A DETERMINISTIC INVENTORY MODEL OF DETERIORATING ITEMS WITH TWO RATES OF PRODUCTION, CONSTANT DEMAND AND SHORTAGES

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Abstract: In this paper, a deterministic inventory model is developed for deteriorating items with a continuous production control when shortages are observed. It is assumed that the production rate is changed to another at a time when the inventory level reaches a prefixed level I_1 and continued until the inventory level reaches the level $S(> I_1)$. The production is started again at a time when the shortage level reaches a prefixed quantity I_2. Two different constant production rates are taken as the multiple of constant demand rate. Finally the total cost per unit time as a function of I_1 , I_2 and S is derived and the optimal decision rules for I_1 , I_2 and S are compared. The solution procedure, numerical examples and sensitivity analysis are presented.

Keywords: Demand, Deterioration, Economic Order Quantity, Inventory, Shortage.

1. INTRODUCTION

In real world problems, several factors play important roles on controlling inventory for deteriorating items. Over the last two decades much more work has been done for it. The effect of deterioration is very important in many inventory models. Deterioration is defined as damage or decay or depletion or spoilage such that the item cannot be used for its original purpose. Most of the physical goods undergo decay or deterioration over time. Commodities such as food stuffs, vegetables, fruits, etc., suffer from depletion by direct storage while kept in store. Food grain, electronics goods, radioactive substances, photographic films, etc., deteriorate through a gradual loss of potential or utility with time. Highly volatile substances such as alcohol, gasoline, turpentine etc. undergo physical depletion with time through the process of evaporation.

The earliest work in this direction is due to Whitin [1] who considered deterioration of fashion goods at the end of a prescribed storage period. Ghare and Schrader [2] developed a simple EOQ (Economic Order Quantity) model with constant rate of decay. Shah and Jaiswal [3] considered an order level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal [4] reconsidered this model by rectifying the error in the work of Shah and Jaiswal [3] in calculating the average inventory holding cost. In all these models, the demand rate and the deterioration rate were constants, the replenishment rate was infinite and no shortage in inventory was allowed. Dave and Patel [5] then reconsidered an inventory model for deteriorating items with time-proportional demand when shortages were prohibited. Sachan [6] further extended the model to allow for shortages. Dave [7] developed another model with variable instantaneous demand, discrete opportunities for replenishment and shortages. Dave and Patel [5], Bahari-Kasani [8], Chung and Ting [9] etc., considered the inventory models for deteriorating items with linearly trended demand and no shortage. Goswami and Chaudhuri [10] developed an EOQ model for deteriorating items with shortages and linear trend in demand. In all these models, the deterioration rate is taken as a constant. However, a new class of inventory models has been developed to show the product in stock deteriorates with time. Covert and Philip [11] the model for variable rate of deterioration assuming a two parameters Weibull distribution. Mishra [12] first developed the production lot size model in which both constant and variable rates of deterioration are considered. Silver [13], Ritchie [14], Deb and Chaudhuri [15], Goel and Aggarwal [16], Hariga and Benkherouf [17] and Jalan et al. [18] developed inventory models with deteriorating items and time dependent demand rates. Goswami and Chaudhuri [19] developed the inventory model where demand rate, production rate and deterioration rate were all taken as time dependent. Nahmias [20] and Raafat [21] have given the detailed information regarding inventory model of deteriorating items in their review articles. Jalan and Chaudhury [22] considered an order level inventory model for deteriorating items without shortages. Ouyang et al. [23] considered the continuous production inventory system with partial backorders. Perumal and Arivarignan [24], Samanta and Roy [25] analyzed the inventory model with two rates of production and shortages.

In the present paper, we have developed a continuous production control inventory model for deteriorating items with shortages in which two different rates of production are taken as multiple of demand rate. It is assumed that production started at one rate and after sometimes it may be switched over to another rate in which second rate of production is greater than that of first and both are multiple of constant demand rate. As a result, a large quantum stock of manufactured items at the initial stage is avoided by starting at a low rate of production and holding cost is reduced. We shall show that this inventory policy taking two rates of production as multiple of a constant demand will lead a better result, i. e., a higher profit. Numerical examples are used to illustrate the solution procedure and the developed models. Sensitivity analysis is carried out to identify the most sensitive parameters in the system.

2. ASSUMPTIONS AND NOTATIONS

The following notations and assumptions are used throughout this paper to develop the proposed model.

(i) D is the constant demand rate .

(ii) $kD(> D)$ and $\gamma D(> kD)$ are the constant production rates started at time 0 and at time $t_1(> 0)$ where k and γ are two arbitrary constants. (Here the first and second production is taken a constant $(= k)$ times and $(= \gamma)$ times multiple of demand rate respectively).

(iii) C_1 is the holding cost per unit time per unit time .

(iv) C_2 is the shortage cost per unit time per unit time.

(v) C_3 is the cost of a deteriorated unit.

(C_1 , C_2 and C_3 are known constants)

(vi) C is the average cost of the system .
(vii) $I(t)$ is the inventory level at time t (≥ 0) .
(viii) Replenishment is instantaneous and lead time is zero.
(ix) T is the fixed duration of a production cycle .
(x) Shortages are allowed and backlogged.
(xi) The distribution of the time of deterioration of an item follows the exponential distribution $g(t)$where $g(t) = \begin{cases} \theta e^{-\theta t}, & for\ t > 0 \\ 0 & , otherwise \end{cases}$,

where θ is called the deterioration rate ; a constant fraction θ $(0 < \theta < 1)$ of the on-hand inventory deteriorates per unit time . It is assumed that no repair or replacement of the deteriorated items takes places during a given cycle.

Here we consider a single commodity deterministic continuous production inventory model with a constant demand rate. The production of the item is started at time 0 , at a rate kD$(> D)$ .Once the inventory level reaches I_1, the rate of production is switched over to γD$(> kD)$ and the production is stopped when the inventory level reaches S $(> I_1)$ and the inventory is depleted at a constant rate D . It is decided to backlogged demands upto I_2 which occur during stockout time. Thus the inventory level reaches '$-I_2$' (backorder level is I_2),the production is started at a (faster) rate γD so as to clear the backlog and when the inventory level reaches 0 (i,e, the backlog is cleared), the next production cycle starts at the (lower) rate kD .

We denote by $[0, t_1]$, the duration of the production at the rate kD,by $[t_1, t_2]$,the duration of the production at the rate γD ,by $[t_2, t_1]$,the duration when there is no production but only consumption by the demand rate D , by $[t_3, t_4]$, the duration of shortage , and by $[t_4, T]$,the duration of time to backlog at the rate γD. The cycle then repeats itself after time T.

This model is represented by the following diagram:

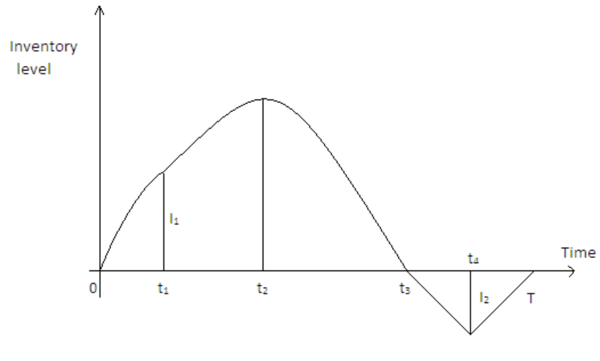

Fig-1. A single commodity deterministic continuous production inventory model.

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3. MATHEMATICAL FORMULATION AND ITS ANALYSIS

Let us consider be the on hand inventory at time $t\,(0 \leq t \leq T)$. Now the two productions are taken as constant multiple of demand rate D, that is, kD and γD where k and γ are positive constants and D is the constant demand rate. Then the differential equations describing the instantaneous states of $I(t)$ in the interval $[0, T\,]$ are given by the following:

$$\frac{dI(t)}{dt} + \theta I(t) = (k-1)D, \qquad 0 \leq t \leq t_1\,, \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = (\gamma-1)D, \qquad t_1 \leq t \leq t_2\,, \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \qquad t_2 \leq t \leq t_3\,, \qquad (3)$$

$$\frac{dI(t)}{dt} = -D, \qquad t_3 \leq t \leq t_4\,, \qquad 4)$$

$$\frac{dI(t)}{dt} = -(\gamma-1)D, \qquad t_4 \leq t \leq T\,. \qquad 5)$$

The boundary conditions are:

$$I(0) = 0,\ I(t_1) = I_1,\ I(t_2) = S,\ I(t_3) = 0,\ I(t_4) = -I_2 \text{ and } I(T) = 0. \qquad (6)$$

The solutions of the equations from (1) to (5) are given by

$$I(t) = \frac{(k-1)D}{\theta}\left(1 - e^{-\theta t}\right), \qquad 0 \leq t \leq t_1, \qquad (7)$$

$$I(t) = \frac{(\gamma-1)D}{\theta} + \left(I_1 - \frac{(\gamma-1)D}{\theta}\right) e^{-\theta(t-t_1)}, \quad t_1 \leq t \leq t_2\,, \qquad (8)$$

$$I(t) = -\frac{D}{\theta} + \left(S + \frac{D}{\theta}\right) e^{-\theta(t-t_2)}, \qquad t_2 \leq t \leq t_3\,, \qquad 9)$$

$$I(t) = -D(t - t_3), \qquad t_3 \leq t \leq t_4\,, \qquad 10)$$

$$I(t) = -I_2 + (\gamma-1)D(t - t_4). \qquad t_4 \leq t \leq T\,. \qquad (11)$$

From equations (6) and (7), we have

$$I_1 = \frac{(k-1)D}{\theta}\left(1 - e^{-\theta t_1}\right),$$

$\therefore\ t_1 = \frac{1}{\theta}\ln\left[1 + \frac{\theta I_1}{(k-1)D} + \frac{\theta^2 {I_1}^2}{(k-1)^2 D^2}\right]$, (neglecting higher powers of θ, $0 < \theta << 1$) (12) From equations (6) and (8), we have

$$S = \frac{(\gamma-1)D}{\theta} + \left[I_1 - \frac{(\gamma-1)D}{\theta}\right] e^{-\theta(t-t_1)},$$

$\therefore\ e^{\theta(t_2-t_1)} = \left[1-\frac{\theta S}{(\gamma-1)D}\right]^{-1}\left[1-\frac{\theta I_1}{(\gamma-1)D}\right].$ (13a)

$= 1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}.$ (13b)

$\therefore\ t_2 = t_1+\frac{1}{\theta}\ln\left[1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right].$ (14)

$$=\frac{1}{\theta}\ln\left[1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}\right]\left[1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right].$$

[Using (12)] (15)

From equations (6) and (9), we have

$\frac{D}{\theta}=\left(S+\frac{D}{\theta}\right)e^{-\theta(t_3-t_2)},$

$\therefore\ t_3 = t_2+\frac{1}{\theta}\ln\left(1+\frac{\theta S}{D}\right),$ (16)

$$=\frac{1}{\theta}\ln\left(1+\frac{\theta S}{D}\right)\left[1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}\right]\left[1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right].$$

[Using (15)] (17)

From equations (6) and (10), we have

$I_2 = D(t_4-t_3),$ (18)

$\therefore\ t_4 = \frac{Q_2}{D}+\frac{1}{\theta}\ln\left(1+\frac{\theta S}{D}\right)\left[1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}\right]\left[1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right],$

[Using (17)] (19)

From equations (6) and (11), we have

$I_2 = (\gamma-1)D(T-t_4).$ (20)

Therefore, the total number of deteriorated items (TDI) in $[0, T]$,

TDI= $[(k-1)Dt_1+(\gamma-1)D(t_2-t_1)-S]+[S-D(t_3-t_2)]$

$$=\frac{(k-1)D}{\theta}\ln\left[1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}\right]+\frac{(\gamma-1)D}{\theta}\ln\left[1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right]-\frac{D}{\theta}\ln\left(1+\frac{\theta S}{D}\right)$$

$$=\frac{(k-1)D}{\theta}\left[\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}-\frac{\theta^2 {I_1}^2}{2(k-1)^2D^2}\right]$$

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$$+\frac{(\gamma-1)D}{\theta}\left[\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}-\frac{\theta^2(S-I_1)^2}{2(\gamma-1)^2D^2}\right]-\frac{D}{\theta}\ln\left(\frac{\theta S}{D}-\frac{\theta^2S^2}{2D^2}\right)$$

(neglecting higher powers of θ, $0 < \theta << 1$)

$$=\frac{\theta}{2D}\left[\frac{1}{(\gamma-1)}\left(\frac{\gamma-k}{k-1}{I_1}^2+\gamma S^2\right)\right]. \tag{21}$$

The deterioration cost (DC) over the period $[0,T]$,

DC= $C_3 \times$ (WI)

$$=\frac{\theta C_3}{2(\gamma-1)D}\left(\frac{(\gamma-k)}{(k-1)}{I_1}^2+\gamma S^2\right). \tag{22}$$

Shortage cost (SC) over the period $[0,T]$

$$SC=-C_2\left[\int_{t_3}^{t_4} I(t)\,dt+\int_{t_4}^{T} I(t)\,dt\right]$$

$$=-C_2\left[\int_{t_3}^{t_4}[-D(t-t_3)]dt+\int_{t_4}^{T}[-I_2+(\gamma-1)D(t-t_4)]dt\right]$$

$$=\frac{\gamma C_2 {I_2}^2}{2(\gamma-1)D}. \tag{23}$$

The inventory carrying cost (CC) over the period $[0,T]$

$$CC=C_1\left[\int_0^{t_1} I(t)\,dt+\int_{t_1}^{t_2} I(t)\,dt+\int_{t_2}^{t_3} I(t)\,dt\right]. \tag{24}$$

Now, $\int_0^{t_1} I(t)\,dt=\int_0^{t_1}\left[\frac{(k-1)D}{\theta}\left(1-e^{-\theta t}\right)\right]dt$ [Using (7)]

$$=\frac{(k-1)D}{\theta}\left(t_1+\frac{1}{\theta}e^{-\theta t_1}-\frac{1}{\theta}\right)$$

$=\frac{(k-1)D}{\theta}\left(\frac{1}{2}\theta {t_1}^2-\frac{1}{6}\theta^2{t_1}^3\right)$ (neglecting higher powers of θ, $0 < \theta << 1$)

$$=\frac{1}{2}(k-1)D\begin{bmatrix}\left[\frac{1}{\theta}\ln\left(1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2{I_1}^2}{(k-1)^2D^2}\right)\right]^2\\-\frac{\theta}{3}\left[\frac{1}{\theta}\ln\left(1+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2{I_1}^2}{(k-1)^2D^2}\right)\right]^3\end{bmatrix} \quad \text{[Using (12)]}$$

$$=\frac{1}{2}(k-1)D\left[\left[\frac{1}{\theta}\left(\frac{\theta I_1}{(k-1)D}+\frac{\theta^2{I_1}^2}{2(k-1)^2D^2}\right)\right]^2-\frac{\theta}{3}\left[\frac{1}{\theta}\left(\frac{\theta I_1}{(k-1)D}+\frac{1}{2}\frac{\theta^2{I_1}^2}{(k-1)^2D^2}\right)\right]^3\right]$$

(neglecting higher powers of θ, $0 < \theta << 1$)

$= \frac{1}{2}\frac{{I_1}^2}{(k-1)D} + \frac{1}{3}\frac{\theta {I_1}^3}{(k-1)^2D^2}$. (neglecting higher powers of θ, $0 < \theta << 1$) (25)

$\int_{t_1}^{t_2} I(t)\,dt = \int_{t_1}^{t_2}\left[\frac{(\gamma-1)D}{\theta} + \left(I_1 - \frac{(\gamma-1)D}{\theta}\right)e^{-\theta(t-t_1)}\right]dt$ [Using (8)]

$$= \frac{(\gamma-1)D}{\theta}(t_2 - t_1) - \frac{1}{\theta}\left(I_1 - \frac{(\gamma-1)D}{\theta}\right)\left(e^{-\theta(t_2-t_1)} - 1\right)$$

$$= \frac{(\gamma-1)D}{\theta^2}\ln\left[1 + \frac{\theta(S-I_1)}{(\gamma-1)D} + \frac{\theta^2S(S-I_1)}{(\gamma-1)^2D^2}\right] - \frac{1}{\theta}\left(I_1 - \frac{(\gamma-1)D}{\theta}\right)\left[\frac{1-\frac{\theta S}{(\gamma-1)D}}{1-\frac{\theta I_1}{(\gamma-1)D}} - 1\right]$$

[Using (13a)]

$$= \frac{(\gamma-1)D}{\theta^2}\left[\frac{\theta(S-I_1)}{(\gamma-1)D} + \frac{\theta^2S(S-I_1)}{(\gamma-1)^2D^2} - \frac{\theta^2(S-I_1)^2}{2(\gamma-1)^2D^2}\right] - \frac{S-I_1}{\theta}$$

(neglecting higher powers of θ, $0 < \theta << 1$)

$$= \frac{1}{2}\frac{(S^2-{I_1}^2)}{(\gamma-1)D}. \quad (26)$$

$\int_{t_2}^{t_3} I(t)\,dt = \int_{t_2}^{t_3}\left[-\frac{D}{\theta} + \left(S + \frac{D}{\theta}\right)e^{-\theta(t-t_2)}\right]dt$ [Using (9)]

$$= -\frac{D}{\theta}(t_3 - t_2) - \frac{1}{\theta}\left(S + \frac{D}{\theta}\right)\left[e^{-\theta(t_3-t_2)} - 1\right]$$

$$= -\frac{D}{\theta^2}\ln\left(1 + \frac{\theta S}{D}\right) - \frac{1}{\theta}\left(S + \frac{D}{\theta}\right)\left[\left(1 + \frac{\theta S}{D}\right)^{-1} - 1\right] \quad \text{[Using (16)]}$$

$$= -\frac{D}{\theta^2}\left(\frac{\theta S}{D} - \frac{\theta^2S^2}{2D^2}\right) - \frac{1}{\theta}\left(S + \frac{D}{\theta}\right)\left(-\frac{\theta S}{D} + \frac{\theta^2S^2}{D^2}\right)$$

(neglecting higher powers of θ, $0 < \theta << 1$)

$$= \frac{S^2}{D}\left(\frac{1}{2} - \frac{\theta S}{D}\right). \quad (27)$$

Therefore, the inventory carrying cost over the cycle $[0, T]$,

$$CC = C_1\left[\frac{{I_1}^2}{2(k-1)D} + \frac{\theta {I_1}^3}{3(k-1)^2D^2} + \frac{(S^2-{I_1}^2)}{2(\gamma-1)D} + \frac{S^2}{D}\left(\frac{1}{2} - \frac{\theta S}{D}\right)\right].$$

[Using (25) (26) & (27)] (28)

From (20), we have

$I_2 = (\gamma - 1)D(T - t_4),$

$= (\gamma - 1)D[(T - t_3) - (t_4 - t_3)]$

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$$= (\gamma-1)D\left[T - \frac{1}{\theta}\ln\left(1+\frac{S\theta}{D}\right)\left(1+\frac{\theta Q_1}{(k-1)D}+\frac{\theta^2 {Q_1}^2}{(k-1)^2D^2}\right)\left(1+\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}\right)-\frac{I_2}{D}\right].$$

[Using (17) and (18)]

$$= (\gamma-1)D\left[T-\frac{1}{\theta}\begin{bmatrix}\frac{S\theta}{D}-\frac{S^2\theta^2}{2D^2}+\frac{\theta I_1}{(k-1)D}+\frac{\theta^2 {I_1}^2}{(k-1)^2D^2}-\frac{\theta^2 {I_1}^2}{2(k-1)^2D^2}\\ +\frac{\theta(S-I_1)}{(\gamma-1)D}+\frac{\theta^2 S(S-I_1)}{(\gamma-1)^2D^2}-\frac{\theta^2(S-I_1)^2}{2(\gamma-1)^2D^2}\end{bmatrix}-\frac{I_2}{D}\right].$$

(neglecting higher powers of θ, $0 < \theta << 1$)

$$= \frac{(\gamma-1)D}{\gamma}\left[T-\frac{S}{D}\left(1-\frac{\theta S}{2D}\right)-\frac{Q_1}{(k-1)D}\left(1+\frac{\theta I_1}{2(k-1)D}\right)-\frac{(S-I_1)}{(\gamma-1)D}\left(1+\frac{\theta(S+I_1)}{2(\gamma-1)D}\right)\right]. \quad (29)$$

The average cost of the system (using (22), (23) & (28)), we have

$$C(I_1,S)=\frac{1}{T}\begin{bmatrix}\frac{\theta C_3}{2D}\left[\frac{1}{\gamma-1}\left(\frac{\gamma-k}{k-1}{I_1}^2+\gamma S^2\right)\right]+\frac{\gamma C_2 {I_2}^2}{2(\gamma-1)D}\\ +C_1\left[\frac{{I_1}^2}{2(k-1)D}+\frac{\theta {I_1}^3}{3(k-1)^2D^2}+\frac{(S^2-{I_1}^2)}{2(\gamma-1)D}+\frac{S^2}{D}\left(\frac{1}{2}-\frac{\theta S}{D}\right)\right]\end{bmatrix}$$

$$=\frac{1}{DT}\begin{bmatrix}\frac{1}{2(\gamma-1)}\left[\theta C_3\left(\frac{\gamma-k}{k-1}I_1^2+\gamma S^2\right)+\gamma C_2 I_2^2\right]\\ +C_1\left[\frac{{I_1}^2}{(k-1)}\left(\frac{1}{2}+\frac{\theta I_1}{3(k-1)D}\right)+\frac{(S^2-{I_1}^2)}{2(\gamma-1)}+S^2\left(\frac{1}{2}-\frac{\theta S}{D}\right)\right]\end{bmatrix} \quad (30)$$

where I_2 is given by (29).

The necessary conditions for $C(I_1,S)$ to be minimum are

$$\frac{\partial C(I_1,S)}{\partial I_1}=\frac{\gamma-k}{(k-1)(\gamma-1)DT}\left[\theta C_3 I_1 + C_1 I_1\left(1+\frac{\theta(\gamma-1)I_1}{(k-1)(\gamma-k)D}\right)-C_2 I_2\left(1+\frac{\theta(k+\gamma-2)I_1}{(k-1)(\gamma-1)D}\right)\right]=0. \quad (31)$$

$$\frac{\partial C(I_1,S)}{\partial S}=\frac{\gamma}{(\gamma-1)DT}\left[\theta C_3 S + C_1 S\left(1-\frac{3\theta(\gamma-1)S}{\gamma D}\right)-C_2 I_2\left(1+\frac{\theta I_1}{(\gamma-1)\gamma D}+\frac{\theta(\gamma-1)S}{\gamma D}\right)\right]=0. \quad (32)$$

Solving equations (31) and (32) and using (29) we get the optimal value ${I_1}^*$, ${I_2}^*$ and S^* of I_1 , I_2 and S respectively which minimizes $C(I_1,S)$ provided they satisfy the sufficient conditions:

$$\frac{\partial^2 C}{\partial {I_1}^2} = \frac{\gamma-k}{DT(k-1)(\gamma-1)}\left[\begin{array}{c} C_1\left(1+\frac{2\theta I_1}{(k-1)D}\right) \\ +C_2\left[\frac{k-\gamma}{\gamma(k-1)}\left(1+\frac{\theta I_1(k+\gamma-2)}{D(k-1)(\gamma-1)}\right)^2 + I_2\left(1+\frac{\theta(k+\gamma-2)}{D(k-1)(\gamma-1)}\right)\right] + \theta C_3 \end{array}\right] > 0.$$

(33) and

$$\left(\frac{\partial^2 C}{\partial {I_1}^2}\right)\left(\frac{\partial^2 C}{\partial S^2}\right) - \left(\frac{\partial^2 C}{\partial I_1\,\partial S}\right)^2 = \frac{\gamma}{D^2T^2(\gamma-1)^2}\left[\frac{(\gamma-k)}{(k-1)}A - \gamma B\right] > 0$$

where $A = C_1\left(1+\frac{2\theta I_1}{(k-1)D}\right) + C_2\left[\frac{k-\gamma}{\gamma(k-1)}\left(1+\frac{\theta I_1(k+\gamma-2)}{D(k-1)(\gamma-1)}\right)^2 + \frac{I_2\theta(k+\gamma-2)}{D(k-1)(\gamma-1)}\right] + C_3\theta$

and $B = C_2^2\left[\left[\frac{(k-\gamma)}{\gamma(k-1)}\left(1+\frac{\theta I_1}{\gamma(\gamma-1)D}+\frac{(\gamma-1)\theta S}{\gamma D}\right)\left(1+\frac{\theta I_1(k+\gamma-2)}{(k-1)(\gamma-1)D}\right)+\frac{\theta I_2}{(\gamma-1)D}\right]^2\right].$ (34)

If the solutions obtained from equations (31) and (32) do not satisfy the sufficient conditions (33) and (34), we may conclude that no feasible solution will be optimal for the set of parameter values taken to solve equations (31) and (32). Such a situation will imply that the parameter values are inconsistent and there is some error in there estimation.

4. NUMERICAL EXAMPLE

Equations (31) and (32) are now solved numerically (for different values of) with the help of a computer using the following values of the parameters:

$C_1 = 1.0$, $C_2 = 2.0$, $C_3 = 25.0$, $D = 3.0$, $\gamma = 3.0$, $k = 2.0$ and $T = 20.0$.

The optimal solutions are found to be (using (29))

Table 1.

θ	${I_1}^*$	S^*	${I_2}^*$	C^*
0.006	19.5767	22.1725	11.3561	11.8475
0.007	19.2788	22.2222	11.4228	12.0008
0.008	18.9970	22.2700	11.4877	12.1536
0.009	18.7299	22.3162	11.5509	12.3058
0.010	18.4760	22.3609	11.6126	12.4575

It is checked that these results satisfy the sufficient conditions (33) and (34) for minimizing C.

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5. SENSITIVITY ANALYSIS

From Table 1, it is clear that as θ increases, ${I_1}^*$decreases while S^*, ${I_2}^*$ and C^* increase. It is also seen that S^* is slightly sensitive, C^* and ${I_2}^*$ are moderately sensitive while ${I_1}^*$ is highly sensitive to changes in the value of .

We now study the effects of changes in the values of the system parameters c_1 , c_2 , c_3 , D, k, γ and T on the optimal minimal total cost. The sensitivity analysis is performed by changing the parameters in increasing order taking one parameter at a time and keeping the remaining parameters unchanged. The analysis is based on above Example at $\theta = 0.006$, then the following points are observed from Table 2.

i. I_1^* and S^* decrease while I_2^* and C^* increase with the increase in value of the parameter c_1. All I_1^* , S^*, I_2^* and C^* are moderately sensitive to changes in c_1.
ii. I_1^* , $S^*$and C^* increase while I_2^* decrease with increase in the value of the parameter c_2. I_1^* , $S^*$and C^* are moderately sensitive while I_2^* is highly sensitive to changes c_2.
iii. I_1^* and S^*decrease while I_2^* and C^* increase with increase in the value of the parameter c_3. I_1^* , and C^* are less sensitive while S^* and I_2^* are moderately sensitive with increase in the value of the parameter c_3.
iv. All I_1^* , S^*, I_2^* and C^* increase with increase in the value of the parameter D. I_1^* , and C^* are highly sensitive while S^* and I_2^* are moderately sensitive with increase in the value of the parameter D.
v. All I_1^* , S^*, I_2^* and C^* increase with increase in the value of the parameter k. I_1^* , and C^* are moderately sensitive while S^* and I_2^* are less sensitive with increase in the value of the parameter k.
vi. All I_1^* , S^*, I_2^* and C^* increase with increase in the value of the parameter γ. All I_1^* , C^*, S^* and I_2^* are less sensitive with increase in the value of the parameter γ.
vii. All I_1^* , S^*, I_2^* and C^* increase with increase in the value of the parameter T. I_1^* , and C^* are moderately sensitive while S^* and I_2^* are less sensitive with increase in the value of the parameter T.

Table -2 ($\theta = 0.006$)

Parameter changing	Change in value	I_1^*	S^*	I_2^*	C^*	Change in C^* %)
c_1	1.0	19.5767	22.1725	11.3561	11.8475	0
	2.0	15.8049	17.5864	17.175	17.6493	+48.9707
	3.0	13.2533	14.5104	21.0893	21.4869	+81.3623
	4.0	11.4019	12.3283	23.8821	24.2072	+104.323
	5.0	9.99863	10.7072	25.9673	26.2339	+121.43
c_2	2.0	19.5767	22.1725	11.3561	11.8475	0
	4.0	22.503	26.0503	6.53482	13.8541	+16.9369
	6.0	23.6581	24.6335	4.58242	14.6775	+23.8869
	8.0	24.2751	28.4922	3.52736	15.1255	+27.6683
	10.0	24.6588	29.0309	2.86687	15.4072	+30.046
c_3	25.0	19.5767	22.1725	11.3561	11.8475	0

	30.0	19.4921	21.9711	11.5829	12.0711	+1.88732
	35.0	19.4053	21.7743	11.806	12.2912	+3.74509
	40.0	19.3166	21.5818	12.0257	12.5078	+5.57333
	45.0	19.2262	21.3935	12.2418	12.7209	+7.37202
D	3.0	19.5767	22.1725	11.3561	11.8475	0
	6.0	39.1533	44.345	22.7123	23.695	+100
	9.0	58.73	66.5175	34.0684	35.5424	+199.999
	12.0	78.3066	88.69	45.4245	47.3899	+299.999
	15.0	97.8833	110.863	56.7802	59.2374	+399.999
k	2.0	19.5767	22.1725	11.3561	11.8475	0
	4.0	26.9413	29.1223	14.3624	15.39	+29.9008
	6.0	27.4868	30.9602	15.1222	16.3438	+37.9515
	8.0	27.944	31.8114	15.4674	16.7873	+41.6949
	10.0	28.2264	32.3014	15.6645	17.0434	+43.8565
γ	2.0	19.5767	22.1725	11.3561	11.8475	0
	4.0	20.8986	24.2251	11.9626	12.5869	+6.24098
	6.0	21.1843	24.8897	12.1163	12.7864	+7.92488
	8.0	21.3093	25.2219	12.1859	12.8793	+8.70901
	10.0	21.3793	25.4216	12.2253	12.9329	+9.16143
T	20.0	19.5767	22.1725	11.3561	11.8475	0
	25.0	24.0222	28.1392	13.9608	14.7451	+24.4575
	30.0	28.2769	34.301	16.4619	17.6168	+48.6963
	35.0	32.3324	40.6739	18.8531	20.4624	+72.7149
	40.0	36.1784	47.2764	21.1272	23.2818	+96.5123

6. CONCLUSION

In this paper, a continuous production inventory model for deteriorating items with shortages is considered in which two different rates of production which are multiple of constant demand rate are taken. It is assumed that production started at one rate and after sometimes it may be switched over to another rate in which second rate of production is greater than that of first and both are multiple of constant demand rate. As a result, a large quantum stock of manufactured items at the initial stage is avoided by starting at a low rate of production and holding cost is reduced. The variation in two production rate leads a better result of consumer satisfaction. As a result higher profit is gained. The proposed model can further be extended in several ways. For example, we may add pricing strategy into consideration. Also, we could extend the deterministic model into stochastic model. Finally, we could generalize the model into variable deterioration instead of constant deterioration.

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