A JACK-KNIFED REGRESSION TYPE ESTIMATOR WITH KNOWN COEFFICIENT OF VARIANTION OF AUXILIARY VARIABLE IN THE PRESENCE OF MEASUREMENT ERRORS

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***Abstract:** Using known coefficient of variation of auxiliary variable, a linear regression type estimator of population mean is considered along with its Jack-knifed version in the presence of measurement errors. Bias and mean square error are found and the properties of the estimators are analyzed for the measurement errors ridden observations.*

***Keywords:** Coefficient of Variation, Regression Type Estimator, Jack-Knife Technique, Measurement Errors.*

1. INTRODUCTION

For a simple random sample of size m, let $\left(x_i, y_i\right)$ be the pair of values instead of the true values $\left(X_i, Y_i\right)$ on the two characteristics $\left(X, Y\right)$ respectively for the i^{th} $\left(i = 1, 2, \ldots, m\right)$ unit in the sample. Let the observational or measurement errors be

$$u_i = y_i - Y_i \tag{1.1}$$

$$v_i = x_i - X_i \tag{1.2}$$

which are stochastic in nature and are uncorrelated with mean zero and variances σ_u^2 and σ_v^2 respectively. Further, let the population means of $\left(X, Y\right)$ be (μ_X, μ_Y), population variances of $\left(X, Y\right)$ be (σ_X^2, σ_Y^2) respectively, σ_{XY} be the population covariance and ρ be the population correlation coefficient between X and Y.

We consider a simple random sample of size $2n$ and split this sample randomly into two sub samples each of size n. Using known coefficient of variation $\left(C_X = \dfrac{\sigma_X}{\mu_X}\right)$ of auxiliary variable X, we can make three regression type estimators $\overline{y}_{lc}^{(3)}$, $\overline{y}_{lc}^{(2)}$ and $\overline{y}_{lc}^{(1)}$ based on the entire sample of size $2n$, second sample of size n and the first sample of size n respectively. Now

$$\overline{y}_{lc}^{(3)} = \overline{y}_{2n}^{(3)} + b_c^{(3)}(\mu_X - \overline{x}_{2n}^{(3)}) \tag{1.3}$$

where $\bar{y}_{2n}^{(3)} = \frac{1}{2n}\sum_{i}^{2n} y_i$, $\bar{x}_{2n}^{(3)} = \frac{1}{2n}\sum_{i}^{2n} x_i$ and

$b_c^{(3)} = \frac{s_{xy_{2n}}^{(3)}}{\mu_X^2 C_X^2}$ for $s_{xy_{2n}}^{(3)} = \frac{1}{2n}\sum_{i=1}^{2n}(x_i - \bar{x}_{2n}^{(3)})(y_i - \bar{y}_{2n}^{(3)})$.

We have

$$\bar{y}_{2n}^{(3)} = \frac{1}{2n}\sum_{i}^{2n} y_i = \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n} y_i\right\}$$

$$= \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(y_i - Y_i + Y_i - \mu_Y + \mu_Y)\right\} = \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(y_i - Y_i)\right.$$

$$\left. + \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(Y_i - \mu_Y)\right\} + \mu_Y$$

$$= \mu_Y + \frac{1}{(2n)^{1/2}}(W_u + W_Y) \tag{1.4}$$

and similarly

$$\bar{x}_{2n}^{(3)} = \mu_X + \frac{1}{(2n)^{1/2}}(W_v + W_X) \tag{1.5}$$

where $W_u = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n} u_i$, $W_Y = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(Y_i - \mu_Y)$

$W_v = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n} v_i$, $W_X = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(X_i - \mu_X)$

which are of order $O_p(1)$.

Similarly $(\bar{x}_n^{(i)}, \bar{y}_n^{(i)})$ being sample means of (X, Y) respectively for the i^{th} $(i = 1,2)$ randomly split into the sub sample of size n , we have

$$\bar{y}_n^{(1)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(1)}} + W_{Y^{(1)}}) \tag{1.6}$$

$$\bar{x}_n^{(1)} = \mu_X + \frac{1}{n^{1/2}}(W_{v^{(1)}} + W_{X^{(1)}}) \tag{1.7}$$

$$\bar{y}_n^{(2)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(2)}} + W_{Y^{(2)}}) \tag{1.8}$$

$$\bar{x}_n^{(2)} = \mu_X + \frac{1}{n^{1/2}}(W_{v^{(2)}} + W_{X^{(2)}}) \tag{1.9}$$

where $W_{u^{(i)}}$, $W_{Y^{(i)}}$, $W_{v^{(i)}}$ *and* $W_{X^{(i)}}$, $i = 1,2$ are defined on parallel lines of W_u, W_Y, W_v and W_X respectively for the i^{th} sub sample of size n .

Further

$$s_{xy_{2n}}^{(3)} = \frac{1}{2n}\sum_{i=1}^{2n} x_i y_i - \bar{x}_{2n}^{(3)} \bar{y}_{2n}^{(3)}$$

$$= \frac{1}{2n}\sum_{i=1}^{2n}(X_i + v_i)(Y_i + u_i)$$

$$-\left\{\frac{1}{(2n)^{1/2}}(W_v + W_X) + \mu_X\right\}\left\{\frac{1}{(2n)^{1/2}}(W_u + W_Y) + \mu_Y\right\}$$

$$= \frac{1}{2n}\sum_{i=1}^{2n}(X_iY_i + v_iY_i + X_iu_i + u_iv_i) - \frac{1}{(2n)}(W_v + W_X)(W_u + W_Y)$$

$$-\frac{1}{(2n)^{1/2}}\mu_X(W_u + W_Y) - \frac{1}{(2n)^{1/2}}\mu_Y(W_v + W_X) - \mu_X\mu_Y$$

$$= \frac{1}{(2n)^{1/2}}\left\{\frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(X_iY_i - \sigma_{XY} - \mu_X\mu_Y)\right.$$

$$\left.+\frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}Y_iv_i + \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}X_iu_i + \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}u_iv_i\right\} + \sigma_{XY} + \mu_X\mu_Y$$

$$-\frac{1}{2n}(W_v + W_X)(W_u + W_Y) - \frac{1}{(2n)^{1/2}}\mu_X(W_u + W_Y)$$

$$-\frac{1}{(2n)^{1/2}}\mu_Y(W_v + W_X) - \mu_X\mu_Y$$

$$= \sigma_{XY} + \frac{1}{(2n)^{1/2}}W_{XY} + \frac{1}{(2n)^{1/2}}W_{Yv} + \frac{1}{(2n)^{1/2}}W_{Xu} + \frac{1}{(2n)^{1/2}}W_{uv}$$

$$-\frac{1}{(2n)}(W_X + W_v)(W_u + W_Y) - \frac{1}{(2n)^{1/2}}\mu_X(W_u + W_Y) - \frac{1}{(2n)^{1/2}}\mu_Y(W_v + W_X) \tag{1.10}$$

where

$$W_{XY} = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}(X_iY_i - \sigma_{XY} - \mu_X\mu_Y)$$

$$W_{Yv} = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}Y_iv_i$$

$$W_{Xu} = \frac{1}{(2n)^{1/2}}\sum_{i=1}^{2n}X_iu_i$$

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$$W_{uv} = \frac{1}{(2n)^{1/2}} \sum_{i=1}^{2n} u_i v_i \ .$$

Using (1.10) in $b_c^{(3)} = \frac{s_{xy_{2n}}^{(3)}}{\mu_X^2 C_X^2}$, we have

$$b_c^{(3)} = \frac{-\mu_X (W_u + W_Y) - \mu_Y (W_v + W_X) - \frac{1}{(2n)^{1/2}} (W_v + W_X)(W_u + W_Y)\}\Big]}{\mu_X^2 C_X^2}$$

$$= \frac{\sigma_{XY}}{\mu_X^2 C_X^2} \Bigg[1 + \frac{1}{\sigma_{XY} (2n)^{1/2}} \{ W_{XY} + W_{Yv} + W_{Xu} + W_{uv} - \mu_X (W_u + W_Y) \, .$$

$$-\mu_Y (W_v + W_X) - \frac{1}{(2n)^{1/2}} (W_v + W_X)(W_u + W_Y) \} \Bigg]$$

or $b_c^{(3)} (\mu_X - \overline{x}_{2n}^{(3)})$

$$= \frac{\sigma_{XY}}{\mu_X^2 C_X^2} \Bigg[1 + \frac{1}{\sigma_{XY} (2n)^{1/2}} \{ W_{XY} + W_{Yv} + W_{Xu} + W_{uv}$$

$$-\mu_X (W_u + W_Y) - \mu_Y (W_v + W_X) - \frac{1}{(2n)^{1/2}} (W_v + W_X) \, .$$

$$(W_u + W_Y) \} \Bigg] \Bigg[-\frac{1}{(2n)^{1/2}} (W_v + W_X) \Bigg]$$

$$= \frac{\sigma_{XY}}{\mu_X^2 C_X^2} \Bigg[-\frac{(W_v + W_X)}{(2n)^{1/2}} - \frac{1}{\sigma_{XY} (2n)} \{ W_v W_{XY} + W_v W_{Yv}$$

$$+W_v W_{Xu} + W_v W_{uv} - \mu_X W_v W_u - \mu_X W_v W_Y - \mu_Y W_v^2 - \mu_Y W_X^2$$

$$-2\mu_Y W_v W_X - \frac{1}{(2n)^{1/2}} W_v^2 W_u - \frac{1}{(2n)^{1/2}} W_v^2 W_Y - \frac{2}{(2n)^{1/2}} W_v W_X W_u$$

$$-\frac{2}{(2n)^{1/2}} W_v W_X W_Y + W_X W_{XY} + W_X W_{Yv} + W_X W_{Xu} + W_X W_{uv}$$

$$-\mu_X W_X W_u - \mu_X W_X W_Y - \frac{1}{(2n)^{1/2}} W_X^2 W_u - \frac{1}{(2n)^{1/2}} W_X^2 W_Y \Bigg\} \Bigg] \qquad (1.11)$$

Substituting the values from (1.4) and (1.11) in (1.3), we have

$$\overline{y}_{lc}^{(3)} = \mu_Y + \frac{1}{(2n)^{1/2}} (W_u + W_Y)$$

$$+\frac{\sigma_{XY}}{\mu_X^2 C_X^2} \Bigg[-\frac{(W_v + W_X)}{(2n)^{1/2}} - \frac{1}{\sigma_{XY} (2n)} \{ W_v W_{XY} + W_v W_{Yv} + W_v W_{Xu} + W_v W_{uv}$$

$$-\mu_X W_v W_u - \mu_X W_v W_Y - \mu_Y W_v^2 - \mu_Y W_X^2 - 2\mu_Y W_v W_X - \frac{1}{(2n)^{1/2}} W_v^2 W_u$$
$$-\frac{1}{(2n)^{1/2}} W_v^2 W_Y - \frac{2}{(2n)^{1/2}} W_v W_X W_u - \frac{2}{(2n)^{1/2}} W_v W_X W_Y + W_X W_{XY}$$
$$\left. \left. + W_X W_{Yv} + W_X W_{Xu} + W_X W_{uv} - \mu_X W_X W_u - \mu_X W_X W_Y - \frac{1}{(2n)^{1/2}} W_X^2 W_u - \frac{1}{(2n)^{1/2}} W_X^2 W_Y \right\} \right] \quad (1.12)$$

Defining $W_{XY^{(i)}}$, $W_{Yv^{(i)}}$, $W_{Xu^{(i)}}$ and $W_{uv^{(i)}}$ for the $i^{th}\left(i=1,2\right)$ sub sample of size n on the similar lines of W_{XY} , W_{Yv} , W_{Xu} and W_{uv} respectively, we can easily show that

$$\bar{y}_{lc}^{(1)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(1)}} + W_{Y^{(1)}})$$
$$+\frac{\sigma_{XY}}{\mu_X^2 C_X^2}\left[-\frac{(W_{v^{(1)}} + W_{X^{(1)}})}{n^{1/2}} - \frac{1}{n\sigma_{XY}}\left\{W_{v^{(1)}} W_{XY^{(1)}} + W_{v^{(1)}} W_{Yv^{(1)}}\right.\right.$$
$$+W_{v^{(1)}} W_{Xu^{(1)}} + W_{v^{(1)}} W_{uv^{(1)}} - \mu_X W_{v^{(1)}} W_{u^{(1)}} - \mu_X W_{v^{(1)}} W_{Y^{(1)}} - \mu_Y W_{v^{(1)}}^2$$
$$-\mu_Y W_{X^{(1)}}^2 - 2\mu_Y W_{v^{(1)}} W_{X^{(1)}} - \frac{1}{n^{1/2}} W_{v^{(1)}}^2 W_{u^{(1)}} - \frac{1}{n^{1/2}} W_{v^{(1)}}^2 W_{Y^{(1)}}$$
$$-\frac{2}{n^{1/2}} W_{v^{(1)}} W_{X^{(1)}} W_{u^{(1)}} - \frac{2}{n^{1/2}} W_{v^{(1)}} W_{X^{(1)}} W_{Y^{(1)}} + W_{X^{(1)}} W_{XY^{(1)}}$$
$$+W_{X^{(1)}} W_{Yv^{(1)}} + W_{X^{(1)}} W_{Xu^{(1)}} + W_{X^{(1)}} W_{uv^{(1)}} - \mu_X W_{X^{(1)}} W_{u^{(1)}} - \mu_X W_{X^{(1)}} W_{Y^{(1)}}$$
$$\left.\left. -\frac{1}{n^{1/2}} W_{X^{(1)}}^2 W_{u^{(1)}} - \frac{1}{n^{1/2}} W_{X^{(1)}}^2 W_{Y^{(1)}} \right\}\right] \quad (1.13)$$

and

$$\bar{y}_{lc}^{(2)} = \mu_Y + \frac{1}{n^{1/2}}(W_{u^{(2)}} + W_{Y^{(2)}})$$
$$+\frac{\sigma_{XY}}{\mu_X^2 C_X^2}\left[-\frac{(W_{v^{(2)}} + W_{X^{(2)}})}{n^{1/2}} - \frac{1}{n.\sigma_{XY}}\left\{W_{v^{(2)}} W_{XY^{(2)}} + W_{v^{(2)}} W_{Yv^{(2)}} + W_{v^{(2)}} W_{Xu^{(2)}}\right.\right.$$
$$+W_{v^{(2)}} W_{uv^{(2)}} - \mu_X W_{v^{(2)}} W_{u^{(2)}} - \mu_X W_{v^{(2)}} W_{Y^{(2)}} - \mu_Y W_{v^{(2)}}^2 - \mu_Y W_{X^{(2)}}^2$$
$$-2\mu_Y W_{v^{(2)}} W_{X^{(2)}} - \frac{1}{n^{1/2}} W_{v^{(2)}}^2 W_{u^{(2)}} - \frac{1}{n^{1/2}} W_{v^{(2)}}^2 W_{Y^{(2)}} - \frac{2}{n^{1/2}} W_{v^{(2)}} W_{X^{(2)}} W_{u^{(2)}}$$
$$-\frac{2}{n^{1/2}} W_{v^{(2)}} W_{X^{(2)}} W_{Y^{(2)}} + W_{X^{(2)}} W_{XY^{(2)}} + W_{X^{(2)}} W_{Yv^{(2)}} + W_{X^{(2)}} W_{Xu^{(2)}}$$

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$$+W_{X^{(2)}}W_{uv^{(2)}} - \mu_X W_{X^{(2)}}W_{u^{(2)}} - \mu_X W_{X^{(2)}}W_{Y^{(2)}} - \frac{1}{n^{1/2}}W^2_{X^{(2)}}W_{u^{(2)}} - \frac{1}{n^{1/2}}W^2_{X^{(2)}}W_{Y^{(2)}}\Big\}\Big]$$

(1.14)

We now consider the Jack-knifed estimator $\bar{y}_{lc}$ given by

$$\bar{y}_{lc} = 2\bar{y}^{(3)}_{lc} - \frac{1}{2}(\bar{y}^{(1)}_{lc} + \bar{y}^{(2)}_{lc}) \tag{1.15}$$

on the lines of Sukhatme and Sukhatme (1997), chapter IV.

2. BIAS AND MEAN SQUARE ERROR

Substituting the values from (1.12), (1.13) and (1.14) in (1.15), we have

$$\bar{y}_{lc} - \mu_Y = \frac{2(W_u + W_Y)}{(2n)^{1/2}} - \frac{1}{2n^{1/2}}\left\{(W_{u^{(1)}} + W_{Y^{(1)}}) + (W_{u^{(2)}} + W_{Y^{(2)}})\right\} - \frac{2\sigma_{XY}(W_v + W_X)}{\mu_X^2 C_X^2 (2n)^{1/2}}$$

$$+\frac{\sigma_{XY}}{2n^{1/2}\mu_X^2 C_X^2}\left\{(W_{v^{(1)}} + W_{X^{(1)}}) + (W_{v^{(2)}} + W_{X^{(2)}})\right\} - \frac{2(W_v W_{XY})}{(2n)\mu_X^2 C_X^2}$$

$$-\frac{2(W_v W_{Yv})}{(2n)\mu_X^2 C_X^2} + \frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{v^{(1)}}W_{Yv^{(1)}}) + (W_{v^{(2)}}W_{Yv^{(2)}})\right\} + \frac{1}{2n\mu_X^2 C_X^2}$$

$$-\frac{2(W_v W_{Xu})}{(2n)\mu_X^2 C_X^2} + \frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{v^{(1)}}W_{Xu^{(1)}}) + (W_{v^{(2)}}W_{Xu^{(2)}})\right\}$$

$$\left\{(W_{v^{(1)}}W_{XY^{(1)}}) + (W_{v^{(2)}}W_{XY^{(2)}})\right\} - \frac{2(W_v W_{uv})}{(2n)\mu_X^2 C_X^2} + \frac{1}{2n\mu_X^2 C_X^2}$$

$$\left\{(W_{V^{(1)}}W_{uv^{(1)}}) + (W_{v^{(2)}}W_{uv^{(2)}})\right\} + \frac{2\mu_X(W_v W_u)}{(2n)\mu_X^2 C_X^2} - \frac{\mu_X}{2n\mu_X^2 C_X^2}$$

$$+\frac{2\mu_X(W_v W_Y)}{(2n)\mu_X^2 C_X^2} - \frac{\mu_X}{2n\mu_X^2 C_X^2}\left\{(W_{v^{(1)}}W_{Y^{(1)}}) + (W_{v^{(2)}}W_{Y^{(2)}})\right\}$$

$$\left\{(W_{V^{(1)}}W_{u^{(1)}}) + (W_{v^{(2)}}W_{u^{(2)}})\right\} + \frac{2\mu_Y(W_v^2)}{(2n)\mu_X^2 C_X^2} - \frac{\mu_Y}{2n\mu_X^2 C_X^2}\left\{(W^2_{v^{(1)}}) + (W^2_{v^{(2)}})\right\}$$

$$+\frac{4\mu_Y(W_X W_v)}{(2n)\mu_X^2 C_X^2} - \frac{\mu_Y}{n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{v^{(1)}}) + (W_{X^{(2)}}W_{v^{(2)}})\right\}$$

$$+\frac{2(W_v^2 W_u)}{(2n)^{1/2}\mu_X^2 C_X^2} - \frac{1}{2n^{1/2}\mu_X^2 C_X^2}\left\{(W^2_{v^{(1)}}W_{u^{(1)}}) + (W^2_{v^{(2)}}W_{u^{(2)}})\right\}$$

$$+\frac{2(W_v^2 W_Y)}{(2n)^{1/2}\mu_X^2 C_X^2} - \frac{1}{2n^{1/2}\mu_X^2 C_X^2}\left\{(W^2_{v^{(1)}}W_{Y^{(1)}}) + (W^2_{v^{(2)}}W_{Y^{(2)}})\right\}$$

$$+\frac{4(W_v W_X W_u)}{(2n)^{1/2}\mu_X^2 C_X^2} - \frac{1}{n^{1/2}\mu_X^2 C_X^2}\left\{(W_{v^{(1)}}W_{X^{(1)}}W_{u^{(1)}}) + (W_{v^{(2)}}W_{X^{(2)}}W_{u^{(2)}})\right\}$$

$$+\frac{4(W_v W_X W_Y)}{(2n)^{1/2}\mu_X^2 C_X^2}-\frac{1}{n^{1/2}\mu_X^2 C_X^2}\left\{(W_{v^{(1)}}W_{X^{(1)}}W_{Y^{(1)}})+(W_{v^{(2)}}W_{X^{(2)}}W_{Y^{(2)}})\right\}$$
$$-\frac{2(W_X W_{XY})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{XY^{(1)}})+(W_{X^{(2)}}W_{XY^{(2)}})\right\}$$
$$-\frac{2(W_X W_{Yv})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{Yv^{(1)}})+(W_{X^{(2)}}W_{Yv^{(2)}})\right\}$$

$$-\frac{2(W_X W_{Xu})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{Xu^{(1)}})+(W_{X^{(2)}}W_{Xu^{(2)}})\right\}$$
$$-\frac{2(W_X W_{uv})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{uv^{(1)}})+(W_{X^{(2)}}W_{uv^{(2)}})\right\}$$
$$+\frac{2\mu_X(W_X W_u)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_X}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{u^{(1)}})+(W_{X^{(2)}}W_{u^{(2)}})\right\}$$
$$+\frac{2\mu_X(W_X W_Y)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_X}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}W_{Y^{(1)}})+(W_{X^{(2)}}W_{Y^{(2)}})\right\}$$
$$+\frac{2\mu_Y(W_X^2)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_Y}{2n\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}^2)+(W_{X^{(2)}}^2)\right\}$$
$$+\frac{2(W_X^2 W_u)}{(2n)^{1/2}\mu_X^2 C_X^2}-\frac{1}{2n^{1/2}\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}^2 W_{u^{(1)}})+(W_{X^{(2)}}^2 W_{u^{(2)}})\right\}$$
$$+\frac{2(W_X^2 W_Y)}{(2n)^{1/2}\mu_X^2 C_X^2}-\frac{1}{2n^{1/2}\mu_X^2 C_X^2}\left\{(W_{X^{(1)}}^2 W_{Y^{(1)}})+(W_{X^{(2)}}^2 W_{Y^{(2)}})\right\}. \quad (2.1)$$

Taking expectation on both sides of (2.1), we have

$$Bias(\bar{y}_{lc})=-\frac{2E(W_v W_{Yv})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{E(W_{v^{(1)}}W_{Yv^{(1)}})+E(W_{v^{(2)}}W_{Yv^{(2)}})\right\}$$
$$+\frac{2\mu_Y E(W_v^2)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_Y}{2n\mu_X^2 C_X^2}\left\{E(W_{v^{(1)}}^2)+E(W_{v^{(2)}}^2)\right\}$$
$$-\frac{2E(W_X W_{XY})}{(2n)\mu_X^2 C_X^2}+\frac{1}{2n\mu_X^2 C_X^2}\left\{E(W_{X^{(1)}}W_{XY^{(1)}})+E(W_{X^{(2)}}W_{XY^{(2)}})\right\}$$
$$+\frac{2\mu_X E(W_X W_Y)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_X}{2n\mu_X^2 C_X^2}\left\{E(W_{X^{(1)}}W_{Y^{(1)}})+E(W_{X^{(2)}}W_{Y^{(2)}})\right\}$$
$$+\frac{2\mu_Y E(W_X^2)}{(2n)\mu_X^2 C_X^2}-\frac{\mu_Y}{2n\mu_X^2 C_X^2}\left\{E(W_{X^{(1)}}^2)+E(W_{X^{(2)}}^2)\right\}$$
$$+\frac{2E(W_X^2 W_Y)}{(2n)^{1/2}\mu_X^2 C_X^2}-\frac{1}{2n^{1/2}\mu_X^2 C_X^2}\left\{E(W_{X^{(1)}}^2 W_{Y^{(1)}})+E(W_{X^{(2)}}^2 W_{Y^{(2)}})\right\}$$

$= 0$ in view of the following results for $i = 1, 2$

$E(W_v W_{Yv}) = E(W_{v^{(i)}} W_{Yv^{(i)}})$, $E(W_v^2) = E(W_{v^{(i)}}^2)$

$E(W_X W_{XY}) = E(W_{X^{(i)}} W_{XY^{(i)}})$, $E(W_X W_Y) = E(W_{X^{(i)}} W_{Y^{(i)}})$,

$E(W_X^2) = E(W_{X^{(i)}}^2)$, $E(W_X^2 W_Y) = E(W_{X^{(i)}}^2 W_{Y^{(i)}})$,

$E(W_u) = E(W_{u^{(i)}}) = 0$, $E(W_v) = E(W_{v^{(i)}}) = 0$,

$E(W_X) = E(W_{X^{(i)}}) = 0$, $E(W_Y) = E(W_{Y^{(i)}}) = 0$,

$E(W_v.W_{XY}) = E(W_{v^{(i)}} W_{XY^{(i)}}) = 0$, $E(W_v.W_{Xu}) = E(W_{v^{(i)}} W_{Xu^{(i)}}) = 0$,

$E(W_v.W_{uv}) = E(W_{v^{(i)}} W_{uv^{(i)}}) = 0$, $E(W_v.W_{uv}) = E(W_{v^{(i)}} W_{uv^{(i)}}) = 0$,

$E(W_v.W_Y) = E(W_{v^{(i)}} W_{Y^{(i)}}) = 0$, $E(W_v.W_X) = E(W_{v^{(i)}} W_{X^{(i)}}) = 0$,

$E(W_X.W_{Yv}) = E(W_{X^{(i)}} W_{Yv^{(i)}}) = 0$, $E(W_X.W_{Xu}) = E(W_{X^{(i)}} W_{Xu^{(i)}}) = 0$,

$E(W_X.W_{uv}) = E(W_{X^{(i)}} W_{uv^{(i)}}) = 0$, $E(W_X.W_u) = E(W_{X^{(i)}} W_{u^{(i)}}) = 0$,

$E(W_v^2 W_Y) = E(W_{v^{(i)}}^2 W_{Y^{(i)}}) = 0$,

$E(W_X W_u W_v) = E(W_{X^{(i)}} W_{u^{(i)}} W_{v^{(i)}}) = 0$,

$E(W_X^2 W_u) = E(W_{X^{(i)}}^2 W_{u^{(i)}}) = 0$,

$E(W_X W_v W_Y) = E(W_{X^{(i)}} W_{v^{(i)}} W_{Y^{(i)}}) = 0$,

$E(W_v^2 W_u) = E(W_{v^{(i)}}^2 W_{u^{(i)}}) = 0$.

Squaring both sides of (2.1) and taking expectation, the mean square error of $\bar{y}_{lc}$ up to order $O(1/n)$ is

$$MSE(\bar{y}_{lc}) = E\left[\frac{4(W_u + W_Y)^2}{2n} + \frac{1}{4n}\{(W_{u^{(1)}} + W_{Y^{(1)}})^2\right.$$
$$+(W_{u^{(2)}} + W_{Y^{(2)}})^2 + 2(W_{u^{(1)}} + W_{Y^{(1)}})(W_{u^{(2)}} + W_{Y^{(2)}})\}$$

$$+\frac{4\sigma_{XY}^2(W_v+W_X)^2}{(2n)\mu_X^4C_X^4}+\frac{1}{4n}\frac{\sigma_{XY}^2}{\mu_X^4C_X^4}\{(W_{v^{(1)}}+W_{X^{(1)}})^2$$
$$+(W_{v^{(2)}}+W_{X^{(2)}})^2+2(W_{v^{(1)}}+W_{X^{(1)}})(W_{v^{(2)}}+W_{X^{(2)}})\}$$
$$-\frac{2(W_u+W_Y)}{(2n)^{1/2}n^{1/2}}\{(W_{u^{(1)}}+W_{Y^{(1)}})+(W_{u^{(2)}}+W_{Y^{(2)}})\}$$
$$+\frac{2\sigma_{XY}(W_v+W_X)}{\mu_X^2C_X^2(2n)^{1/2}n^{1/2}}\{(W_{u^{(1)}}+W_{Y^{(1)}})+(W_{u^{(2)}}+W_{Y^{(2)}})\}$$
$$-\frac{2\sigma_{XY}^2(W_v+W_X)}{\mu_X^4C_X^4(2n)^{1/2}n^{1/2}}\{(W_{v^{(1)}}+W_{X^{(1)}})+(W_{v^{(2)}}+W_{X^{(2)}})\}$$
$$+\frac{2\sigma_{XY}(W_u+W_Y)}{\mu_X^2C_X^2(2n)^{1/2}n^{1/2}}\{(W_{v^{(1)}}+W_{X^{(1)}})+(W_{v^{(2)}}+W_{X^{(2)}})\}$$
$$-\frac{8\sigma_{XY}(W_u+W_Y)(W_v+W_X)}{(2n)\mu_X^2C_X^2}-\frac{1}{2n}\frac{\sigma_{XY}}{\mu_X^2C_X^2}\{(W_{u^{(1)}}+W_{Y^{(1)}})$$
$$\left.+(W_{u^{(2)}}+W_{Y^{(2)}})\}\{(W_{v^{(1)}}+W_{X^{(1)}})+(W_{v^{(2)}}+W_{X^{(2)}})\}\right]$$

Solving this expression, we get

$$MSE(\bar{y}_{lc})=\frac{1}{2n}\left\{\sigma_u^2+\sigma_Y^2+\frac{\sigma_{XY}^2}{\mu_X^4C_X^4}(\sigma_v^2+\sigma_X^2)-\frac{2\sigma_{XY}^2}{\mu_X^2C_X^2}\right\}$$
$$=\frac{\sigma_Y^2}{2n}(1-\rho^2)+\frac{1}{2n}\left\{\sigma_u^2+\frac{\rho^2\sigma_Y^2\sigma_v^2}{\sigma_X^2}\right\}$$
$$=M_1+M_2 \qquad (2.2)$$

where $M_1=\frac{\sigma_Y^2}{2n}(1-\rho^2)$

is the mean square error without measurement error

and $M_2=\frac{1}{2n}\left\{\sigma_u^2+\frac{\rho^2\sigma_Y^2\sigma_v^2}{\sigma_X^2}\right\}$

is the contribution of measurement error to the mean square error.

3. CONCLUDING REMARKS

a. We see from the last section that the jack-knifed regression type estimator $\bar{y}_{lc}$ is unbiased estimator of population mean μ_Y of the study variable Y whereas from Maneesha and Singh (2002), the usual linear regression estimator

$$\bar{y}_{lr} = \bar{y} + b(\mu_X - \bar{x}) \qquad (3.1)$$

where $\bar{y} = \frac{1}{2n}\sum_{i=1}^{2n} y_i$, $\bar{x} = \frac{1}{2n}\sum_{i=1}^{2n} x_i$, $b = \frac{s_{yx}}{s_x^2}$, $s_{yx} = \frac{1}{2n}\sum_{i}^{2n}(y_i - \bar{y})(x_i - \bar{x})$,

$s_x^2 = \frac{1}{2n}\sum_{i}^{2n}(x_i - \bar{x})^2$

in the presence of measurement errors, is a biased estimator of population mean μ_Y having its mean square error to the first degree approximation, to be

$$MSE(\bar{y}_{lr}) = \frac{\sigma_Y^2}{2n}(1-\rho^2) + \frac{1}{2n}\left(\sigma_u^2 + \frac{\rho^2\sigma_Y^2\sigma_v^2}{\sigma_X^2}\right) \qquad (3.2)$$

$$= MSE(\bar{y}_{lc}) \text{ \{from (2.2)\}}. \qquad (3.3)$$

Thus, in the light of the linear regression estimator $\bar{y}_{lr}$ being biased and the jack-knifed regression type estimator $\bar{y}_{lc}$ being unbiased, and both the estimators $\bar{y}_{lr}$ and $\bar{y}_{lc}$ having the same mean square error given by (3.2) and (3.3), the jack-knifed regression type estimator $\bar{y}_{lc}$ based on coefficient of variation of auxiliary variable X may be preferred to the usual linear regression estimator $\bar{y}_{lr}$ in the presence of observational or measurement errors.

b. We know that the mean square error of the mean per unit estimator $\bar{y}$ in the presence of measurement errors is

$$MSE(\bar{y}) = \frac{\sigma_Y^2}{2n} + \frac{\sigma_u^2}{2n} \qquad (3.4)$$

and from (2.2), the mean square error of the jack-knifed regression type estimator using coefficient of variation of auxiliary variable X, to the first degree of approximation, is

$$MSE(\bar{y}_{lc}) = \frac{\sigma_Y^2}{2n} + \frac{\sigma_u^2}{2n} + \frac{\rho^2\sigma_Y^2}{2n}\left(\frac{\sigma_v^2}{\sigma_X^2} - 1\right) \qquad (3.5)$$

Comparing (3.4) with (3.5), we see that $\bar{y}_{lc}$ is more efficient than $\bar{y}$ in the sense of having lesser mean square error if

$$\frac{\sigma_v^2}{\sigma_X^2} - 1 < 0 \text{ or } \frac{\sigma_v^2}{\sigma_X^2} < 1 \tag{3.6}$$

showing that as long as variation in the measurement error variable v is less than the variation in the auxiliary variable X, $\bar{y}_{lc}$ is more efficient than $\bar{y}$; otherwise $\bar{y}_{lc}$ becomes less efficient than $\bar{y}$. From (3.3) to (3.5), it may be mentioned here again that the same efficiency condition (3.6) for $\bar{y}_{lc}$ holds true for $\bar{y}_{lr}$ also, but $\bar{y}_{lr}$ being biased estimator is not preferable to the unbiased estimator $\bar{y}_{lc}$.

4. REFERENCES

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