MATRICES DIVISION

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Abstract: The paper presents Division of two matrices and what are the methods to divide two matrices.

1. INTRODUCTION

Let's assume two matrices J & K having m-rows & n-columns

$$J = \begin{bmatrix} j11\ j12\ \ldots.j1n \\ j21\ j22\ \ldots.j2n \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ jm1\ jm2\ \ldots.jmn \end{bmatrix} \qquad K = \begin{bmatrix} k11\ k12\ \ldots.k1n \\ k21\ k22\ \ldots.k2n \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ km1\ km2\ \ldots.kmn \end{bmatrix}$$

j_mXj_n k_mXk_n

- **The product of two matrices is [J][K]=[S]**

$$\begin{bmatrix} j11\ j12\ \ldots.j1n \\ j21\ j22\ \ldots.j2n \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ jm1\ jm2\ \ldots.jmn \end{bmatrix} \begin{bmatrix} k11\ k12\ \ldots.k1n \\ k21\ k22\ \ldots.k2n \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ km1\ km2\ \ldots.kmn \end{bmatrix} = \begin{bmatrix} s11\ s12\ \ldots.s1n \\ s21\ s22\ \ldots.s2n \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ sm1\ sm2\ \ldots.smn \end{bmatrix}$$

j_mXj_n k_mXk_n = $s_mxs_n=j_mxk_n$

- The law To multiply two matrices is
- **Multiplicand matrix Column no. = Multiplier Row no.**
- To Divide two matrices, matrices have to satisfy below condition the condition is

Either divisor row/column must be equal to dividend row/column

Because in matrix multiplication

Multiplicand is unknown i.e., [J] is unknown.

To find order matrix –J =[J]

J_M X J_N * K_M X K_N = S_M X S_N = J_M X K_N

[J]=[S]/[K]

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J_M X J_N= (S_M X S_N)/ (K_M X K_N)= (J_M X K_N)/ (K_M X K_N)

In this case Dividend and Divisor columns must be Equal to get Quotient (i.e.,[J]).

The order of Quotient matrix is

dividend row x **divisor row**

Multiplier is unknown i.e., [K] is unknown.

To find order matrix –K =[K]

J_M X J_N * K_M X K_N = S_M X S_N = J_M X K_N

K_M X K_N= (S_M X S_N)/ (J_M X J_N)= (J_M X K_N)/ (J_M X J_N)

In this case Dividend and Divisor rows must be Equal to get Quotient(i.e.,[K]).

The order of Quotient matrix is

divisor column x **dividend column**

Wrong Matrix Division gives wrong Quotient (this is possible only in case of Square Matrices).

2. METHODS TO DIVIDE MATRICES

Matrices can be divided by 4-methods,they are

i) Division by Determinants

ii) Division by Linear Solving Equations

iii) Division by Matrix Transformation Method

iv) Division by Inverse Matrix Method

Division by Inverse Matrix Method is currently using method.

i) Division by Determinants (m =n)

(This method is only applicable for finding quotient of square matrices but not for non square matrices because finding determination is not possible for non square matrices)

when Rows of Dividend and Divisor are Equal (i.e., [J]) the elements of [J]

$$\blacktriangleright \; Js_m, sm \;\; km = \left| \begin{matrix} [K]sm, sm \rightarrow km \\ - - - - - - - \\ [K]km, kn \end{matrix} \right|$$

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Here $sm \rightarrow km$ means by Substituting/Replacing **m**[th] row of [S] in **m**[th] row of [K]

(Determinant of replacing matrix by rows) **/** (Determinant of actual/original/primary matrix)

when Columns of Dividend and Divisor are Equal (i.e., [K])

the elements of [K]

$$\rightarrow \quad Ks_n \; j_n, s_n = \left| \frac{[J] sn \rightarrow jn, sn}{[J] jm, jn} \right|$$

Here $sn \rightarrow jn$ means by Substituting/Replacing **n**[th] column of [S] in **n**[th] column of [J]

(Determinant of replacing matrix by column) **/** (Determinant of actual/original/primary matrix)

Application and use:

Easy method to solve linear equations

$$\begin{bmatrix} 2 & 6 & 7 \\ 6 & 8 & 3 \\ 9 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 89 \\ 35 \end{bmatrix}$$

3x3 3x1 3x1

Sn=1 3,Jm=Jn=3 ►

$$x = \left| \frac{\begin{matrix} 12 & 6 & 7 \\ 89 & 8 & 3 \\ 35 & 1 & 7 \end{matrix}}{\begin{matrix} 2 & 6 & 7 \\ 6 & 8 & 3 \\ 9 & 1 & 7 \end{matrix}} \right| = \frac{-3809}{-446} = 8.54035$$

$$y = \left| \frac{\begin{matrix} 2 & 12 & 7 \\ 6 & 89 & 3 \\ 9 & 35 & 7 \end{matrix}}{\begin{matrix} 2 & 6 & 7 \\ 6 & 8 & 3 \\ 9 & 1 & 7 \end{matrix}} \right| = \frac{-3281}{-446} = 7.35650$$

$$x = \begin{vmatrix} 2\ 6\ 12 \\ 6\ 8\ 89 \\ 9\ 1\ 35 \\ ---- \\ 2\ 6\ 7 \\ 6\ 8\ 3 \\ 9\ 1\ 7 \end{vmatrix} = \begin{matrix} 3136 \\ ---- \\ -446 \end{matrix} = -7.03139$$

Advantages:

- Finding determinants is easy
- Division is easily done by determinants.

Disadvantage:

- If denominator det is zero, it is highly impossible to find quotient matrix elements.
- This method is only applicable for square matrices because finding determinants for non square matrices is not possible if determinant is possible then this method is used to find quotient matrix elements of non square matrix.

ii) Division by Solving Linear Equations (m=n/m≠n)

(This method is applicable for finding quotient of square matrices and non square matrices)

In this method, Form matrix with unknown elements and multiply with known elements and equate it to product of the matrix.

- Product = Dividend
- Unknown matrix is Multiplicand when rows are equal of Divisor and Dividend and unknown matrix is Multiplier when columns are equal of Divisor and Dividend.
- Known matrix is Multiplier when rows are equal of Divisor and Dividend and Known matrix is Multiplicand when columns are equal of Divisor and Dividend

when Rows of Dividend and Divisor are Equal (i.e., [J])

$$\begin{bmatrix} x11 & x12 & \dots. & x1n \\ x21 & x22 & \dots. & x2n \\ : & : & & : \\ : & : & & : \\ xm1 & xm2 & \dots. & xmn \end{bmatrix} \begin{bmatrix} k11 & k12 & \dots. & k1n \\ k21 & k22 & \dots. & k2n \\ : & : & & : \\ : & : & & : \\ km1 & km2 & \dots. & kmn \end{bmatrix} = \begin{bmatrix} s11 & s12 & \dots. & s1n \\ s21 & s22 & \dots. & s2n \\ : & : & & : \\ : & : & & : \\ sm1 & sm2 & \dots. & smn \end{bmatrix}$$

[K] and [S] are known matrices. [X] is unknown matrix.

Now solve the unknown elements of unknown matrix.

$$x_{11}k_{11}+x_{12}k_{21}+x_{13}k_{31}+\dots\dots\dots+x_{1n}k_{m1}=s_{11}$$

$$x_{11}k_{12}+x_{12}k_{22}+x_{13}k_{32}+\dots\dots\dots+x_{1n}k_{m2}=s_{12}$$

$$x_{11}k_{13}+x_{12}k_{23}+x_{13}k_{33}+\dots\dots\dots+x_{1n}k_{m3}=s_{13}$$

to

$$x_{11}k_{1n}+x_{12}k_{2n}+x_{13}k_{3n}+\dots\dots\dots+x_{1n}k_{mn}=s_{1n}$$

Find x_{11} to x_{1n} values.

And repeat same procedure to find remaining

x_{21} to x_{2n} values

x_{31} to x_{3n} values

to

x_{m1} to x_{mn} values final matrix

$$\begin{bmatrix} x11 & x12 & \dots . & x1n \\ x21 & x22 & \dots . & x2n \\ : & : & & : \\ : & : & & : \\ xm1 & xm2 & \dots . & xmn \end{bmatrix} = \begin{bmatrix} j11 & j12 & \dots . & j1n \\ j21 & j22 & \dots . & j2n \\ : & : & & : \\ : & : & & : \\ jm1 & jm2 & \dots . & jmn \end{bmatrix}$$

when Columns of Dividend and Divisor are Equal (i.e., [K])

$$\begin{bmatrix} j11 & j12 & \dots . & j1n \\ j21 & j22 & \dots . & j2n \\ : & : & & : \\ : & : & & : \\ jm1 & jm2 & \dots . & jmn \end{bmatrix} \begin{bmatrix} x11 & x12 & \dots . & x1n \\ x21 & x22 & \dots . & x2n \\ : & : & & : \\ : & : & & : \\ xm1 & xm2 & \dots . & xmn \end{bmatrix} = \begin{bmatrix} s11 & s12 & \dots . & s1n \\ s21 & s22 & \dots . & s2n \\ : & : & & : \\ : & : & & : \\ sm1 & sm2 & \dots . & smn \end{bmatrix}$$

[J] and [S] are known matrices. [X] is unknown matrix.

j11x11+j12x21+j13x31+…….+j1nxm1=s11

j21x11+j22x21+j23x31+…….+j2nxm1=s21

j31x11+j32x21+j33x31+…….+j3nxm1=s31

: : : : :

: : : : :

jm1x11+jm2x21+jm3x31+…….+jmnxm1=sm1

Find $\mathbf{x_{11}}$ to $\mathbf{x_{1n}}$ values.

And repeat same procedure to find remaining

$\mathbf{x_{21}}$ to $\mathbf{x_{2n}}$ values

$\mathbf{x_{31}}$ to $\mathbf{x_{3n}}$ values

to

$\mathbf{x_{m1}}$ to $\mathbf{x_{mn}}$ values

final matrix

$$\begin{bmatrix} x11\ x12\ \ldots.x1n \\ x21\ x22\ \ldots.x2n \\ :\quad :\quad : \\ :\quad :\quad : \\ xm1\ xm2\ \ldots.xmn \end{bmatrix} = \begin{bmatrix} k11\ k12\ \ldots.k1n \\ k21\ k22\ \ldots.k2n \\ :\quad :\quad : \\ :\quad :\quad : \\ km1\ km2\ \ldots.kmn \end{bmatrix}$$

Advantages:

There is no difficult to find non square matrix elements

iii Division by Matrix Transformation Method (m=n)

In this method, Divisor Transformed to Dividend by using Addition, Subtraction, Multiplication and Division operations on rows and columns.

When Rows of Dividend and Divisor are Equal (i.e., [J])

$$\begin{bmatrix} 1\ 0\ \ldots.0 \\ 0\ 1\ \ldots.0 \\ :\ :\quad : \\ :\ :\quad : \\ 0\ 0\ \ldots.1 \end{bmatrix} : \begin{bmatrix} k11\ k12\ \ldots.k1n \\ k21\ k22\ \ldots.k2n \\ :\quad :\quad : \\ :\quad :\quad : \\ km1\ km2\ \ldots.kmn \end{bmatrix} = \begin{bmatrix} s11\ s12\ \ldots.s1n \\ s21\ s22\ \ldots.s2n \\ :\quad :\quad : \\ :\quad :\quad : \\ sm1\ sm2\ \ldots.smn \end{bmatrix}$$

Transform [K] to [S] by using Gauss-Jordan method on ROWS. Finally the [I] becomes [J] and [K] becomes [S]

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$$\begin{bmatrix} j11\ j12 \dots .\ j1n \\ j21\ j22 \dots .\ j2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ jm1\ jm2 \dots .\ jmn \end{bmatrix} \begin{bmatrix} s11\ s12 \dots .\ s1n \\ s21\ s22 \dots .\ s2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ sm1\ sm2 \dots .\ smn \end{bmatrix} = \begin{bmatrix} s11\ s12 \dots .\ s1n \\ s21\ s22 \dots .\ s2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ sm1\ sm2 \dots .\ smn \end{bmatrix}$$

when Columns of Dividend and Divisor are Equal (i.e., [K])

$$\frac{\begin{bmatrix} 1 & 0 & \dots \dots . & 0 \\ 0 & 1 & \dots \dots . & 0 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots \dots . & 1 \end{bmatrix}}{\begin{bmatrix} j11\ j12 \dots .\ j1n \\ j21\ j22 \dots .\ j2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ jm1\ jm2 \dots .\ jmn \end{bmatrix}} = \begin{bmatrix} s11\ s12 \dots .\ s1n \\ s21\ s22 \dots .\ s2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ sm1\ sm2 \dots .\ smn \end{bmatrix}$$

Transform [J] to [S] by using Gauss-Jordan method on COLUMNS. Finally the [I] becomes [K] and [J] becomes [S]

$$\frac{\begin{bmatrix} k11\ k12 \dots .\ k1n \\ k21\ k22 \dots .\ k2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ km1\ km2 \dots .\ kmn \end{bmatrix}}{\begin{bmatrix} s11\ s12 \dots .\ s1n \\ s21\ s22 \dots .\ s2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ sm1\ sm2 \dots .\ smn \end{bmatrix}} = \begin{bmatrix} s11\ s12 \dots .\ s1n \\ s21\ s22 \dots .\ s2n \\ \vdots \quad \vdots \quad\quad \vdots \\ \vdots \quad \vdots \quad\quad \vdots \\ sm1\ sm2 \dots .\ smn \end{bmatrix}$$

Advantages:

Square Matrices can transform easily

Disadvantage:

Not applicable for non square matrices

iii) Division by Inverse Matrix Method (m=n)

This method is currently using in the world.

to find [J] elements

$[J]=[S][K]^{-1}$

Inverse Divisor (i.e., [K]) and multiply with [S] by placing right of [S] to find [K] elements

$[K] = [J][S]^{-1}$

Inverse Divisor (i.e., [J]) and multiply with [S] by placing Left of [S]

Notes:

- **Matrix-[J]** is row operated Matrix and **Matrix-[K]** is column operated Matrix.
- For Square Matrices we can use any one of four methods and for non-square matrices we can use only 2^{nd} method i.e., Division By Solving Linear Equation Method.
- When given matrices are **cumulative matrices,** we can use either method to find Quotient Matrix
- For example $[A][A]^{-1}=[I]$

If Matrix-[A] is known Matrix. $[A]^{-1}$ is unknown Matrix. We can take Matrix-[A] as Row operated matrix or Column operated matrix.

Verification:

1) We know

(det of J)*(det of K) = (det of S)

$$|\mathbf{J}||\mathbf{K}| = |\mathbf{S}|$$

After Division by following any of these above methods

(det of Divisor Matrix)*(det of Quotient) = (det of Dividend Matrix)

2) By **The Law of Matrix Multiplication:**

The law of matrix multiplication is

The Product of sum of Multiplicand Matrix n^{th} column elements and sum of Multiplier Matrix m^{th} row elements is equal to the sum of product Matrix elements

$$\sum_{m=n=1}^{Jn=Km} CjnRkm = \sum_{\substack{m=1,\\ n=1}}^{\substack{m=Jm\\ n=Kn}} Smn$$

Where $\mathbf{C_{jn}}$=Sum of n^{th} Column elements of Multiplicand Matrix (i.e., Matrix-J = [J])

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R_{km}= Sum of m^{th} Row elements of Multiplier Matrix (i.e., Matrix-K = [K])
S_{mn}= Sum of all elements of Product Matrix (i.e., Matrix-S = [S]

3. CONCLUSION

In Square Matrices rows and columns values are same so it is very difficulty differentiate whether the divisor is matrix [J] or [K]. if we know the given Divisor is[J] we can easily find quotient matrix(i.e., [K]) by any one of four methods. In case of Non-Square matrices order of the Matrices are Different to each other to it is easy to get Quotient by using **Division By Solving Linear Equation Method.**

4. REFERENCES

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