τ_1-g*s open in (X, τ_1,τ_2). We have X-G is τ_1-g*s closed in (X, τ_1,τ_2).

Now,X-G=X-[A $\cup${ X-[τ_2-cl(A)]}]

=(X-A)$\cap${ X-[τ_2-cl(A)]}

=(X-A)$\cap$[τ_2-cl(A)]

=[τ_2-cl(A)]-A.

Therefore τ_2-cl(A)]-A is τ_1- g*s closed in(X, τ_1,τ_2).

(d)⟹(c):

Suppose that τ_2-cl(A)-A is τ_1-g*s closed in (X, τ_1,τ_2). Let F= τ_2-cl(A)-A. Then F is τ_1-g*s open in (X, τ_1,τ_2).

Now, X-F=X-{[τ_2-cl(A)]-A}
=X∩{[τ_2-cl(A)]-A}^{c}
=X∩{[τ_2-cl(A)∩ A^c}^{c}
=X∩{[τ_2-cl(A)]^c∪ $(A^c)^c$}
= X∩[τ_2-cl(A)]^c∪ {X∩ A}.
=[τ_2-cl(A)]^c∪A
={X-[τ_2-cl(A)]} ∪A.

Hence A∪[τ_2-cl(A)] is τ_1-g*s open in (X, τ_1, τ_2).

Theorem:3.13 In a bitopological space (X, τ_1, τ_2) the following are equivalent.

(a)A-[τ_1-int(A)] is τ_2-g*s open in (X, τ_1, τ_2).

(b) [τ_1-int(A)] ∪[X-A] is τ_2-g*s closed in (X, τ_1, τ_2).

(c)G∪[τ_1-int(A)]=A for some τ_2-g*s open set G in (X, τ_1, τ_2).

Proof:(a)⟹(b):

Now,

X-{A- [τ_1-int(A)]}
=X∩{A- [τ_1-int(A)]}^{c}
= X∩{A ∩[τ_1-int(A)}^{c}]^c
= X∩{A^c ∪ [{ τ_1-int(A)}^{c}]^c}
= X∩{A^c ∪ [τ_1-int(A)]}
={A^c∪[τ_1-int(A)]}
=[τ_1-int(A)] ∪[X-A].

Since A-[τ_1-int(A)] is τ_2-g*s open, we have ,

x-{A-[τ_1-int(A)]}=[τ_1-int(A)] ∪[X-A] is τ_2-g*s closed in (X, τ_1, τ_2).

(b)⟹(a):

Suppose that [τ_1-int(A)] ∪[X-A] is τ_2-g*s closed in (X, τ_1, τ_2). Since [τ_1-int(A)] ∪[X-A] is τ_2-g*s closed, we have X-{[τ_1-int(A)] ∪[X-A]} is τ_2 -g*s open.

Now,

X-{[τ_1-int(A)] ∪[X-A]}
= X∩{[τ_1-int(A)] ∪ [X-A]^c
= X∩{[τ_1-int(A)] ∪A^c}^{c}
=X∩{[τ_1-int(A)]^c ∩A}

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=A ∩[τ_1-int(A)]^c
=A-[τ_1-int(A)]

Therefore A-[τ_1-int(A)] is τ_2-g*s open in (X, τ_1, τ_2).

(b)⟹(c):

Suppose that [τ_1-int(A)] ∪[X-A] is τ_2-g*s closed. Let U={ τ_1-int(A)] ∪[X-A]. Then ∪ is τ_2-g*s closed. Then U^c is τ_2-g*s open. Now,

U^c U[τ_1-int(A)]=
{[τ_1-int(A)]∪[X-A]}^{c}∪ [τ_1-int(A)]
={[τ_1-int(A)]^c∩(A^c)^c}∪[τ_1-int(A)]
={[τ_1-int(A)]^c∩A}∪[τ_1-int(A)]
={[τ_1-int(A)]^c∪ [$\tau_1 -$ int(A)]} ∩ { A∪[τ_1-int(A)]}
=X∩ A
=A

Then G=U^c.Then A=G∪[τ_1-int(A)]=A for some τ_2-g*s open set in (X, τ_1, τ_2).

(c)⟹(b):

Suppose that A=G∪[τ_1-int(A)]}=A for some τ_2 −g*s open set in G in (X, τ_1, τ_2).

Now,

[τ_1-int(A)]∪[X-A]

= $\tau_1 -$ int(A) ∪ A^c
= [$\tau_1 -$ int(A) ∪ {G ∪ [$\tau_1 -$ int(A)]}^{c}
= $\tau_1 -$ int(A)]∪{G^c ∩[τ_1-int(A)]^c}
={[τ_1-int(A)} ∪G^c}∩{[τ_1-int(A)] ∪[τ_1-int(A)]^c}
={[τ_1-int(A)] ∪G^c}∩X}
={[τ_1-int(A)] ∪G^c}

=X-G

Since G is τ_2 −g*s open in (X, τ_1, τ_2).We have X-G is τ_2 −g*s closed in (X, τ_1, τ_2).

Therefore [τ_1-int(A)]∪[X-A] is τ_2 −g*s closed in (X, τ_1, τ_2).

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