A JACK KNIFED GENERALIZED ESTITMATOR IN THE PRESENCE OF MEASUREMENT ERRORS

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Abstract: In this paper, using a Jack-knifed version of Quenouille's method, the generalized estimator of population mean of Srivastava (1971) using single auxiliary variable is considered and its properties are analyzed in the presence of measurement errors.

Keywords: Jack – knife technique, measurement error, auxiliary variable.

1. INTRODUCTION

For a simple random sample of size m, let (x_i, y_i) be the pair of values instead of the true values (X_i, Y_i) on the two characteristics (X, Y) respectively for the i^{th} $(i = 1, 2, \ldots, m)$ unit in the sample. Let the observational or measurement errors be

$$u_i = y_i - Y_i \tag{1.1}$$

$$v_i = x_i - X_i \tag{1.2}$$

which are stochastic in nature and are uncorrelated with mean zero and variances σ_u^2 and σ_v^2 respectively. Further, let the population means of (X, Y) be (μ_X, μ_Y), population variances of (X, Y) be (σ_X^2, σ_Y^2) respectively, σ_{XY} be the population covariance and ρ be the population correlation coefficient between X and Y.

We consider a simple random sample of size $2n$ and split this sample randomly into two sub-samples each of size n. Let $(\bar{y}_{2n}, \bar{x}_{2n})$ be the sample means of values on (Y, X) respectively for the entire sample of size $2n$ and $(\bar{y}_n^{(i)}, \bar{x}_n^{(i)})$ be the sample means of values on (Y, X) respectively for i^{th} $(i = 1, 2)$ sub-sample of size n

From Srivastava (1971), the generalized estimator of population mean μ_Y using mean μ_X of single auxiliary variable X is

$$\bar{y}_f = \bar{y}_m f\left(\frac{\bar{x}_m}{\mu_X}\right) = \bar{y}_m f(u_m) \tag{1.3}$$

where $(\bar{y}_m, \bar{x}_m)$ are the sample means of (Y, X) respectively for a simple random sample of size m, $u_m = \frac{\bar{x}_m}{\mu_X}$ and $f(u_m)$ satisfying the validity conditions of Taylor's series expansion is a bounded function of u_m such that $f(u_m = 1) = 1$.

Let $\bar{y}_f^{(3)}$ is the generalized estimator of Srivastava (1971) given by (1.3) for the entire sample of size 2n, $\bar{y}_f^{(1)}$ and $\bar{y}_f^{(2)}$ be the generalized estimators of Srivastava for the two randomly splitted sub-samples of size n each.

Thus

$$\bar{y}_f^{(3)} = \bar{y}_{2n} f\left(\frac{\bar{x}_{2n}}{\mu_X}\right) = \bar{y}_{2n} f(z_3) \tag{1.4}$$

$$\bar{y}_f^{(1)} = \bar{y}_n^{(1)} f\left(\frac{\bar{x}_n^{(1)}}{\mu_X}\right) = \bar{y}_n^{(1)} f(z_1) \tag{1.5}$$

$$\bar{y}_f^{(2)} = \bar{y}_n^{(2)} f\left(\frac{\bar{x}_n^{(2)}}{\mu_X}\right) = \bar{y}_n^{(2)} f(z_2) \tag{1.6}$$

where $z_3 = \frac{\bar{x}_{2n}}{\mu_X}$, $z_1 = \frac{\bar{x}_n^{(1)}}{\mu_X}$, $z_2 = \frac{\bar{x}_n^{(2)}}{\mu_X}$, $f(z_3)$, $f(z_1)$ and $f(z_2)$ are the bounded functions of z_3, z_2 and z_1 respectively satisfying the regularity conditions of $f(u_m)$ in (1.3) involved in Srivastava (1971) generalized estimator $\bar{y}_f$.

We have

$$\bar{y}_{2n} = \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_i^{2n} y_i\right\}$$

$$= \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_i^{2n} (y_i - Y_i + Y_i - \mu_Y + \mu_Y)\right\}$$

$$= \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_i^{2n} [u_i + (Y_i - \mu_Y) + \mu_Y]\right\}$$

$$= \mu_Y + \frac{1}{(2n)^{1/2}}(W_u + W_Y) \tag{1.7}$$

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where $W_u = \frac{1}{(2n)^{1/2}} \sum_{i}^{2n} u_i$ and $W_Y = \frac{1}{(2n)^{1/2}} \sum_{i}^{2n} (Y_i - \mu_Y)$ are of order $O_p(1)$.

Defining W_v, W_X, $W_{u^{(i)}}$, $W_{Y^{(i)}}$, $W_{v^{(i)}}$, $W_{X^{(i)}}$ ($i = 1, 2$) similarly as for W_u and W_Y, we have

$$\bar{x}_{2n} = \mu_X + \frac{1}{(2n)^{1/2}}(W_v + W_X) \quad (1.8)$$

$$\bar{y}_n^{(1)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(1)}} + W_{Y^{(1)}}) \quad (1.9)$$

$$\bar{x}_n^{(1)} = \mu_X + \frac{1}{n^{1/2}}(W_{v^{(1)}} + W_{X^{(1)}}) \quad (1.10)$$

$$\bar{y}_n^{(2)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(2)}} + W_{Y^{(2)}}) \quad (1.11)$$

$$\bar{x}_n^{(2)} = \mu_X + \frac{1}{n^{1/2}}(W_{v^{(2)}} + W_{X^{(2)}}) . \quad (1.12)$$

Expanding $f(z_3)$ in third order Taylor's series about the point $z_3 = 1$ and noting that $f(z_3 = 1) = 1$, we have

$$\bar{y}_f^{(3)} = \bar{y}_{2n}\left[f(1) + (z_3 - 1) f'(1) + \frac{1}{2!}(z_3 - 1)^2 f''(1) + \frac{1}{3!}(z_3 - 1)^3 f'''(z_3^*) \right],$$ (where $z_3{}^* = 1 + \theta(z_3 - 1)$, $0 < \theta < 1$)

$$= \left\{ \mu_Y + \frac{1}{(2n)^{1/2}}(W_u + W_Y) \right\} \left\{ 1 + \left(\frac{\bar{x}_{2n}}{\mu_X} - 1 \right) f'(1) \right.$$
$$\left. + \frac{1}{2!}\left(\frac{\bar{x}_{2n}}{\mu_X} - 1 \right)^2 f''(1) + \frac{1}{3!}\left(\frac{\bar{x}_{2n}}{\mu_X} - 1 \right)^3 f'''(z_3^*) \right.$$

$$= \left\{ \mu_Y + \frac{1}{(2n)^{1/2}}(W_u + W_Y) \right\} \left\{ 1 + \frac{(W_v + W_X)}{(2n)^{1/2} \mu_X} f'(1) \right\}$$
$$+ \frac{1}{2!}\left(\frac{(W_v + W_X)}{(2n)^{1/2} \mu_X} \right)^2 f''(1) + \frac{1}{3!}\left(\frac{(W_v + W_X)}{(2n)^{1/2} \mu_X} \right)^3 f'''(z_3^*)$$

$$= \left\{ \mu_Y + \frac{(W_u + W_Y)}{(2n)^{1/2}} \right\} \left\{ 1 + \frac{(W_v + W_X)}{(2n)^{1/2} \mu_X} f'(1) \right\} + \frac{1}{2!} \frac{(W_v^2 + W_X^2 + 2W_v W_X)}{(2n)\ \mu_X^2} f''(1)$$

$$+ \frac{1}{3!} \frac{(W_v^3 + W_X^3 + 3W_v W_X (W_v + W_X))}{(2n)^{3/2} \mu_X^3} f'''(z_3^*) \Bigg\}$$

$$= \mu_Y + \frac{1}{(2n)^{1/2}} \frac{\mu_Y}{\mu_X} (W_v + W_X) f'(1) + \frac{1}{2!(2n)} \frac{\mu_Y}{\mu_X^2} (W_v^2 + W_X^2 + 2W_v W_X) f''(1)$$

$$+ \frac{(W_u + W_Y)}{(2n)^{1/2}} + \frac{1}{(2n)\mu_X} (W_u W_v + W_u W_X + W_Y W_v + W_Y W_X) f'(1) + O_p\left(\frac{1}{n^{3/2}}\right). \quad (1.13)$$

Similarly

$$\bar{y}_f^{(1)} = \mu_Y + \frac{1}{n^{1/2}} \frac{\mu_Y}{\mu_X} (W_{v^{(1)}} + W_{X^{(1)}}) f'(1) + \frac{1}{2!n} \frac{\mu_Y}{\mu_X^2} (W_{v^{(1)}}^2 + W_{X^{(1)}}^2 + 2W_{v^{(1)}} W_{X^{(1)}}) f''(1)$$

$$+ \frac{(W_{u^{(1)}} + W_{Y^{(1)}})}{n^{1/2}} + \frac{1}{n\mu_X} (W_{u^{(1)}} W_{v^{(1)}} + W_{u^{(1)}} W_{X^{(1)}}$$

$$+ W_{Y^{(1)}} W_{v^{(1)}} + W_{Y^{(1)}} W_{X^{(1)}}) f'(1) + O_p\left(\frac{1}{n^{3/2}}\right) \quad (1.14)$$

and

$$\bar{y}_f^{(2)} = \mu_Y + \frac{1}{n^{1/2}} \frac{\mu_Y}{\mu_X} (W_{v^{(2)}} + W_{X^{(2)}}) f'(1) + \frac{1}{2!n} \frac{\mu_Y}{\mu_X^2} (W_{v^{(2)}}^2 + W_{X^{(2)}}^2 + 2W_{v^{(2)}} W_{X^{(2)}}) f''(1)$$

$$+ \frac{(W_{u^{(2)}} + W_{Y^{(2)}})}{n^{1/2}} + \frac{1}{n\mu_X} (W_{u^{(2)}} W_{v^{(2)}} + W_{u^{(2)}} W_{X^{(2)}}$$

$$+ W_{Y^{(2)}} W_{v^{(2)}} + W_{Y^{(2)}} W_{X^{(2)}}) f'(1) + O_p\left(\frac{1}{n^{3/2}}\right) \quad (1.15)$$

On the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for N to be large for simplicity, the Jack-knifed generalized estimator $\bar{y}_{gw}$ is

$$\bar{y}_{gw} = 2\bar{y}_f^{(3)} - \frac{1}{2}\left(y_f^{(1)} + y_f^{(2)} \right) \quad (1.16)$$

Substituting the values of $y_f^{(3)}$, $y_f^{(1)}$ and $y_f^{(2)}$ in (1.16) from (1.13) to (1.15), we have

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$$\bar{y}_{gw} - \mu_Y = \frac{2\mu_Y}{(2n)^{1/2}\mu_X}(W_v + W_X)f'(1) \qquad -\frac{1}{2}\left[\frac{1}{n^{1/2}}\frac{\mu_Y}{\mu_X}(W_{v^{(1)}} + W_{X^{(1)}})f'(1)\right.$$

$$\left. + \frac{1}{n^{1/2}}\frac{\mu_Y}{\mu_X}(W_{v^{(2)}} + W_{X^{(2)}})f'(1)\right]$$

$$+\frac{2\mu_Y}{2!(2n)\mu_X^2}(W_v^2 + W_X^2 + 2W_vW_X)f''(1)$$

$$-\frac{1}{2}\left[\frac{1}{2!n}\frac{\mu_Y}{\mu_X^2}(W_{v^{(1)}}^2 + W_{X^{(1)}}^2 + 2W_{v^{(1)}}W_{X^{(1)}})f''(1)\right.$$

$$\left.+\frac{\mu_Y}{2!n\mu_X^2}(W_{v^{(2)}}^2 + W_{X^{(2)}}^2 + 2W_{v^{(2)}}W_{X^{(2)}})f''(1)\right]$$

$$+\frac{2(W_u + W_Y)}{(2n)^{1/2}} - \frac{1}{2}\left[\frac{(W_{u^{(1)}} + W_{Y^{(1)}})}{n^{1/2}} + \frac{(W_{u^{(2)}} + W_{Y^{(2)}})}{n^{1/2}}\right]$$

$$+\frac{2}{(2n)\mu_X}(W_uW_v + W_uW_X + W_YW_v + W_YW_X)f'(1)$$

$$-\frac{1}{2}\left[\frac{1}{n\mu_X}(W_{u^{(1)}}W_{v^{(1)}} + W_{u^{(1)}}W_{X^{(1)}} + W_{Y^{(1)}}W_{v^{(1)}}\right.$$

$$+W_{Y^{(1)}}W_{X^{(1)}})f'(1) + \frac{1}{n\mu_X}(W_{u^{(2)}}W_{v^{(2)}} + W_{u^{(2)}}W_{X^{(2)}}$$

$$\left.+W_{Y^{(2)}}W_{v^{(2)}} + W_{Y^{(2)}}W_{X^{(2)}})f'(1)\right] \qquad (1.17)$$

2. BIAS AND MEAN SQUARE ERROR

Taking expectation on both sides of (1.17), the bias of $\bar{y}_{gw}$ up to terms of order $O(1/n)$, is

$$Bias(\bar{y}_{gw}) = \frac{\mu_Y}{(2n)\mu_X^2}(\sigma_v^2 + \sigma_X^2)f''(1) - \frac{1}{2}\frac{1}{2!}\frac{\mu_Y}{n\mu_X^2}[(\sigma_v^2 + \sigma_X^2)f''(1)$$

$$+(\sigma_v^2 + \sigma_X^2)f''(1)] + \frac{2}{(2n)\mu_X}\sigma_{XY}f'(1)$$

$$-\frac{1}{2n\mu_X}\left[\sigma_{XY}f'(1) + \sigma_{XY}f'(1)\right]$$

$$= 0. \qquad (2.1)$$

Squaring both sides of (1.17) and taking expectation, the mean square error of $\bar{y}_{gw}$ up to terms of order $O(1/n)$, is

$$MSE(\overline{y}_{gw}) = E\Bigg[\frac{4\mu_Y^2}{(2n)\mu_X^2}(W_v + W_X)^2\{f'(1)\}^2 + \frac{1}{4}\frac{\mu_Y^2}{n\mu_X^2}\{(W_{v^{(1)}} + W_{X^{(1)}})f'(1)$$

$$+(W_{v^{(2)}} + W_{X^{(2)}})f'(1)\}^2 + \frac{4(W_u + W_Y)^2}{(2n)} + \frac{1}{4n}\{(W_{u^{(1)}} + W_{Y^{(1)}}) + (W_{u^{(2)}} + W_{Y^{(2)}})\}^2$$

$$-\frac{2\mu_Y^2}{(2n)^{1/2}n^{1/2}\mu_X^2}\{(W_v + W_X)f'(1)\}.\{(W_{v^{(1)}} + W_{X^{(1)}})f'(1) + (W_{v^{(2)}} + W_{X^{(2)}})f'(1)\}$$

$$-\frac{2(W_u + W_Y)}{(2n)^{1/2}n^{1/2}}\frac{\mu_Y}{\mu_X}\{(W_{v^{(1)}} + W_{X^{(1)}})f'(1) + (W_{v^{(2)}} + W_{X^{(2)}})f'(1)\}$$

$$-\frac{2(W_u + W_Y)}{(2n)^{1/2}n^{1/2}}\Big\{ (W_{u^{(1)}} + W_{Y^{(1)}}) + (W_{u^{(2)}} + W_{Y^{(2)}}) \Big\} - \frac{2\mu_Y}{(2n)^{1/2}n^{1/2}\mu_X}(W_v + W_X)f'(1)$$

$$\{(W_{u^{(1)}} + W_{Y^{(1)}} + (W_{u^{(2)}} + W_{Y^{(2)}})\}$$

$$+\frac{8\mu_Y}{(2n)\mu_X}(W_v + W_X)(W_u + W_Y)f'(1) + \frac{1}{(2n)}\frac{\mu_Y}{\mu_X}\Big\{(W_{v^{(1)}} + W_{X^{(1)}})f'(1)$$

$$+(W_{v^{(2)}} + W_{X^{(2)}})f'(1)\Big\}\{(W_{u^{(1)}} + W_{Y^{(1)}}) + (W_{u^{(2)}} + W_{Y^{(2)}})\}\Bigg]$$

$$= \frac{4\mu_Y^2}{(2n)\mu_X^2}(\sigma_v^2 + \sigma_X^2)\{f'(1)\}^2 + \frac{2\mu_Y^2}{4n\mu_X^2}(\sigma_v^2 + \sigma_X^2)\{f'(1)\}^2$$

$$+\frac{4}{2n}(\sigma_u^2 + \sigma_Y^2) + \frac{2}{4n}(\sigma_u^2 + \sigma_Y^2) - \frac{2\mu_Y^2}{(2n)^{1/2}n^{1/2}\mu_X^2}\frac{2}{\sqrt{2}}(\sigma_v^2 + \sigma_X^2)\{f'(1)\}^2$$

$$-\frac{2}{(2n)^{1/2}n^{1/2}}\frac{2}{\sqrt{2}}(\sigma_u^2 + \sigma_Y^2) - \frac{2\mu_Y}{(2n)^{1/2}n^{1/2}\mu_X}\frac{2}{\sqrt{2}}\sigma_{XY}f'(1)$$

$$-\frac{2\mu_Y}{(2n)^{1/2}n^{1/2}\mu_X}\frac{2}{\sqrt{2}}\sigma_{XY}f'(1) + \frac{8\mu_Y}{(2n)\mu_X}\sigma_{XY}f'(1) + \frac{1}{(2n)}\frac{\mu_Y}{\mu_X}2\sigma_{XY}f'(1)$$

$$= \frac{\mu_Y^2}{2n\mu_X^2}(\sigma_v^2 + \sigma_X^2)\{f'(1)\}^2 + \frac{2\mu_Y}{2n\mu_X}\sigma_{XY}f'(1) + \frac{2}{4n}(\sigma_u^2 + \sigma_Y^2)$$

$$= \frac{\mu_Y^2}{2n}\left[\frac{\sigma_Y^2}{\mu_Y^2} + \frac{\sigma_X^2\{f'(1)\}^2}{\mu_X^2} + \frac{2\sigma_{XY}}{\mu_X\mu_Y}f'(1)\right]$$

$$+\frac{1}{2n}\left[\frac{\mu_Y^2}{\mu_X^2}\sigma_v^2\{f'(1)\}^2 + \sigma_u^2\right] \qquad (2.2)$$

3. CONCLUDING REMARKS

(a) We can easily see that bias and mean square error of $\overline{y}_f^{(3)}$ to the terms of order $O(1/n)$ are

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$$Bias(\bar{y}_f^{(3)}) = \frac{1}{2!(2n)}\frac{\mu_Y}{\mu_X^2}(\sigma_v^2+\sigma_X^2)f''(1) + \frac{1}{(2n)\mu_X}\sigma_{XY} \tag{3.1}$$

and
$$MSE(\bar{y}_f^{(3)}) = \frac{\mu_Y^2}{2n}\left[\frac{\sigma_Y^2}{\mu_Y^2} + \frac{\sigma_X^2\{f'(1)\}^2}{\mu_X^2} + \frac{2\sigma_{XY}}{\mu_X\mu_Y}f'(1)\right] + \frac{1}{2n}\left[\frac{\mu_Y^2}{\mu_X^2}\sigma_v^2\{f'(1)\}^2 + \sigma_u^2\right] \tag{3.2}$$

Further, from (2.1) and (2.2), the bias and mean square error of the Jack-knifed estimator $\bar{y}_{gw}$ are

$$Bias(\bar{y}_{gw}) = 0 \tag{3.3}$$

and

$$MSE(\bar{y}_{gw}) = \frac{\mu_Y^2}{2n}\left[\frac{\sigma_Y^2}{\mu_Y^2} + \frac{\sigma_X^2\{f'(1)\}^2}{\mu_X^2} + \frac{2\sigma_{XY}}{\mu_X\mu_Y}f'(1)\right] + \frac{1}{2n}\left[\frac{\mu_Y^2}{\mu_X^2}\sigma_v^2\{f'(1)\}^2 + \sigma_u^2\right]. \tag{3.4}$$

From (3.2) and (3.4), we see that both the estimators $\bar{y}_f^{(3)}$ and $\bar{y}_{gw}$ have the same mean square error but from (3.1) bias of $\bar{y}_f^{(3)}$ is not zero whereas from (3.3) bias of the Jack-knifed estimator $\bar{y}_{gw}$ is zero; hence, in the sense of bias($\bar{y}_{gw}$)=0 and both the estimators $\bar{y}_f^{(3)}$ and $\bar{y}_{gw}$ having the same mean square error to the order of our approximation, the Jack-knifed estimator $\bar{y}_{gw}$ may be preferred to the estimator $\bar{y}_f^{(3)}$ in the presence of measurement errors also.

(b) Unbiasedness property of the jack-knifed generalized estimator helps in finding unbiased or almost unbiased jack-knifed estimators in the special cases without increasing the mean square error. For example, as a special case of $\bar{y}_f^{(3)}$, the estimator by Manisha and Singh (2001) in the notations of this paper, is

$$\bar{y}_\theta^{(3)} = \theta\bar{y}_{2n}\frac{\mu_X}{\bar{x}_{2n}} + (1-\theta)\bar{y}_{2n} \qquad = \bar{y}_{2n}\left[\theta\left(\frac{\bar{x}_{2n}}{\mu_X}\right)^{-1} + (1-\theta)\right]$$

$$= \bar{y}_{2n}(\theta z_3^{-1} + 1 - \theta)\,. \tag{3.5}$$

with $f(z_3) = \theta z_3^{-1} + (1-\theta)$ having its value unity at the point $z_3 = 1$ satisfying the condition of the generalized class $\overline{y}_f^{(3)}$. Further, for the estimator $\overline{y}_\theta^{(3)}$ by Manisha and Singh (2001), $\overline{y}_f^{(1)}$ and $\overline{y}_f^{(2)}$ are respectively

$$\overline{y}_\theta^{(1)} = \overline{y}_n^{(1)}(\theta z_1^{-1} + 1 - \theta) \tag{3.6}$$

and $$\overline{y}_\theta^{(2)} = \overline{y}_n^{(2)}(\theta z_2^{-1} + 1 - \theta) \tag{3.7}$$

so that the jack-knifed estimator as a special case of $\overline{y}_{gw}$ corresponding to the estimator $\overline{y}_\theta^{(3)}$ is

$$\overline{y}_{\theta w} = 2\overline{y}_\theta^{(3)} - \frac{1}{2}\left(\overline{y}_\theta^{(1)} + \overline{y}_\theta^{(2)}\right) \tag{3.8}$$

with $$\text{Bias}\left(\overline{y}_{\theta w}\right) = 0. \tag{3.9}$$

From (3.5) to (3.7), we see that $f\left(z_3\right) = \theta z_3^{-1} + 1 - \theta$, $f\left(z_2\right) = \theta z_2^{-1} + 1 - \theta$ and $f\left(z_1\right) = \theta z_1^{-1} + 1 - \theta$ have the value of $f^{'}(1) = -\theta$, which when substituted in (2.2) gives the mean square error of $\overline{y}_{\theta w}$ as a special case of that of $\overline{y}_{gw}$ to be

$$MSE\left(\overline{y}_{\theta w}\right) = \frac{\mu_y^2}{2n}\left[\frac{\sigma_y^2}{\mu_y^2} + \theta^2\frac{\sigma_x^2}{\mu_x^2} - 2\theta\frac{\sigma_{xy}}{\mu_x\mu_y}\right] + \frac{1}{2n}\left[\theta^2\frac{\mu_y^2}{\mu_x^2}\sigma_v^2 + \sigma_u^2\right]$$

$$= \frac{\sigma_y^2}{2n}\left[1 - \theta\frac{C_x}{C_y}\left(2\rho - \theta\frac{C_x}{C_y}\right)\right] + \frac{1}{2n}\left[\theta^2\frac{\mu_y^2}{\mu_x^2}\sigma_v^2 + \sigma_u^2\right] \tag{3.10}$$

where $C_x = \sigma_x / \mu_x$ and $C_y = \sigma_y / \mu_y$.

From Manisha and Singh (2001), the bias and mean square error of $\overline{y}_\theta^{(3)}$ are respectively

$$Bias\left(\overline{y}_\theta^{(3)}\right) = \theta\left[\frac{\mu_y}{2n\mu_x^2}\left(\sigma_x^2 + \sigma_v^2\right) - \frac{1}{2n\mu_x}\rho\sigma_x\sigma_y\right] \tag{3.11}$$

and $$MSE\left(\overline{y}_\theta^{(3)}\right) = \frac{\sigma_y^2}{2n}\left[1 - \theta\frac{C_x}{C_y}\left(2\rho - \theta\frac{C_x}{C_y}\right)\right] + \frac{1}{2n}\left[\theta^2\frac{\mu_y^2}{\mu_x^2}\sigma_v^2 + \sigma_u^2\right]. \tag{3.12}$$

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From (3.9) to (3.12), in the light of the estimator $\overline{y}_{\theta}^{(3)}$ being biased and the jack-knifed estimator $\overline{y}_{\theta w}$ having bias to be zero, and both the estimators $\overline{y}_{\theta}^{(3)}$ and $\overline{y}_{\theta w}$ having the same mean square error, the jack-knifed estimator $\overline{y}_{\theta w}$ may be preferred to $\overline{y}_{\theta}^{(3)}$ of Manisha and Singh (2001). Similar remarks hold for other estimators of the class $\overline{y}_{f}^{(3)}$ or $\overline{y}_{f}$.

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