ON SOME MATHEMATICAL ASPECTS OF FUZZY ANALYTIC HIERARCHY PROCESS

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Abstract: ***Fuzzy Analytic Hierarchy Process (FAHP) is applied to a problem taken from the literature. The ranking obtained by the elements of the priority vector is same as that obtained by this method. It is observed that the alternative A stands first in ranking and B stands lasting in ranking.***

Keywords: Multi-Criteria Decision-Making, Analytic Hierarchy Process, Pair wise comparisons.

I MOTIVATION

AHP is applied to many diverse fields and FAHP is also applied to many fields, but not to such a problem where a computer system is selected .

II METHOD OF SOLUTION

A. Analytic Hierarchy Process (AHP)

The pair-wise comparison method and the hierarchical model were developed in 1980 by T.L.Saaty in the context of the Analytic Hierarchy Process (AHP) . Evangelos Triantaphyllou et al used the analytic Hierarchy Process for decision making in engineering applications[1].Kousalya P et al used AHP to find alternatives to curb student absenteeism in engineering colleges .Kousalya P et al selected a student for All Round Excellence Award using fuzzy AHP and TOPSIS methods . The steps for implementing the AHP process are illustrated [4] as follows:

- Define the Objectives.
- . Identify the Criteria/Attributes.
- Choose the Alternatives.
- Establish the Hierarchy.
- Design Questionnaire and survey
- Construct the Pairwise Comparison matrices using Satty's 9-point scale.

B Algorithm of FAHP method

The opinions obtained by the experts are converted to fuzzy numbers using fuzzy scale given in Table 1.

Table 1:Pairwise Comparison scale

The fuzzy analytic hierarchy Process is explained below.

TFN	***Inverse TFN***	***Definition***	***Explanation***
(1,1,1)	(1,1,1)	Equal importance	Two elements contribute equally to the property
(1,3,5)	(1/5,1/3,1)	Moderate importance of one over another	Experience and judgment slightly favor one over the other
(3,5,7)	(1/7,1/5,1/3)	Essential or strong importance	Experience and judgment strongly favor one over another
(5,7,9)	(1/9,1/7,1/5)	Very strong importance	An element is strongly favored and its dominance is demonstrated in practice.
(7,9,11)	(1/11,1/9,1/7)	Extreme importance	The evidence favoring one element over another is one of the highest possible order of affirmation

Step 1: The value of fuzzy synthetic extent with respect to the i[th] object is defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \tag{1}$$

Step 2: As $\tilde{M}_1 = (l_1, m_1, u_1)$ and $\tilde{M}_2 = (l_2, m_2, u_2)$ are two TFNs, the degree of possibility of $\tilde{M}_2 = (l_2, m_2, u_2) \geq \tilde{M}_1 = (l_1, m_1, u_1)$ is defined as

$$V(\tilde{M}_2 \geq \tilde{M}_1) = \sup_{y \geq x} \left[\min \left(\mu_{\tilde{M}_1}(x), \mu_{\tilde{M}_2}(y) \right) \right] \tag{2}$$

$\mu \quad V(\tilde{M}_2 \geq \tilde{M}_1)$

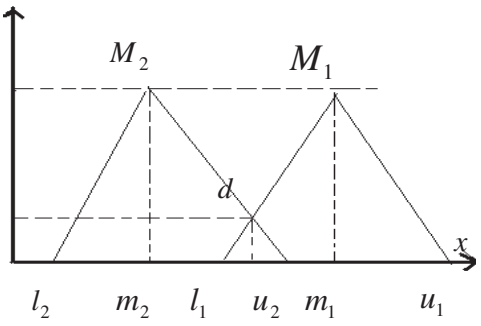

Fig. 1: Intersection between M_1 and M_2

Step 3: The degree of possibility for convex fuzzy number can be defined by

$$V(M \geq M_1, M_2, M_3 \ldots M_K) = V\left[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } (M \geq M_3) \text{ and} \ldots (M \geq M_k)\right]$$
$$= Min\ \mathrm{V}(M \geq M_i),\ \mathrm{i} = 1,2,3\ldots,\ \mathrm{k} \quad (3)$$

Assume that $d'(A_i) = \min \mathrm{V}(S_1 \geq S_k)\ \text{for}\ k = 1,2,3,\ldots n; k \neq i$. Then the weight vector is given by

$$W' = \left(d'(A_1), d'(A_2), d'(A_3), \ldots\ldots d'(A_n)\right)^T \quad (4)$$

Step 4: Via normalization, the normalized vectors are given by

$$W = \left(d(A_i), d(A_2), d(A_3), \ldots\ldots d(A_n)\right)^T \quad (5)$$

where W is a non fuzzy number.

C Illustration

Suppose consider the example of selecting the best computer system [1], there are three alternative configurations, say A, B, and C. Also, suppose that one of the decision criteria is hardware expandability (i.e., the flexibility of attaching to the system other related peripheral devices, such as printers, new memory, etc.). Suppose that system A is much better than system B, and system C is the least desired one as far as the hardware expandability criterion is concerned.

Table 2: Fuzzy opinions showing weights of alternatives w.r.t C1

C1 Hardware Expandability	**A**	**B**	**C**
A	(1 ,1 ,1)	(4,6 ,8)	(6 ,8 ,10)
B	(1/4,1/6,1/8)	(1,1 ,1)	(2,4 ,6)
C	(1/6,1/8,1/10)	(1/2,1/4,1/6)	(1,1 ,1)

Table 3: Fuzzy opinions showing weights of alternatives w.r.t C2

C2 Hardware Maintainability	**A**	**B**	**C**
A	(1 ,1 ,1)	(5,,7,9)	(1/7.1/5,1/3)
B	(1/9,1/7 ,1/5)	(1,1 ,1)	(1/10,1/8 ,1/6)
C	(1/8,1/6 ,1/4)	(2,4 ,6)	(1,1 ,1)

Table 4 : Fuzzy opinions showing weights of alternatives w.r.t C3

C3 Financing Available	A	B	C
A	(1 ,1 ,1)	(6,8 ,10)	(4 ,6 ,8)
B	(1/7,1/5 ,1/3)	(1,1 ,1)	(1/5,1/3 ,1)
C	(1/6,1/4 ,1/2)	(1,3 ,5)	(1,1 ,1)

Table 5: Fuzzy opinions showing weights of alternatives w.r.t C4

C4 User Friendly	A	B	C
A	(1 ,1 ,1)	(3,5 ,7)	(2 ,4 ,6)
B	(1/7,1/5 ,1/3)	(1,1 ,1)	(1/5,1/3 ,1)
C	(1/6,1/4 ,1/2)	(1/7,1/5 ,1/3)	(1,1 ,1)

Table 6: Opinions of criteria

	C_1	C_2	C_3	C_4
C_1	(1 ,1 ,1)	(3,5 ,7)	(1 ,3 ,5)	(5,7 ,9)
C_2	(1/7,1/5 ,1/3)	(1,1 ,1)	(1/5,1/3 ,1)	(3,5 ,7)
C_3	(1/9,1/7 ,1/5)	(1,3 ,5)	(1,1 ,1)	(4,6,8)
C_4	(1/9,1/7 ,1/5)	(1/7,1/5 ,1/3)	(1/8,1/6 ,1/4)	(1,1,1)

Table7 : Ranking of alternatives

	C1 0.465549	C2 0.201117	C3 0.333333	C4 0	Priority vector	Rank
A	0.984252	0.610501	0.91659	1	0.88653	1
B	0.015748	0	0	0	0.007331	3
C	0	0.389499	0.08341	0	0.106138	2

3 CONCLUSIONS

It can be concluded that Alternative A is ranked first and Alternative C is ranked next. Alternative B is ranked the last.

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