INTUITIONISTIC FUZZY ROUGH SET ON TWO UNIVERSAL SETS AND KNOWLEDGE REPRESENTATION

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Abstract: Intuitionistic fuzzy approximation space is a generalization of fuzzy approximation space. In this paper, we define intuitionistic fuzzy rough set for two universal sets U and V. We define the concept of solitary set with respect to intuitionistic fuzzy relation from U to V. Further based on solitary set, we study the algebraic properties that are interesting in the theory of rough sets. Further, we present an application of intuitionistic fuzzy rough set on two universal sets model for better knowledge representation.

Keywords: Fuzzy rough set, intuitionistic fuzzy rough set, lower and upper approximation, Rough set.

1. INTRODUCTION

At the present age of internet, a huge repository of the data is available across various domains. Therefore, it is very hard to extract useful information from the voluminous of data available in the universe. So, information retrieval and knowledge representation has become one of the most popular areas of recent research. Information retrieval and acquisition of knowledge is one of the important components of an information system. In order to transform the processed data intelligently and automatically into useful information and knowledge, there is a need of techniques and tools. Classical tools can be used to some extent to deal with problems arising in economics, social science, medical service, and engineering when the data are crisp, deterministic and precise in character. However, these tools fail to solve problems when the data are inconsistent, ambiguous and precise in character. In order to overcome this, fuzzy set by Zadeh [16] and rough set by Pawlak [26], [29], soft set by Molodtsov [7] are developed. Development of these techniques and tools are studied under different domains like knowledge discovery in database, computational intelligence, knowledge engineering, granular computing etc [18], [19], [21].

The rough set [26], [29] philosophy is based on the concept that there is some information associated with each object of the universe. So, there is a need to classify objects of the universe based on the indiscernibility relation between them. The basic idea of rough set is based upon the approximation of sets by pair of sets known as lower approximation and upper approximation. Here, the lower and upper approximation operators are based on equivalence relation. However, the requirement of equivalence relation is a failure in many real life problems. In order to achieve this, rough set is generalized to some extent. For instance, the equivalence relation is generalized to binary relations [17], [25], [27], neighborhood systems [22], coverings [23], Boolean algebras [14], [28], fuzzy lattices [12], and completely distributive lattices [5].

On the other hand, rough set is generalized to fuzzy environment such as fuzzy rough set [6], and rough fuzzy set [24]. Further, the indiscernibility relation is generalized to almost indiscernibility relation to study many real life problems. Based on fuzzy proximity relation the concept of rough set on fuzzy approximation spaces and its applications are studied by Tripathy and Acharjya [3], [9]. This fuzzy proximity relation is further generalized to intuitionistic fuzzy proximity relation, and the concept of rough set on intuitionistic fuzzy approximation space is studied by Tripathy [1]. The different applications are also studied by Tripathy and Acharjya [2], [4], [8], [10], [11]. Further rough set models on two universal sets are generalized with generalized approximation spaces and interval structure [20]. We continue a further study in the same direction.

The rest of the paper is organized as follows: Section 2 presents the foundations of rough set based on two universal sets. In Section 3, we study fuzzy rough sets on two universal sets and its algebraic properties that are defined with the help of solitary set with respect to a fuzzy relation. Also, we define topological characterization of fuzzy rough set on two universal sets. Fuzzy sets are intuitionistic fuzzy sets but the converse is not necessarily true. Therefore, intuitionistic fuzzy approximation space is a generalization of fuzzy approximation space. Thus, we generalize the concepts of fuzzy rough set on two universal sets to intuitionistic fuzzy rough set based on two universal sets in Section 4. In Section 5, we discuss knowledge representation with an example. This is further followed by a conclusion in Section 6.

2. ROUGH SET BASED ON TWO UNIVERSAL SETS

The rough set model is generalized using two distinct but related universal sets by Wong et al. [20]. Let *U* and *V* be two universal sets and $R \subseteq (U \times V)$ be a binary relation. By a knowledge base, we understand the relational system (*U*, *V*, *R*) an approximation space. For an element $x \in U$, we define the right neighborhood or the *R*-relative set of x in *U*, $r(x)$ as $r(x) = \bigcup \{y \in V : (x, y) \in R\}$. Similarly for an element $y \in V$, we define the left neighborhood or the *R*-relative set of y in *V*, $l(y)$ as $l(y) = \bigcup \{x \in U : (x, y) \in R\}$.

For any two elements $x_1, x_2 \in U$, we say x_1 and x_2 are equivalent if $r(x_1) = r(x_2)$. Therefore, $(x_1, x_2) \in E_U$ if and only if $r(x_1) = r(x_2)$, where E_U denote the equivalence relation on *U*. Hence, E_U partitions the universal set *U* into disjoint subsets and is denoted as U / E_U. Similarly for any two elements $y_1, y_2 \in V$, we say y_1 and y_2 are equivalent if $l(y_1) = l(y_2)$. Thus, $(y_1, y_2) \in E_V$ if and only if $l(y_1) = l(y_2)$, where E_V denote the equivalence relation on *V* and partitions the universal set *V* into disjoint subsets. Therefore for the approximation space (*U*, *V*, *R*), it is clear that $E_V \circ R = R = R \circ E_U$, where $E_V \circ R$ is the composition of *R* and E_V.

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For any $Y \subseteq V$ and the binary relation R, we associate two subsets $\underline{R}Y$ and $\overline{R}Y$ called the R-lower and R-upper approximations of Y respectively, which are given by:

$$\underline{R}Y = \bigcup\{x \in U : r(x) \subseteq Y\} \tag{1}$$

$$\overline{R}Y = \bigcup\{x \in U : r(x) \cap Y \neq \phi\} \tag{2}$$

The R-boundary of Y is denoted as $BN_R(Y)$ and is given as $BN_R(Y) = \overline{R}Y - \underline{R}Y$. The pair $(\underline{R}Y, \overline{R}Y)$ is called as the rough set of $Y \subseteq V$ if $\underline{R}Y \neq \overline{R}Y$ or equivalently $BN_R(Y) \neq \phi$. Further, if U and V are finite sets, then the binary relation R from U to V can be represented as $R(x, y)$, where

$$R(x, y) = \begin{cases} 1 & \text{if } (x, y) \in R \\ 0 & \text{if } (x, y) \notin R \end{cases}$$

The characteristic function of $X \subseteq U$ is defined for each $x \in U$ as follows:

$$X(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{if } x \notin X \end{cases}$$

Therefore, the R - lower and R - upper approximations can be also presented in an equivalent form as shown below, where $\wedge$ and $\vee$ denotes minimum and maximum operators respectively.

$$(\underline{R}Y)x = \wedge_{y \in V}((1 - R(x, y)) \vee Y(y)) \tag{3}$$

$$(\overline{R}Y)x = \vee_{y \in V}(R(x, y) \wedge Y(y)) \tag{4}$$

3. FUZZY ROUGH SET BASED ON TWO UNIVERSAL SETS

The basic idea of rough sets, introduced by Pawlak [26], [29] depends upon the notion of equivalence relations defined over a universe *U*. However, equivalence relations in real life problems are relatively rare in practice. Therefore, efforts have been made to make the relations less significant by removing one or more of the three requirements of an equivalence relation. A fuzzy relation is an extension of the concept of a relation on any set U. Therefore, fuzzy rough sets by Dubois and Prade [6] generalizes the concepts of Pawlak rough sets. Further it is generalized to fuzzy rough sets in two universal sets by Guilong Liu [13]. However, for completeness of the paper we state the definitions and basic concepts of fuzzy rough sets in two universal sets.

Let U be an universe of discourse and x is a particular element of U. A fuzzy set X of U is defined as a collection of ordered pairs $(x, \mu_X(x))$, where $\mu_X(x): U \to [0, 1]$ is a mapping known as the membership function of *X*. The family of all fuzzy sets in *U* is denoted as $F(U)$.

Let U and V be two non empty universal sets. Let R_F be a fuzzy binary relation from $U \to V$. Therefore, (U, V, R_F) is called a fuzzy approximation space. For any $Y \in F(V)$ and the fuzzy binary relation R_F, we associate two subsets $\underline{R_F}Y$ and $\overline{R_F}Y$ called the R_F-lower and R_F-upper approximations of *Y* respectively. A fuzzy rough set [162] is a pair $(\underline{R_F}Y, \overline{R_F}Y)$ of fuzzy sets on U such that for every $x \in U$

$$(\underline{R_F}Y)x = \wedge_{y \in V}((1-\mu_{R_F}(x, y)) \vee Y(y)) \tag{5}$$

$$(\overline{R_F}Y)x = \vee_{y \in V}(\mu_{R_F}(x, y) \wedge Y(y)) \tag{6}$$

Definition 3.1: Let *U* and *V* are two universal sets and R_F is a fuzzy relation from *U* to *V*. If $x \in U$ and $\mu_{R_F}(x, y) = 0$ for all $y \in V$, then we call x is a solitary element with respect to R_F. The set, S of all solitary elements with respect to the fuzzy relation R_F is called as solitary set and is given as:

$$S = \{x : x \in U, \mu_{R_F}(x, y) = 0 \quad \forall\ y \in V\} \text{ and}$$

$$\mu_{R_F}(x, \ y) = \text{Minimum } (\mu_U(x), \mu_V(y))$$

3.1. Algebraic Properties: In this section, we state the algebraic properties of fuzzy rough sets on two universal sets without proof as established by Guilong Liu [13] and are important in the context of rough sets. Let U and V be two universal sets. Let R_F be a fuzzy relation from *U* to *V*, and *S* the solitary set with respect to R_F. Then for $X, Y \in F(V)$, the following properties hold:

(i) $\underline{R_F}\phi \subseteq S$, $\overline{R_F}\phi = \phi$, $\underline{R_F}V = U$ and $\overline{R_F}V = S'$, where S' denotes the complement of *S* in *U*.

(ii) $S \subseteq \underline{R_F}X$ and $\overline{R_F}X \subseteq S'$

(iii) If $S \neq \phi$, then $\underline{R_F}X \neq \overline{R_F}X$

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(iv) For any given index set I, $X_i \in F(V)$, $\underline{R_F}(\bigcap_{i\in I} X_i) = \bigcap_{i\in I} \underline{R_F} X_i$ and $\overline{R_F}(\bigcup_{i\in I} X_i) = \bigcup_{i\in I} \overline{R_F} X_i$.

(v) If $X \subseteq Y$, then $\underline{R_F} X \subseteq \underline{R_F} Y$ and $\overline{R_F} X \subseteq \overline{R_F} Y$

(vi) $\underline{R_F} X \cup \underline{R_F} Y \subseteq \underline{R_F}(X \cup Y)$, and $\overline{R_F}(X \cap Y) \subseteq \overline{R_F} X \cap \overline{R_F} Y$

(vii) $(\underline{R_F} X)' = \overline{R_F} X'$ and $(\overline{R_F} X)' = \underline{R_F} X'$

3.2. Types of Fuzzy Rough Set Based on Two Universal Sets: In this section, we introduce an interesting characterization of fuzzy rough set on two universal sets employing the notion of the lower and upper approximation. It results four important and different types of fuzzy rough sets on two universal sets as shown below. Here $A > 0$ represents those elements of A having positive membership value.

Type 1: If $\left(\underline{R_F} Y\right)_{>0} \neq \phi$ and $\left(\overline{R_F} Y\right)_{>0} \neq U$, then we say that Y is roughly R_F -definable on two universal sets.

Type 2: If $\left(\underline{R_F} Y\right)_{>0} = \phi$ and $\left(\overline{R_F} Y\right)_{>0} \neq U$, then we say that Y is internally R_F -undefinable on two universal sets.

Type 3: If $\left(\underline{R_F} Y\right)_{>0} \neq \phi$ and $\left(\overline{R_F} Y\right)_{>0} = U$, then we say that Y is externally R_F -undefinable on two universal sets.

Type 4: If $\left(\underline{R_F} Y\right)_{>0} = \phi$ and $\overline{R_F} Y = U$, then we say that Y is totally R_F -undefinable on two universal sets.

4. INTUITIONISTIC FUZZY ROUGH SET ON TWO UNIVERSAL SETS

In the previous section we have studied Guilong Liu's [13] fuzzy rough set based on two universal sets. In fuzzy set theory we do not consider the non membership values and we assume that membership values of all elements exists. However, it is not true in many real life problems due to the presence of hesitation. In fuzzy set theory, if $\mu(x)$ be the degree of membership of an element x , then the degree of non membership of x is calculated using mathematical formula $(1-\mu(x))$ with the assumption that full part of the degree of membership is determinism and in-deterministic part is zero. This is not always applicable in real life and hence intuitionistic fuzzy set theory is better. At the same time, intuitionistic fuzzy set theory reduces to fuzzy set theory if in-deterministic part is zero. It indicates that intuitionistic fuzzy set model is a generalized model over fuzzy set model. Therefore, intuitionistic fuzzy rough set on two universal sets is a better model than fuzzy rough set on two universal sets.

Now, we present the definitions, notations and results of intuitionistic fuzzy rough set on two universal sets. We define the basic concepts leading to intuitionistic fuzzy rough set on two universal sets in which we denote μ for membership and ν for non membership functions that is associated with an intuitionistic fuzzy set.

Definition 4.1 [15] Let *U* be a universe of discourse and x is a particular element of *U*. An intuitionistic fuzzy set *X* of *U* is defined as $< x, \mu_X(x), \nu_X(x) >$, where the function $\mu_X : U \to [0,1]$ and $\nu_X : U \to [0,1]$ define the degree of membership and degree of non membership respectively of the element $x \in U$ to the set *X*, and for every $x \in U$, $0 \le \mu_X(x) + \nu_X(x) \le 1$. The amount $\pi_X(x) = 1 - (\mu_X(x) + \nu_X(x))$ is called the hesitation part, which may cater either membership value or non-membership value or the both. For simply, we will use (μ_X, ν_X) to denote the intuitionistic fuzzy set *X*. The family of all intuitionistic fuzzy subsets of *U* is denoted by $IF(U)$. The complement of an intuitionistic fuzzy set *X* is denoted by

$$X' = \{< x, \nu_X(x), \mu_X(x) > \mid x \in U\}$$

Definition 4.2 [15] Let *U* and *V* be two non empty universal sets. An intuitionistic fuzzy relation R_{IF} from $U \to V$ is an intuitionistic fuzzy set of $(U \times V)$ characterized by the membership function μ_R and non-membership function ν_R, where

$$R_{IF} = \{< (x, y), \mu_{R_{IF}}(x, y), \nu_{R_{IF}}(x, y) > \mid x \in U, y \in V\}$$

with $0 \le \mu_{R_{IF}}(x, y) + \nu_{R_{IF}}(x, y) \le 1$ for every $(x, y) \in U \times V$.

Definition 4.3 Let *U* and *V* be two universal sets and R_{IF} be an intuitionistic fuzzy relation from *U* to *V*. If $x \in U$, $\mu_{R_{IF}}(x, y) = 0$ and $\nu_{R_{IF}}(x, y) = 1$ for all $y \in V$, we call x is a solitary element with respect to R_{IF}. The set of all solitary elements with respect to the relation R_{IF} is called solitary set *S*. i.e.,

$$S = \{x \mid x \in U, \mu_{R_{IF}}(x, y) = 0, \nu_{R_{IF}}(x, y) = 1 \ \forall \ y \in V\}$$

Definition 4.4 Let *U* and *V* be two non empty universal sets and R_{IF} be an intuitionistic fuzzy relation from *U* to *V*. Therefore, (U, V, R_{IF}) is called an

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intuitionistic fuzzy approximation space. For $Y \in IF(V)$, an intuitionistic fuzzy rough set is a pair $(\underline{R_{IF}}Y, \overline{R_{IF}}Y)$ of intuitionistic fuzzy set on U such that for every $x \in U$,

$$\underline{R_{IF}}Y = \{< x,\ \mu_{\underline{R_{IF}}(Y)}(x), \nu_{\underline{R_{IF}}(Y)}(x) > | x \in U\} \tag{7}$$

$$\overline{R_{IF}}Y = \{< x,\ \mu_{\overline{R_{IF}}(Y)}(x), \nu_{\overline{R_{IF}}(Y)}(x) > | x \in U\} \tag{8}$$

where

$$\mu_{\underline{R_{IF}}(Y)}(x) = \bigwedge_{y \in V}\left[\nu_{R_{IF}}(x,y) \vee \mu_Y(y)\right]$$

$$\nu_{\underline{R_{IF}}(Y)}(x) = \bigvee_{y \in V}\left[\mu_{R_{IF}}(x,y) \wedge \nu_Y(y)\right]$$

$$\mu_{\overline{R_{IF}}(Y)}(x) = \bigvee_{y \in V}\left[\mu_{R_{IF}}(x,y) \wedge \mu_Y(y)\right]$$

$$\nu_{\overline{R_{IF}}(Y)}(x) = \bigwedge_{y \in V}\left[\nu_{R_{IF}}(x,y) \vee \nu_Y(y)\right]$$

The pair $(\underline{R_{IF}}Y, \overline{R_{IF}}Y)$ is called the intuitionistic fuzzy rough set of Y with respect to (U, V, R_{IF}), where $\underline{R_{IF}}, \overline{R_{IF}} : IF(U) \rightarrow IF(V)$ are referred as lower and upper intuitionistic fuzzy rough approximation operators on two universal sets.

4.1 Algebraic Properties of Intuitionistic Fuzzy Rough Set on Two Universal Sets: In this section, we establish the algebraic properties of intuitionistic fuzzy rough set on two universal sets through solitary set that are interesting and valuable in the study of intuitionistic fuzzy rough sets on two universal sets. These properties are useful in finding knowledge from the universe.

Proposition 4.1 Let U and V be two universal sets. Let R_{IF} be an intuitionistic fuzzy relation from U to V and further let S be the solitary set with respect to R_{IF}. Then for $X, Y \in F(V)$, the following properties holds:

(i) $\underline{R_{IF}}(V) = U$ and $\overline{R_{IF}}(\phi) = \phi$.

(ii) If $X \subseteq Y$, then $\underline{R_{IF}}(X) \subseteq \underline{R_{IF}}(Y)$ and $\overline{R_{IF}}(X) \subseteq \overline{R_{IF}}(Y)$.

(iii) $\underline{R_{IF}}(X) = (\overline{R_{IF}}(X'))'$ and $\overline{R_{IF}}(X) = (\underline{R_{IF}}(X'))'$

(iv) $\underline{R_{IF}}\phi \supseteq S$ and $\overline{R_{IF}}V \subseteq S'$, where S' denotes the complement of S in U.

(v) For any given index set J, $X_i \in IF(V)$, $\underline{R_{IF}}(\bigcup_{i\in J} X_i) \supseteq \bigcup_{i\in J} \underline{R_{IF}} X_i$ and $\overline{R_{IF}}(\bigcap_{i\in J} X_i) \subseteq \bigcap_{i\in J} \overline{R_{IF}} X_i$.

(vi) For any given index set J, $X_i \in IF(V)$, $\underline{R_{IF}}(\bigcap_{i\in J} X_i) = \bigcap_{i\in J} \underline{R_{IF}} X_i$ and $\overline{R_{IF}}(\bigcup_{i\in J} X_i) = \bigcup_{i\in J} \overline{R_{IF}} X_i$.

Proof (i) First note that V is an intuitionistic fuzzy set satisfying $\mu_V(x) = 1$ and $\nu_V(x) = 0$ for all $x \in V$. Thus, V can be represented as

$$V = \{< x, 1, 0 > \mid x \in V\}$$

Now, by definition we have

$$\mu_{\underline{R_{IF}}(Y)}(x) = \wedge_{y\in V}\left[\nu_{R_{IF}}(x,y) \vee \mu_Y(y)\right] = 1 \text{ and}$$

$$\nu_{\underline{R_{IF}}(Y)}(x) = \vee_{y\in V}\left[\mu_{R_{IF}}(x,y) \wedge \nu_Y(y)\right] = 0$$

Therefore we get,

$$\underline{R_{IF}}(V) = \{< x,\ \mu_{\underline{R_{IF}}(V)}(x), \nu_{\underline{R_{IF}}(V)}(x) > \mid x \in U\}$$
$$= \{< x,\ 1, 0 > \mid x \in U\} \quad = U$$

Similarly, ϕ is an intuitionistic fuzzy set satisfying $\mu_V(x) = 0$ and $\nu_V(x) = 1$ for all $x \in V$. Thus, ϕ can be represented as

$$\phi = \{< x, 0, 1 > \mid x \in V\}$$

Now, by definition we have

$$\mu_{\overline{R_{IF}}(Y)}(x) = \vee_{y\in V}\left[\mu_{R_{IF}}(x,y) \wedge \mu_Y(y)\right] = 0 \text{ and}$$

$$\nu_{\overline{R_{IF}}(Y)}(x) = \wedge_{y\in V}\left[\nu_{R_{IF}}(x,y) \vee \nu_Y(y)\right] = 1$$

Therefore we get,

$$\overline{R_{IF}}(\phi) = \{< x,\ \mu_{\overline{R_{IF}}(\phi)}(x), \nu_{\overline{R_{IF}}(\phi)}(x) > \mid x \in U\}$$
$$= \{< x,\ 0, 1 > \mid x \in U\} \ = \phi$$

(ii) First note that $X \subseteq Y$ if and only if $\mu_X(x) \le \mu_Y(x)$ and $\nu_X(x) \ge \nu_Y(x)$ for all $x \in V$. Therefore, we have

$$\mu_{\underline{R_{IF}}(X)}(x) = \wedge_{y\in V}\left[\nu_{R_{IF}}(x,y) \vee \mu_X(y)\right]$$

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$$\leq \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\mu_Y(y)\right] = \mu_{\underline{R_{IF}}(Y)}(x) \text{ and}$$

$$\nu_{\underline{R_{IF}}(X)}(x) = \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\nu_X(y)\right]$$

$$\geq \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\nu_Y(y)\right] = \nu_{\underline{R_{IF}}(Y)}(x)$$

Therefore, $\underline{R_{IF}}X \subseteq \underline{R_{IF}}Y$.

Similarly, we have

$$\mu_{\overline{R_{IF}}(X)}(x) = \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\mu_X(y)\right]$$

$$\leq \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\mu_Y(y)\right] = \mu_{\overline{R_{IF}}(Y)}(x) \text{ and}$$

$$\nu_{\overline{R_{IF}}(X)}(x) = \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\nu_X(y)\right]$$

$$\geq \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\nu_Y(y)\right] = \nu_{\overline{R_{IF}}(Y)}(x)$$

Therefore, $\overline{R_{IF}}X \subseteq \overline{R_{IF}}Y$

(iii) We know that $\overline{R_{IF}}(X') = \{< x,\ \mu_{\overline{R_{IF}}(X')}(x), \nu_{\overline{R_{IF}}(X')}(x) > | \, x \in U\}$, where

$$\mu_{\overline{R_{IF}}(X')}(x) = \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\mu_{X'}(y)\right]$$

$$= \underset{y\in V}{\vee}\left[\mu_{R_{IF}}(x,y)\wedge\nu_X(y)\right] = \nu_{\underline{R_{IF}}(X)}(x) \text{ and}$$

$$\nu_{\overline{R_{IF}}(X')}(x) = \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\nu_{X'}(y)\right]$$

$$= \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\mu_X(y)\right] = \mu_{\underline{R_{IF}}(X)}(x).$$

Therefore, we have

$$\overline{R_{IF}}(X') = \{< x,\ \mu_{\overline{R_{IF}}(X')}(x), \nu_{\overline{R_{IF}}(X')}(x) > \big| x\in U\}$$

$$= \{< x,\ \nu_{\underline{R_{IF}}(X)}(x),\ \mu_{\underline{R_{IF}}(X)}(x) > \big| x\in U\}$$

It indicates that $(\overline{R_{IF}}(X'))' = \{< x, \mu_{\underline{R_{IF}}(X)}(x), \nu_{\underline{R_{IF}}(X)}(x) > | \, x\in U\}$ and consequently $(\overline{R_{IF}}(X'))' = \underline{R_{IF}}(X)$.

Similarly, $\underline{R_{IF}}(X') = \{< x,\ \mu_{\underline{R_{IF}}(X')}(x),\ \nu_{\underline{R_{IF}}(X')}(x) > \big| x\in U\}$, where

$$\mu_{\underline{R_{IF}}(X')}(x) = \underset{y\in V}{\wedge}\left[\nu_{R_{IF}}(x,y)\vee\mu_{X'}(y)\right]$$

$$= \wedge_{y\in V}\left[\nu_{R_{IF}}(x,y)\vee\nu_X(y)\right] = \nu_{\overline{R_{IF}}(X)}(x) \text{ and}$$

$$\nu_{\underline{R_{IF}}(X')}(x) = \vee_{y\in V}\left[\mu_{R_{IF}}(x,y)\wedge\nu_{X'}(y)\right]$$

$$= \vee_{y\in V}\left[\mu_{R_{IF}}(x,y)\wedge\mu_X(y)\right] = \mu_{\overline{R_{IF}}(X)}(x)$$

Therefore, we have

$$\underline{R_{IF}}(X') = \{< x, \mu_{\underline{R_{IF}}(X')}(x), \nu_{\underline{R_{IF}}(X')}(x) > \mid x\in U\}$$

$$= \{< x, \nu_{\overline{R_{IF}}(X)}(x), \mu_{\overline{R_{IF}}(X)}(x) > \mid x\in U\}$$

It indicates that $(\underline{R_{IF}}(X'))' = \{< x, \mu_{\overline{R_{IF}}(X)}(x), \nu_{\overline{R_{IF}}(X)}(x) > \mid x\in U\}$ and consequently $(\underline{R_{IF}}(X'))' = \overline{R_{IF}}(X)$.

(iv) First note that, ϕ is an intuitionistic fuzzy set satisfying $\mu_V(x)=0$ and $\nu_V(x)=1$ for all $x\in V$. Thus, ϕ can be represented as

$$\phi = \{< x, 0, 1 > \mid x\in V\}$$

Also note that, S is a solitary set. This indicates that

$$S = \{x \mid x\in U, \mu_{R_{IF}}(x,y)=0, \nu_{R_{IF}}(x,y)=1 \ \forall \ y\in V\}.$$

Therefore, we have for all $x\in S$

$$\mu_{\underline{R_{IF}}(\phi)}(x) = \wedge_{y\in V}\left[\nu_{R_{IF}}(x,y)\vee\mu_\phi(y)\right] = \wedge_{y\in V}\left[1\vee 0\right]=1 \text{ and}$$

$$\nu_{\underline{R_{IF}}(\phi)}(x) = \vee_{y\in V}\left[\mu_{R_{IF}}(x,y)\wedge\nu_\phi(y)\right] = \vee_{y\in V}\left[0\wedge 1\right]=0$$

Hence, it is clear that $\mu_{\underline{R_{IF}}(\phi)}(x) \ge \mu_{R_{IF}}(x,y)$ and $\nu_{\underline{R_{IF}}(\phi)}(x) \le \nu_{R_{IF}}(x,y)$ for $x\in S$. Therefore, by proposition (ii) we have $\underline{R_{IF}}\phi \supseteq S$.

Similarly, by proposition (iii) we have $\overline{R_{IF}}(X) = (\underline{R_{IF}}(X'))'$. On taking $X=V$ we get $\overline{R_{IF}}V = (\underline{R_{IF}}(V'))'$. But $V'=\phi$ and hence $\overline{R_{IF}}V = (\underline{R_{IF}}\phi)'$. Again by proposition (iv) we have $\underline{R_{IF}}\phi \supseteq S$. It implies that $(\underline{R_{IF}}\phi)' \subseteq S'$. Therefore, we get $\overline{R_{IF}}V \subseteq S'$.

(v) From the properties of union, for any index set $J = \{1, 2, 3, \ldots, n\}$,

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$X_1 \subseteq \bigcup_{i\in J} X_i; X_2 \subseteq \bigcup_{i\in J} X_i; X_3 \subseteq \bigcup_{i\in J} X_i; \cdots ; X_n \subseteq \bigcup_{i\in J} X_i$

Therefore, by proposition (ii) we have

$$\underline{R_{IF}} X_1 \subseteq \underline{R_{IF}}(\bigcup_{i\in J} X_i); \cdots ; \underline{R_{IF}} X_n \subseteq \underline{R_{IF}}(\bigcup_{i\in J} X_i)$$

It indicates that,

$$\bigcup_{i\in J} \underline{R_{IF}} X_i \subseteq \underline{R_{IF}}(\bigcup_{i\in J} X_i) \text{ i.e., } \underline{R_{IF}}(\bigcup_{i\in J} X_i) \supseteq \bigcup_{i\in J} \underline{R_{IF}} X_i$$

Similarly for any index set $J = \{1, 2, 3, \cdots, n\}$

$$\bigcap_{i\in J} X_i \subseteq X_1; \bigcap_{i\in J} X_i \subseteq X_2; \bigcap_{i\in J} X_i \subseteq X_3; \cdots ; \bigcap_{i\in J} X_i \subseteq X_n$$

Therefore, by proposition (ii) we have

$$\overline{R_{IF}}(\bigcap_{i\in J} X_i) \subseteq \overline{R_{IF}} X_1; \cdots ; \overline{R_{IF}}(\bigcap_{i\in J} X_i) \subseteq \overline{R_{IF}} X_n$$

It indicates that, $\overline{R_{IF}}(\bigcap_{i\in J} X_i) \subseteq \bigcap_{i\in J} \overline{R_{IF}} X_i$

(vi) For any index set $J = \{1, 2, 3, \cdots, n\}$, $X_i \in IF(V)$,

$$\underline{R_{IF}}(\bigcap_{i\in j} X_i) = \{< x, \mu_{\underline{R_{IF}}(\bigcap_{i\in j} X_i)}(x), \nu_{\underline{R_{IF}}(\bigcap_{i\in j} X_i)}(x) > \big| x \in U\} \text{. But,}$$

$$\mu_{\underline{R_{IF}}(\bigcap_{i\in j} X_i)}(x) = \bigwedge_{y\in V} [\nu_{R_{IF}}(x,y) \vee \mu_{(\bigcap_{i\in j} X_i)}(y)]$$

$$= \bigwedge_{y\in V} [\nu_{R_{IF}}(x,y) \vee (\mu_{X_1}(y) \wedge \mu_{X_2}(y) \wedge \cdots \wedge \mu_{X_n}(y))]$$

$$= \bigwedge_{y\in V} [(\nu_{R_{IF}}(x,y) \vee \mu_{X_1}(y)) \wedge \cdots \wedge (\nu_{R_{IF}}(x,y) \vee \mu_{X_n}(y))]$$

$$= \mu_{\underline{R_{IF}} X_1}(x) \wedge \mu_{\underline{R_{IF}} X_2}(x) \wedge \cdots \wedge \mu_{\underline{R_{IF}} X_n}(x)$$

$$= \text{Min } \{\mu_{\underline{R_{IF}} X_i}(x)\}$$

$$\nu_{\underline{R_{IF}}(\bigcap_{i\in j} X_i)}(x) = \bigvee_{y\in V} [\mu_{R_{IF}}(x,y) \wedge \nu_{(\bigcap_{i\in j} X_i)}(y)]$$

$$= \bigvee_{y\in V} [\mu_{R_{IF}}(x,y) \wedge (\nu_{X_1}(y) \vee \nu_{X_2}(y) \vee \nu_{X_3}(y) \vee \cdots \vee \nu_{X_n}(y))]$$

$$= \bigvee_{y\in V} [(\mu_{R_{IF}}(x,y) \wedge \nu_{X_1}(y)) \vee \cdots \vee (\mu_{R_{IF}}(x,y) \wedge \nu_{X_n}(y))]$$

$$= \nu_{\underline{R_{IF}} X_1}(x) \vee \nu_{\underline{R_{IF}} X_2}(x) \vee \cdots \vee \nu_{\underline{R_{IF}} X_n}(x)$$

$$= \text{Max } \{\nu_{\underline{R_{IF}} X_i}(x)\}$$

Again, for any index set $J = \{1, 2, 3, \cdots, n\}$, $X_i \in IF(V)$,

$\underline{R_{IF}} X_1 = \{< x,\ \mu_{\underline{R_{IF}} X_1}(x), \nu_{\underline{R_{IF}} X_1}(x) > \big| x \in U\}$;

$\underline{R_{IF}} X_2 = \{< x,\ \mu_{\underline{R_{IF}} X_2}(x), \nu_{\underline{R_{IF}} X_2}(x) > \big| x \in U\}$;

…… …… …… …… ……

$\underline{R_{IF}} X_n = \{< x,\ \mu_{\underline{R_{IF}} X_n}(x), \nu_{\underline{R_{IF}} X_n}(x) > \big| x \in U\}$

Therefore, we have

$$\bigcap_{i\in j} \underline{R_{IF}} X_i = \{< x,\ \text{Min}\ \{\mu_{\underline{R_{IF}} X_i}(x)\}, \text{Max}\ \{\nu_{\underline{R_{IF}} X_i}(x)\} > \big| x \in U\}$$

Hence, it is clear that $\underline{R_{IF}}(\bigcap_{i\in J} X_i) = \bigcap_{i\in J} \underline{R_{IF}} X_i$.

Similarly, for any index set $J = \{1, 2, 3, \cdots, n\}$, $X_i \in IF(V)$,

$$\overline{R_{IF}}(\bigcup_{i\in J} X_i) = \{< x,\ \mu_{\overline{R_{IF}}(\bigcup_{i\in J} X_i)}(x), \nu_{\overline{R_{IF}}(\bigcup_{i\in J} X_i)}(x) > \big| x \in U\} \text{ But,}$$

$$\mu_{\overline{R_{IF}}(\bigcup_{i\in j} X_i)}(x) = \bigwedge_{y\in V} [\mu_{R_{IF}}(x,y) \wedge \mu_{(\bigcup_{i\in j} X_i)}(y)]$$

$$= \bigwedge_{y\in V} [\mu_{R_{IF}}(x,y) \wedge (\mu_{X_1}(y) \vee \mu_{X_2}(y) \vee \cdots \vee \mu_{X_n}(y))]$$

$$= \bigwedge_{y\in V} [(\mu_{R_{IF}}(x,y) \wedge \mu_{X_1}(y)) \vee \cdots \vee (\mu_{R_{IF}}(x,y) \vee \mu_{X_n}(y))]$$

$$= \mu_{\overline{R_{IF}} X_1}(x) \vee \mu_{\overline{R_{IF}} X_2}(x) \vee \cdots \vee \mu_{\overline{R_{IF}} X_n}(x) = \text{Max}\ \{\mu_{\overline{R_{IF}} X_i}(x)\}$$

$$\nu_{\overline{R_{IF}}(\bigcup_{i\in j} X_i)}(x) = \bigvee_{y\in V} \left[\nu_{R_{IF}}(x,y) \vee \nu_{(\bigcup_{i\in j} X_i)}(y)\right]$$

$$= \bigvee_{y\in V} \left[\nu_{R_{IF}}(x,y) \vee (\nu_{X_1}(y) \wedge \nu_{X_2}(y) \wedge \nu_{X_3}(y) \wedge \cdots \wedge \nu_{X_n}(y))\right]$$

$$= \bigvee_{y\in V} \left[(\nu_{R_{IF}}(x,y) \vee \nu_{X_1}(y)) \wedge \cdots \wedge (\nu_{R_{IF}}(x,y) \wedge \nu_{X_n}(y))\right]$$

$$= \nu_{\overline{R_{IF}} X_1}(x) \wedge \nu_{\overline{R_{IF}} X_2}(x) \wedge \cdots \wedge \nu_{\overline{R_{IF}} X_n}(x) = \text{Min}\ \{\nu_{\overline{R_{IF}} X_i}(x)\}$$

Again, for any index set $J = \{1, 2, 3, \cdots, n\}$, $X_i \in IF(V)$,

$\overline{R_{IF}} X_1 = \{< x,\ \mu_{\overline{R_{IF}} X_1}(x), \nu_{\overline{R_{IF}} X_1}(x) > \big| x \in U\}$;

$\overline{R_{IF}} X_2 = \{< x,\ \mu_{\overline{R_{IF}} X_2}(x), \nu_{\overline{R_{IF}} X_2}(x) > \big| x \in U\}$;

…… …… …… …… ……

$\overline{R_{IF}} X_n = \{< x,\ \mu_{\overline{R_{IF}} X_n}(x), \nu_{\overline{R_{IF}} X_n}(x) > \big| x \in U\}$

Therefore, we have

$$\bigcup_{i\in j} \overline{R_{IF}} X_i = \{< x,\ \text{Max}\ \{\mu_{\overline{R_{IF}} X_i}(x)\}, \text{Min}\ \{\nu_{\overline{R_{IF}} X_i}(x)\} > \big| x \in U\}$$

Hence, it is clear that $\overline{R_{IF}}(\bigcup_{i\in J} X_i) = \bigcup_{i\in J} \overline{R_{IF}} X_i$.

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5. KNOWLEDGE REPRESENTATION

In this section, we demonstrate how the above concepts can be applied to real life problems. We consider an example in which we study the relation between customers and the supermarkets in a particular metropolitan city. In general, supermarket takes care of customer's everyday household needs and more. Therefore, it spread across a wide range of products of food and non food items, ranging from basic necessities such as, fruits and vegetables, staples, personal care, home care, household care products, general merchandise, and dairy products. Hence it is a one stop solution for the customers to fulfill daily shopping needs at a convenient location close to the customer. Apart from this the best possible value for customer's money, quality of the product, style and the behaviour of supporting staff play a vital role in choosing a supermarket. However, there exist many other factors to choose a supermarket. For this reason, in general, customer has to depend on more than one supermarket in a city. Therefore, it is essential to establish the relation between the customers and the supermarkets in a city. Therefore, from customer behaviour it is clear that they are not happy with one supermarket. Hence, intuitionistic fuzzy relation better depicts the relation between the customers and supermarkets. To make our analysis simple, we consider a small universe U of 5 customers and another small universe V of 6 supermarkets in a particular city. Therefore, (U, V, R_{IF}) be an intuitionistic fuzzy approximation space, where $U = \{c_1, c_2, c_3, c_4, c_5\}$ and $V = \{m_1, m_2, m_3, m_4, m_5, m_6\}$. We define the intuitionistic fuzzy relation $R_{IF} \in IF(U \times V)$ by the following matrix.

$$R_{IF} = \begin{array}{c} \\ \end{array}\begin{array}{cccccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 \\ \end{array}$$

$$R_{IF} = \begin{bmatrix} 0.8, 0.1 & 0.2, 0.5 & 0.6, 0.2 & 0.4, 0.4 & 0,1 & 0.4, 0.2 \\ 0.7, 0.3 & 0.9, 0.1 & 0.5, 0.4 & 1,0 & 0.4, 0.5 & 0.5, 0.4 \\ 0.3, 0.6 & 0.6, 0.2 & 0.4, 0.5 & 0.3, 0.4 & 0.2, 0.4 & 0.3, 0.3 \\ 0.5, 0.5 & 0.3, 0.6 & 0.7, 0.2 & 0.6, 0.3 & 0.2, 0.4 & 0.6, 0.4 \\ 0.7, 0.2 & 0.1, 0.8 & 0.4, 0.3 & 0.5, 0.4 & 0, 0.6 & 0.5, 0.2 \end{bmatrix}$$

Let us consider an intuitionistic fuzzy set $Y = \{< m_1, 0.6, 0.2 >, < m_2, 0.5, 0.4 >, < m_3, 0.3, 0.5 >, < m_4, 0.8, 0.1 >, < m_5, 1, 0 >, < m_6, 0.7, 0.2 >\}$. Therefore, by Definition 6.8.4 we have

$$\mu_{\underline{R_{IF}}(Y)}(c_1) = \underset{y \in V}{\wedge}\left[\nu_{R_{IF}}(c_1, y) \vee \mu_Y(y)\right]$$
$$= [0.1 \vee 0.6] \wedge [0.5 \vee 0.5] \wedge [0.2 \vee 0.3] \wedge [0.4 \vee 0.8] \wedge [1 \vee 1] \wedge [0.2 \vee 0.7]$$
$$= 0.6 \wedge 0.5 \wedge 0.3 \wedge 0.8 \wedge 1 \wedge 0.7 = 0.3$$
$$\nu_{\underline{R_{IF}}(Y)}(c_1) = \underset{y \in V}{\vee}\left[\mu_{R_{IF}}(c_1, y) \wedge \nu_Y(y)\right]$$
$$= 0.2 \vee 0.2 \vee 0.5 \vee 0.1 \vee 0 \vee 0.2 = 0.5$$

Similarly, $\mu_{\underline{R_{IF}}(Y)}(c_2) = 0.4$, $\nu_{\underline{R_{IF}}(Y)}(c_2) = 0.5$, $\mu_{\underline{R_{IF}}(Y)}(c_3) = 0.5$, $\nu_{\underline{R_{IF}}(Y)}(c_3) = 0.4; \mu_{\underline{R_{IF}}(Y)}(c_4) = 0.3; \nu_{\underline{R_{IF}}(Y)}(c_4) = 0.5; \mu_{\underline{R_{IF}}(Y)}(c_5) = 0.3$ and $\nu_{\underline{R_{IF}}(Y)}(c_5) = 0.4$.

Hence, the lower approximation is given as:

$$\underline{R_{IF}}Y = \{< c_1, 0.3, 0.5 >, < c_2, 0.4, 0.5 >, < c_3, 0.5, 0.4 >, < c_4, 0.3, 0.5 >, < c_5, 0.3, 0.4 >\}$$

Similarly, we obtain the upper approximation as:

$$\overline{R_{IF}}Y = \{< c_1, 0.6, 0.2 >, < c_2, 0.8, 0.1 >, < c_3, 0.5, 0.3 >, < c_4, 0.6, 0.3 >, < c_5, 0.6, 0.2 >\}$$

6. CONCLUSION

In this paper we extended the study of fuzzy rough sets on two universal sets further by defining their topological characterization. Also, we have introduced intuitionistic fuzzy rough set on two universal sets and its properties are studied. We have also provided a real life example to verify the concepts developed for their application in knowledge extraction and in designing knowledge bases.

7. REFERENCES

1. B. K. Tripathy, "Rough sets on intuitionistic fuzzy approximation spaces," Proceedings of 3rd International IEEE Conference on Intelligent Systems (IS06), London, pp. 776-779, 2006.
2. B. K. Tripathy, and D. P. Acharjya, "Association rule granulation using rough sets on intuitionistic fuzzy approximation spaces and granular computing," Annals. Computer Science Series, vol. 9 (1), pp. 125-144, 2011.
3. B. K. Tripathy, and D. P. Acharjya, "Knowledge mining using ordering rules and rough sets on fuzzy approximation spaces," International Journal of Advances in Science and Technology, vol. 1 (3), pp. 41-50, 2010.
4. B. K. Tripathy, D. P. Acharjya, and V. Cynthya, "A framework for intelligent medical diagnosis using rough set with formal concept analysis," International Journal of Artificial Intelligence & Applications, vol. 2 (2), pp. 45-66, 2011.
5. D. Chen, W. Zhang, D. Yeung, and E. C. C. Tsang, "Rough approximations on a complete completely distributive lattice with applications to generalized rough sets," Information Sciences, vol. 176, pp. 1829 - 1848, 2006.
6. D. Dubois, and H. Prade, "Rough fuzzy sets and fuzzy rough sets," International Journal of General System, vol. 17, pp. 191-209, 1990.
7. D. Molodtsov, "Soft set theory-first results," Computers and Mathematics with Applications, vol. 37, pp. 19–31, 1999.
8. D. P. Acharjya, "Comparative study of rough sets on fuzzy approximation spaces and intuitionistic fuzzy approximation spaces," International Journal of Computational and Applied Mathematics, vol. 4 (2), pp. 95-106, 2009.
9. D. P. Acharjya, and B. K. Tripathy, "Rough sets on fuzzy approximation space and application to distributed knowledge systems," International Journal of Artificial Intelligence and Soft Computing, vol. 1 (1), pp. 1- 14, 2008.
10. D. P. Acharjya, and B. K. Tripathy, "Rough sets on intuitionistic fuzzy approximation spaces and knowledge representation," International Journal of Artificial Intelligence and Computational Research, vol. 1 (1), pp. 29–36, 2009.

11. D. P. Acharjya, and L. Ezhilarasi, "A knowledge mining model for ranking institutions using rough computing with ordering rules and formal concept analysis," International Journal of Computer Science Issues, vol. 8 (2), pp. 417-425, 2011.
12. G. L. Liu, "Generalized rough sets over fuzzy lattices," Information Sciences, vol. 178, pp. 1651 – 1662, 2008.
13. G. L. Liu, "Rough set theory based on two universal sets and its applications," Knowledge Based Systems, vol. 23, pp. 110–115, 2010.
14. G. L. Liu, "Rough sets over the Boolean algebras," Lecture Notes in Artificial Intelligence, vol. 3641, pp. 24–131, 2005.
15. K. T. Atanasov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, pp. 87–96, 1986.
16. L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, pp. 338–353, 1965.
17. M. Kondo, "Algebraic approach to generalized rough sets," Lecture Notes in Artificial Intelligence, vol. 3641, pp. 132 – 140, 2005.
18. M. Kryszkiewlcz, "Rough set approach to incomplete information systems," Information Sciences, vol. 112, pp. 39 – 49, 1998.
19. P. K. Maji, and A. R. Roy, "An application of soft sets in a decision making problem," Computers and Mathematics with Applications, vol. 44, pp. 1077–1083, 2002.
20. S. K. M. Wong, L. S. Wang, and Y. Y. Yao, "Interval structure: a framework for representing uncertain information," Proceedings of the 8th Conference on Uncertainty in Artificial Intelligence, pp. 336 –343, 1993.
21. T. Y. Lin, "Granular Computing: Examples, intuitions and modeling," Proceeding of the IEEE International Conference on Granular Computing, Beijing, China, pp. 40-44, 2005.
22. T. Y. Lin, "Neighborhood systems and approximation in database and knowledge base systems," Proceedings of the Fourth International Symposium on Methodologies of Intelligent Systems, pp. 75 – 86, 1989.
23. W. Zhu, and F. Y. Wang, "On three types of covering rough sets," IEEE Transactions on Knowledge and Data Engineering, vol. 19 (8), pp. 1131-1144, 2007.
24. Y. Y. Yao, "Two views of the theory of rough sets in finite universes," International Journal of Approximation Reasoning, vol. 15, pp. 291–317, 1996.
25. Z. Bonikowski, "Aggebraic structure of rough sets," in Rough sets, Fuzzy sets and Knowledge Discovery, W. P. Ziarko, Ed. New York: Springer Verlag, 1994, pp. 242-247.
26. Z. Pawlak, "Rough sets," International Journal of Computer Information Science, vol. 11, pp. 341 – 356, 1982.
27. Z. Pawlak, and A. Skowron, "Rough sets: Some extensions," Information Sciences, Elsevier, vol. 177 (1), pp. 28–40, 2007.
28. Z. Pawlak, and A. Skowron, "Rough sets and Boolean reasoning," Information Sciences, Elsevier, vol. 177 (1), pp. 41–73, 2007.
29. Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning about Data, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1991.

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