GENERALIZED HAHN – BANCH THEOREM IN GENERALIZD 2 – NORMED SPACES

Dr. A. Singadurai[1], S. Meera Sudharsana Revathi[2]

Abstract: In this paper we prove the Generalized Hahn Banach theorem in the context of generalized complex 2- normed spaces.

Keywords: Generalized 2-norm, bounded 2-linear operator, Hahn – Banach Theorem.

1. INTRODUCTION

The concept of 2-normed spaces was introduced and studied by Siegfried Gahler, in 1984. Z.Lewandowska introduced the generalization of Gahler 2-normed space in [7]. In [10] the Hahn Banach Theorem was studied by Z.Lewandowska, Mohammad Sal Moslehian, Assie Saadatpour Moghaddam. They have taken only real 2- normed space X and study the real extension of a real 2-linear functional in a subspace M_0 of X. The generalized 2-normed space was studied by Sh. Rezapour, I.Kupka [12]. Here we attempt to study Generalized Hahn Banach Theorem for the complex extension of a complex 2-linear functional on a subspace M_0 of X.

2. PRELIMINARIES

Definition 2.1 Let X and Y be the complex linear spaces. Denote D a non-empty subset of $X \times Y$ such that for every $x \in X, y \in Y$ the sets $D_x = \{y \in Y; (x,y) \in D\}$ and $D^y = \{x \in X; (x,y) \in D\}$ are linear subspaces of the spaces Y and X, respectively.

A function $\|.,.\|: D \longrightarrow [0,\infty)$ will be called a generalized 2-norm on D if it satisfies the following conditions.

(i)$\|x,\ \alpha y\| = |\alpha|.\|x,y\| = \|\alpha x,\ y\|$ for any scalar α and all $(x,y) \in D$.

(ii)$\|x, y+z\| \leq \|x,y\| + \|x,z\|\ for\ x \in X,\ y,z \in Y$ with $(x,y),(x,z) \in D$.

(iii)$\|x+y, z\| \leq \|x,z\| + \|y,z\|\ for\ x,y \in X, z \in Y$ with $(x,z),(y,z) \in D$.

The set D is called a 2-normed set. In particular, if $D = X \times Y$, the function $\|.,.\|$ is said to be a generalized 2-norm on $X \times Y$ and the pair $(X \times Y, \|.,.\|)$ is called a generalized complex 2-normed space.

If $X = Y$, then the generalized complex 2-normed space $(X \times Y, \|.,.\|)$ is denoted by $(X, \|.,.\|)$. In the case that $X = Y$, $D = D^{-1}$, where $D^{-1} = \{(y,x): (x,y) \in D\}$ and $\|x,y\| = \|y,x\|$ for all $(x,y) \in D$, we call $\|.,.\|$ a generalized symmetric 2-

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norm and D a symmetric 2-normed set, unless, otherwise stated, a Generalized 2-normed space $(X, \|.,.\|)$ we mean the complex 2-normed space.

Example 2.2 Let X be a complex linear space having two seminorms $\|.\|_1$ and $\|.\|_2$. Then $(X, \|.,.\|)$ is a generalized 2-normed space with 2-norm defined by

$\|x, y\| = \|x\|_1 . \|y\|_2 ; x, y \in X.$

Example 2.3 Let X be a complex inner product space. Then X is a symmetric generalized 2-normed space under the 2-norm $\|x, y\| = |< x, y >|$ for all $x, y \in X$.

Example 2.4[10] Suppose that *s* be the linear space of all sequences of real numbers. Put $\|x, y\| = \sum_{n=1}^{\infty} |x_n||y_n|$, where $x = \{x_n\}, y = \{y_n\} \in s$. Then $D = \{(x, y) \in s \times s : \|x, y\| < \infty\}$ is a symmetric 2-normed set and the function $\|.,.\|: D \longrightarrow [0, \infty)$ is a generalized symmetric 2-norm on D.

In [10] Z.Lewandowska introduced bounded 2-linear operator. In similar way here we introduce bounded 2-linear operator on a generalized complex 2-linear space.

Definition 2.5 Let X be complex linear space, $D \subseteq X \times X$ be a 2-normed set, Y a normed space. An operator $F: D \longrightarrow Y$ is said to be 2-linear if it satisfies the following conditions.

(i) $F(a + c, b + d) = F(a, b) + F(a, d) + F(c, b) + F(c, d)$ for all $a, b, c, d \in X$ such that $a, c \in D^b \cap D^d$.
(ii) $F(\alpha a, \beta b) = \alpha . \beta F(a, b)$ for all $\alpha, \beta \in \mathbb{C}$ and for all $(a, b) \in D$.

Definition: 2.6 Let X be a complex linear space, $D \subseteq X \times X$ be a 2-normed set, Y a normed space. A 2-linear operator $F: D \longrightarrow Y$ is said to be bounded if there is a positive number K such that $\|F(a, b)\| \leq K\|a, b\|; (a, b) \in D$.Then the number $\|F\| = \inf \{K > 0: \|F(a, b)\| \leq K. \|a, b\|; (a, b) \in D\}$ is called the norm of the 2 – linear operator F.

Example 2.7 Consider $(X, \|.,.\|)$ in the example 2.3 and define $F: X \times X \longrightarrow \mathbb{R}$ by $F(x, y) =< x, y >$. Then F is a bounded 2-linear operator and $\|F\| = 1$.

3. MAIN RESULTS

Theorem 3.1 Let $(X, \|.,.\|)$ be a generalized 2- normed linear space and M be a linear subspace of $X \times X$. If (a_0, b_0) is a vector not in M_0 and if $M_0 = M + \{(a_0, b_0)\}$ is the linear subspace spanned by M and (a_0, b_0). If F is a real bounded 2- linear functional, there is an extension F_0 of F on M_0 such that $\|F\| = \|F_0\|$.

Proof: Case (i) Suppose $(X,\|.,.\|)$ is a generalized real 2- normed linear space.

Let $F: M \longrightarrow \mathbb{R}$ is a real bounded 2- linear functional.

Without loss of generality assume that, $\|F\| = 1$.

Now, $M_0 = M + \{(a_0, b_0)\} = \{(a,b) + \left(\alpha a_{0\,,}\, \beta b_0\right) : \alpha, \beta \in \mathbb{R}\}$

$= \{(a + \alpha a_0\,,\; b + \beta b_0) : \alpha, \beta \in \mathbb{R}\}$

Define F_0as, $F_0(a + \alpha a_0\,,\; b + \beta b_0) = F(a,b) + \alpha.\beta.r_0$, r_0 is real.

To prove: $F_0$is 2- linear.

Let $(a_1 + \alpha_1 a_0\,,\; b_1 + \beta_1 b_0), (a_2 + \alpha_2 a_0\,,\; b_2 + \beta_2 b_0) \in M_0$ and $x_1 = a_1 + \alpha_1 a_0\,, x_2 = a_2 + \alpha_2 a_0,$

$y_1 = b_1 + \beta_1 b_0\,, y_2 = b_2 + \beta_2 b_0 \in X.$

(i) $F_0(x_1 + x_2, y_1 + y_2) = F_0((a_1 + a_2) + (\alpha_1 + \alpha_2)a_0\,, (b_1 + b_2) + (\beta_1 + \beta_2)b_0)$

$= F(a_1 + a_2, b_1 + b_2) + (\alpha_1 + \alpha_2).(\beta_1 + \beta_2).r_0$

$= F(a_1, b_1) + F(a_2, b_1) + F(a_1, b_2) + F(a_2, b_2) + \alpha_1\beta_1 r_0 + \alpha_2\beta_1 r_0 + \alpha_1\beta_2 r_0 + \alpha_2\beta_2 r_0$

$= [F(a_1, b_1) + \alpha_1\beta_1 r_0] + [F(a_2, b_1) + \alpha_2\beta_1 r_0] + [F(a_1, b_2) + \alpha_1\beta_2 r_0] + [F(a_2, b_2) + \alpha_2\beta_2 r_0]$

$= F_0(a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0) + F_0(a_1 + \alpha_1 a_0, b_2 + \beta_2 b_0) + F_0(a_2 + \alpha_2 a_0, b_1 + \beta_1 b_0)$

$+F_0(a_2 + \alpha_2 a_0, b_2 + \beta_2 b_0)$

$= F_0(x_1, y_1) + F_0(x_1, y_2) + F_0(x_2, y_1) + F_0(x_2, y_2).$

(ii) Let $k_1, k_2 \in \mathbb{R}$ and let $x_1 = a_1 + \alpha_1 a_0, y_1 = b_1 + \beta_1 b_0$.

$$F_0(k_1 x_1, k_2 y_1) = F_0(k_1(a_1 + \alpha_1 a_0), k_2(b_1 + \beta_1 b_0))$$
$$= F_0(k_1 a_1 + k_1\alpha_1 a_0, k_2 b_1 + k_2\beta_1 b_0)$$

$= F(k_1 a_1, k_2 b_1) + (k_1\alpha_1)(k_2\beta_1)\, r_0 = k_1 k_2 F(a_1, b_1) + (k_1 k_2)(\alpha_1\beta_1 r_0)$

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$$= k_1 k_2 [F(\alpha_1, b_1) + \alpha_1 \beta_1 r_0] = k_1 k_2 F_0(a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0)$$

$$= k_1 k_2 F_0(x_1, y_1)$$

Hence F_0 is 2-linear.

To find r_0 such that, $\|F_0\| = 1$.

Now, $\|F_0\| = \inf\{k > 0 : \|F_0(x, y)\| \leq k \|x, y\| : (x, y) \in M_0\}$. Let $x_1 = a_1 + \alpha_1 a_0, y_1 = b_1 + \beta_1 b_0$

We have to choose r_0 such that, $\|F_0(x_1, y_1)\| \leq \| x_1, y_1\|$

$\Rightarrow |F(a_1, b_1) + \alpha_1 \beta_1 r_0| \leq \| a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0\|$

$\Rightarrow -\| a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0\| \leq F(a_1, b_1) + \alpha_1 \beta_1 r_0 \leq \|a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0\|$

$\Rightarrow -F(a_1, b_1) - \|a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0\| \leq \alpha_1 \beta_1 r_0 \leq -F(a_1, b_1) + \|a_1 + \alpha_1 a_0, b_1 + \beta_1 b_0\|$

$\Rightarrow -F\left(\frac{a_1}{\alpha_1}, \frac{b_1}{\beta_1}\right) - \left\|\frac{a_1}{\alpha_1} + a_0, \frac{b_1}{\beta_1} + b_0\right\| \leq r_0 \leq -F\left(\frac{a_1}{\alpha_1}, \frac{b_1}{\beta_1}\right) + \left\|\frac{a_1}{\alpha_1} + a_0, \frac{b_1}{\beta_1} + b_0\right\|$

Let $(a_1, b_1), (a_2, b_2) \in M$.

Now $(a_1, b_1) - (a_2, b_2) = (a_1 - a_2, b_1 - b_2)$

$\Rightarrow F(a_1 - a_2, b_1 - b_2) = F(a_1, b_1) + F(a_1, -b_2) + F(-a_2, b_1) + F(-a_2, -b_2)$

$$= F(a_1, b_1) - F(a_1, b_2) - F(a_2, b_1) + F(a_2, b_2)$$

Also, $F(a_1 - a_2, b_1 - b_2) \leq |F(a_1 - a_2, b_1 - b_2)|$

$\Rightarrow F(a_1, b_1) - F(a_1, b_2) - F(a_2, b_1) + F(a_2, b_2) \leq \|F\| \|a_1 - a_2, b_1 - b_2\| = \|a_1 - a_2, b_1 - b_2\|$

$$= \left\|a_1 + a_0 - (a_2 + a_0), b_1 + b_0 - (b_2 + b_0)\right\|$$

$\leq \|a_1 + a_0, b_1 + b_0 - (b_2 + b_0)\| + \|a_2 + a_0,\ b_1 + b_0 - (b_2 + b_0)\|$

$\leq \|a_1 + a_0, b_1 + b_0\| + \|a_1 + a_0, b_2 + b_0\| + \|a_2 + a_0, b_1 + b_0\| + \|a_2 + a_0, b_2 + b_0\|$

$\Rightarrow -F(a_1, b_2) - F(a_2, b_1) - \|a_1 + a_0, b_2 + b_0\| - \|a_2 + a_0, b_1 + b_0\|$

$\leq -F(a_1, b_1) - F(a_2, b_2) + \|a_1 + a_0, b_1 + b_0\| + \|a_2 + a_0, b_2 + b_0\|$

$\Rightarrow \{-\mathrm{F}(a_1, b_2) - \|a_1 + a_0, b_2 + b_0\|\} + \{-\mathrm{F}(a_2, b_1) \pm \|a_2 + a_0, b_1 + b_0\|\}$

$\leq \{-F(a_1, b_1) + \|a_1 + a_0, b_1 + b_0\|\} + \{-F(a_2, b_2) + \|a_2 + a_0, b_2 + b_0\|\}$

Since (a_1, b_1), $(a_2, b_2) \in M$ are arbitrary, we can write,

$-\mathrm{F}(a, b) - \|a + a_0, b + b_0\| \leq -F(a, b) + \|a + a_0, b + b_0\|$; for all $(a, b) \in M$.

Now consider, $p = \{-\mathrm{F}(a, b) - \|a + a_0, b + b_0\|;\ \forall\ (a, b) \in M\}$ and
$q = \{-\mathrm{F}(a, b) + \|a + a_0, b + b_0\|;\ \forall\ (a, b) \in M\}$.
Hence $p \leq q$.

Take r_0 such that $p \leq r_0 \leq q$ and hence $\|F\| = 1$.
Suppose $\|F\| \neq 1$, consider $G = \frac{F}{\|F\|}$.

Hence there is an extension G_0 of G on M_0 such that $\|G_0\| = \|G\| = 1$.

That is $G_0(a, b) = G(a, b)\ \forall\ (a, b) \in M$.

That means that $G_0(a, b) = \frac{F(a,b)}{\|F\|}\ \forall\ (a, b) \in M$.

$\Rightarrow F(a, b) = \|F\| . G_0(a, b)\ \forall\ (a, b) \in M$

Hence we can define $F_0: M_0 \longrightarrow \mathbb{R}$ as $F_0(x, y) = \|F\| . G_0(x, y)$ is the extension of F on M_0 with $\|F_0\| = \|F\|$.

Case (ii) Suppose $(X, \|.,.\|)$ is a generalized complex 2 – normed linear space. Hence we get M is a complex 2-linear subspace of $X \times X$ and hence $F : M \longrightarrow \mathbb{C}$ is a complex valued 2-linear functional on M.

Assume that $\|F\| = 1$. Now, define $F(a, b) = G(a, b) + i\,H(a, b)\ \forall\ (a, b) \in M$ where G and H are real 2 – linear functional on M.

Now, $F(ia, b) = i\,F(a, b)$

$\Rightarrow G(ia, b) + i\,H(ia, b) = i\,[G(a, b) + i\,H(a, b)]$

$\Rightarrow G(ia, b) + i\,H(ia, b) = i\,G(a, b) - H(a, b)$

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$\Rightarrow G(ia,b) = -H(a,b)$

$\Rightarrow H(a,b) = -G(ia,b)$

Also, $F(a,ib) = i\,F(a,b)$

$\Rightarrow G(a,ib) + i\,H(a,ib) = i\,[G(a,b) + i\,H(a,b)]$

$\Rightarrow G(a,ib) + i\,H(a,ib) = i\,G(a,b) - H(a,b)$

$\Rightarrow G(a,ib) = -H(a,b)$

$\Rightarrow H(a,b) = -G(a,ib)$

Hence $H(a,b) = -G(ia,b) = -G(a,ib)$.

Since $\|F\| = 1$, $\|G\| \leq 1$ and G is a real 2-linear functional defined on M.

By **case (i)**, G can be extended to G_0 on a linear space M_0 with $\|G_0\| = \|G\|$.

Now, define $F_0(x,y) = G_0(x,y) - i\,G_0(ix,y)\ \forall (x,y) \in M_0$.

Claim: F_0 is the extension of F from M to M_0

If $(x,y) \in M$, then $G_0(x,y) = G(x,y)$ and hence if $(x,y) \in M$, then $F_0(x,y) = G(x,y) - iG(ix,y)$

That is if $(x,y) \in M$, then $F_0(x,y) = F(x,y)$. Hence we get F_0 is the extension of F from M to M_0.

To prove: F_0 is 2-linear

(i) Let $x_1 = a_1 + \alpha_1 a_0$, $x_2 = a_2 + \alpha_2 a_0$, $y_1 = b_1 + \beta_1 b_0$, $y_2 = b_2 + \beta_2 b_0 \in X$.

$$F_0(x_1 + x_2\,, y_1 + y_2) = G_0(x_1 + x_2\,, y_1 + y_2) - iG_0(i(x_1 + x_2)\,, y_1 + y_2)$$

$$= G_0(x_1,y_1) + G_0(x_1,y_2) + G_0(x_2,y_1) + G_0(x_2,y_2) - i[G_0(ix_1,y_1) + G_0(ix_1,y_2) + G_0(ix_2,y_1) + G_0(ix_2,y_2)]$$

$$= [G_0(x_1,y_1) - iG_0(ix_1,y_1)] + [G_0(x_1,y_2) - iG_0(ix_1,y_2)] + [G_0(x_2,y_1) - iG_0(ix_2,y_1)] + [G_0(x_2,y_2) - iG_0(ix_2,y_2)]$$

$$= F_0(x_1,y_1) + F_0(x_1,y_2) + F_0(x_2,y_1) + F_0(x_2,y_2)$$

(ii) $F_0(ix,y) = G_0(ix,y) - iG_0(i.ix,y)$

$= G_0(ix,y) - iG_0(-x,y)$

$= G_0(ix,y) + iG_0(x,y)$

$= i[G_0(x,y) - iG_0(ix,y)]$

$= i\,F_0(x,y)$

$F_0(x,iy) = G_0(x,iy) - iG_0(ix,iy)$

$= G_0(x,iy) - iG_0(i.ix,y)$

$= G_0(x,iy) - iG_0(-x,y)$

$= G_0(ix, y) + iG_0(x, y)$

$= i[G_0(x, y) - iG_0(ix, y)]$

$= i F_0(x, y)$

$F_0(ix, iy) = G_0(ix, iy) - iG_0(i.ix, iy)$

$= G_0(i.ix, y) - iG_0(i.(-x), y)$

$= G_0(-x, y) - iG_0(ix, y)$

$= -G_0(x, y) + iG_0(ix, y)$

$= -[G_0(x, y) - iG_0(ix, y)]$

$= - F_0(x, y)$

Now, $F_0((a_1 + i b_1)x, (a_2 + i b_2)y) = F_0(a_1x + i b_1x , a_2y + i b_2y)$

$= G_0(a_1x + ib_1x, a_2y + i b_2y) - iG_0(i(a_1x + i b_1x), (a_2y + i b_2y)$

$= G_0(a_1x, a_2y) + G_0(a_1x, ib_2y) + G_0(ib_1x, a_2y) + G_0(ib_1x, ib_2y) -$
$iG_0(i a_1x, a_2y)$

$-iG_0(ia_1x, ib_2y) - iG_0(i.ib_1x, a_2y) - i G_0(i.ib_1x, ib_2y).$

$= a_1a_2 G_0(x, y) + a_1b_2 G_0(x, iy) + a_2b_1 G_0(ix, y) + b_1b_2 G_0(ix, iy)$

$-i a_1a_2G_0(ix, y) - i a_1b_2G_0(ix, iy) - i b_1a_2 G_0(-x, y) - ib_1b_2G_0(-x, iy)$

$= a_1a_2 G_0(x, y) + a_1b_2 G_0(ix, y) + a_2b_1 G_0(ix, y) - b_1b_2 G_0(x, y)$

$- i a_1a_2 G_0(ix, y) + i a_1b_2G_0(ix, iy) + i a_2b_1 G_0(x, y) + ib_1b_2G_0(ix, y)$

$= G_0(x, y)[a_1a_2 - b_1b_2 + i a_1b_2 + i a_2b_1] + G_0(ix, y)[a_1b_2 +$
$a_2b_1 - ia_1a_2 + i b_1b_2]$

$= G_0(x, y) [(a_1 + i b_1) (a_2 + i b_2)] - i G_0(ix, y) [i a_1b_2 + i a_2b_1 +$
$a_1a_2 - b_1b_2]$

$= G_0(x, y) [(a_1 + i b_1) (a_2 + i b_2)] - i G_0(ix, y) [(a_1 + i b_1) (a_2 + i b_2)]$

$= (a_1 + i b_1) (a_2 + i b_2) [G_0(x, y) - i G_0(ix, y)]$

$= (a_1 + i b_1) (a_2 + i b_2) F_0(x, y)$

Hence F_0 is 2-linear.

Claim : $\|\boldsymbol{F_0}\| = \mathbf{1}$

Now, $\|F_0\| = \sup\{|F_0 (x, y)| : \|x, y\| = 1\}$

Suppose $(x, y) \in M_0$, $\|x, y\| = 1$ and $F_0(x, y)$ is real, then $F_0(x, y) = G_0(x, y)$.

$\Longrightarrow |F_0(x, y)| = |G_0(x, y)| \leq 1$ [Since $\|G_0\| = \sup\{|G_0(x, y)| : \|x, y\| = 1\} \leq 1$]

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Hence $\|F\| = 1$.

Suppose $F_0(x,y)$ is complex, then $F_0(x,y) = r\, e^{i\theta}, r > 0$

That is $e^{-i\theta}.F_0(x,y) = e^{-i\theta}.r.e^{i\theta} \Rightarrow F_0(xe^{-i\theta}, y) = r$, which is real.

Since $\|x.e^{-i\theta}, y\| = 1$ and $F_0(xe^{-i\theta}, y)$ is real, $|F_0(xe^{-i\theta}, y)| \leq 1$

$$\begin{aligned} \Rightarrow |F_0(x,y)| \leq 1 \; [\because |F_0(xe^{-i\theta}, y)| &= |e^{-i\theta}||F_0(x,y)| \\ &= 1.|F_0(x,y)| \\ &= |F_0(x,y)|. \end{aligned}$$

Hence $\|F_0\| = 1$.

Theorem: 3.2 (Generalized Hahn Banach Theorem)

Let $(X, \|.,.\|)$ be a generalized 2-normed space and m be a linear subspace of $X \times X$. If F is a bounded 2-linear functional on M, then there exists a bounded 2-linear extension F_0 on $X \times X$ such that $F_0(a,b) = F(a,b)\ \forall\ (a,b) \in M$ and $\|F\| = \|F_0\|$.

Proof: If $M = X \times X$ or $\|F\| = 0$, then take $F = F_0$, otherwise without loss of generality we assume that $\|F\| = 1$.

Let (x, y) $\notin M$. Then there is a bounded 2-linear functional G on $M_0 = M + \{(x,y)\}$ such that G is the extension of F and $\|F\| = \|G\|$.

Consider $\mathcal{A}$ is the set of all such extensions. This means that the set of all pairs (G, L) where L is a linear subsapace of $X \times X$ contains M and G is a bounded 2-linear functional such that $G(a,b) = F(a,b)\ \forall\ (a,b) \in M$ and $\|G\| = 1$.

Define a relation " $\leq$ " on $\mathcal{A}$ as follows.
Let $(G_1, L_1),\ (G_2, L_2) \in \mathcal{A}$. Then $G_1 \leq G_2$ if $L_1 \subseteq L_2$.
That is $G_1(a,b) = G_2(a,b)\ \forall\ (a,b) \in L_1$.

Hence $(\mathcal{A}, \leq)$ is a partially ordered set.

Now, consider a chain $\{(G_i, L_i)\}$ in $\mathcal{A}$. This chain has maximal element $\cup\, G_i$ whose domain is the union of L_i's.
By Zorn's lemma, G has a maximal element. Let it be F_0, $\|F_0\| = \|F\|$.

Claim: F_0 is the extension of F on $\times X$.

Suppose F_0 is a bounded 2-linear functional on $M_0 \neq X \times X$, then there is a vector $(x_0, y_0) \notin M_0$.

Therefore By **theorem3.1**, F_0 has a functional extension on $M_0 + \{(x_0, y_0)\}$.
Hence F_0 is not a maximal element in $\mathcal{A}$ which is a contradiction.

Therefore the bounded 2-linear function F_0 is the extension of F on $X \times X$ such that $F(a,b) = F_0(a,b)\ \forall\ (a,b) \in M$ and $\|F_0\| = \|F\|$. It completes the proof of Hahn Banach theorem.

4. CONCLUSION

In this paper we have proved Generalized Hahn Banach Theorem in a Complex 2-normed space. This idea will help researchers to concentrate more on complex 2-normed spaces.

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[1.] *A.Singadurai, Associate Professor of Maths,T.D.M.N.S. College, T.Kallikulam – 627 113.*
singadurai_59@yahoo.co.in
[2] *S. Meera Sudharsana Revathi, Assistant Professor in Maths, T.D.M.S. College, T.Kallikulam – 627 113.*
meera54376@gmail.com