EXPLORING COMPLEX 3x+1 MAPPING: AN ALGORITHMIC VIEW

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Abstract: This is a sequel to the paper,'"Exploring Complex $3x+1$ Mapping "',where an Algorithmic analysis is applied on the methods that continue to apply:

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1. INTRODUCTION

The $3x+1$ problem is a popular problem in Number theory concerning iteration of the function $f:Z \to Z$ defined as

$$f(x)=\begin{cases}3x+1 & \text{if x is odd}\\ \dfrac{x}{2} & \text{if x is even}\end{cases} \tag{1}$$

The $3x+1$ conjecture asserts that, for each $x \in Z^+$, there is a $n \in Z^+$ such that $f^n(x)=1$. This problem is also known as Collatz problem.

The trajectory of x under f or the orbit of x is the sequence of iterates, $x, f(x), f^2(x), \ldots, f^n(x), \ldots$ where f^n denotes n-fold composition of f with itself.

The $3x+1$ function f in (1) is generalized to the complex function $F:C \to C$ defined as

$$F(z)=\begin{cases}3z+1 & \text{if ceiling(| z |) is odd}\\ \dfrac{z}{2} & \text{if ceiling(| z |) is even}\end{cases} \tag{2}$$

where $|z|=[Re(z)^2+Im(z)^2]^{\frac{1}{2}}$ and ceiling($|z|$) represents the least integer greater than or equal to $|z|$ [3]. This function is called as complex Collatz function.

The equivalent conjecture corresponding to (2) is "The triconvergence property holds for all n ",where the triconvergence property of z is " The trajectory of z under F can be partitioned into three subsequences a,b,c such that $a \to 1, b \to 4, c \to 2$ " [3].

It is also remarked that to obtain a generalisation one looks for sets of complex numbers satisfying the triconvergence property and one need not look for to find complex numbers that probably do not satisfy the triconvergence property.The initial value $3+5i$ have grown to about 1.25 $\times 10^{12}+1.42\times 10^{12}i$, after 10^5 iterations of F, is very unlikely that $3+5i$ satisfies the triconvergence property.[3]

In this paper, we generalise, the $3x+1$ function $f: Z \to Z$ in (1) to complex function $F: C \to C$ for whole complex plane, with no constraints, based on investigating the divisors of ceiling($|z|$), which are integers, where z is a complex number. The equivalent conjecture to the complex function F is formulated and the main result namely "For each complex number z, applying succesive iterations of F, eventually reaches 1", is discussed

Complex $3x+1$ *Function*

We consider (2) here, that is the function $F: C \to C$ defined as

$$F(z) = \begin{cases} 3z+1 & \text{if ceiling}(|z|) \text{ is odd} \\ \dfrac{z}{2} & \text{if ceiling}(|z|) \text{ is even} \end{cases}$$

Also the orbit of z under F is the same as the orbit of x under f [3].

Throughout our discussions, z_0 represents a complex number and n represents a positive integer.

To start with, the set C of complex numbers is partitioned into the following residue classes. [4]. $(a) := \{z_0 \in C : \text{ceiling}(|z_0|) \text{is odd and is equal to } 1 \bmod 4\}$
$(b) := \{z_0 \in C : \text{ceiling}(|z_0|) \text{is odd and is equal to } -1 \bmod 4\}$
$(c) := \{z_0 \in C : \text{ceiling}(|z_0|) \text{iseven}\}$

The residue classes (a),(b) and (c) are called respectively orbits A,B and C.

Previous results on Complex $3x+1$ ***Function***

We provide the results obtained in the paper[5] in this section

Lemma 1: *If* z_0 *is in (a) and ceiling(*$|z_0|$*)* $\geq$ *5 then { ceiling*$(|z_0|)-1$*} is divisible by at least 4 .*

In order to apply succesive iterations of F, we rewrite $F(z)$ as $F(z) = 3z+1 = 3(z-1)+4$ when ceiling($|z|$) is odd. (3)

Let $z_0 \in$ (a). Then by Lemma 1, we get

$$\frac{3(z_0-1)}{4}+1=\frac{F(z_0)}{4}\approx F^3(z_0) \qquad (4)$$

which we denote by $G(z_0)$.

Hence $$G(z_0)=\frac{3(z_0-1)}{4}+1 \text{ if } z_0 \in (a) \qquad (5)$$

The orbit A at z_0 consists z_0, $G(z_0), G^2(z_0)$ .... such that $G^i(z_0)$ for all i are in (a).

Theorem:1 Let $z_0 \in (a)$ .Then ceiling $|G(z_0)| \equiv$ 0 mod 3 or 1 mod 3 or 2 mod 3.

Lemma:**2** If z_0 is in (b) and ceiling $(|z_0|) \geq 3$ then $\frac{\text{ceiling}(|z_0|)-1}{2}$ is odd.

$$F(z_0)=\frac{3(z_0-1)}{2}+2=\frac{F(z_0)}{2}=F^2(z_0) \qquad (6)$$

which we denote by $H(z_0)$.

Hence $$H(z_0)=\frac{3(z_0-1)}{2}+2 \text{ if } z_0 \text{ is in (b)} \qquad (7)$$

The orbit B at z_0 consists $z_0, H(z_0), H^2(z_0)\ldots$ such that $H^i(z_0)$ for all i are in (b).

Theorem:2 *Let $z_0 \in$ (b). Then ceiling $(|H(z_0)|) \equiv$ 0 mod 3 or 1 mod 3 or 2 mod 3.*

Theorem:3 *Let $z_0 \in (b)$. If ceiling $(|z_0|)+1$ is divisible by 2^n not by 2^{n+1} and each $H^i(z_0), 1 \leq i \leq n-2$ is in (b) then $H^{n-1}(z_0)$ is in (a).*

Remark: In the above Theorem 3, the condition each $H^i(z_0)$, $1 \leq i \leq n-2$ isnecessary.Otherwise $H^j(z_0) \in (c)$ for some $j, 1 \leq j \leq n-2$.

Main Result:

Here we discuss the main result which is equivalent to $3x+1$ conjecture corresponding to the mapping (2).

Orbits: The residue classes (a) (b) and (c) are called orbits A, B, and C respectively. The orbit A of z_0 under G consists z_0, $G(z_0)$, $G^2(z_0),\ldots$ such that $G^i(z_0) \forall i$ are in (a). Similarly the orbit B of z_0 under H consists

z_0, $H(z_0)$, $H^2(z_0)$ such that $H^i(z_0)\forall i$ are in (b). The orbit C of z_0 under F consists z_0, $F(z_0)$, $F^2(z_0)$, ... such that $F^i(z_0)\forall$ i are in (c).

We remark that if z_0 is such that ceiling($|z_0|$) $\equiv$ 0 mod3 or 1mod3 or 2mod3 then $z_0 \in$ any one of the residue classes (a),(b),(c).

Theorem:4 Let z_0 be in (a). Then the successive iterations of $G(z_0)$ where $G(z_0)$ is given by (5), lead to any of the following cases.
z_0 in (a) implies ceiling ($|z_0|$) is odd and is equal to 1 mod 4.

case:1 If $G(z_0)$, $G^2(z_0)$, ...,$G^{i-1}(z_0)$ are in (a) then

$$G^i(z_0) = \frac{3^i(z_0-1)}{4^i}+1 \tag{8}$$

ByTheorem1,we have ceiling $|G^i(z_0)|) \equiv 0$ mod3 or 1mod3 or 2mod3. That is ceiling($|G^i(z_0)|$) is any of the forms $3k+1, 3k+2, 3k$. Evidently ceiling ($|G^i(z_0)|$) may be odd or even integers as z_0 is in (a).
case:2 $G^i(z_0)$ is in (b). Then the iteration of H lead to

$$H^{n-1}(G^i(z_0)) = \frac{3^{n-1}(\frac{3^i(z_0-1)}{4^i}+2)}{2^{n-1}}-1 \tag{9}$$

provided ceiling ($|G^i(z_0)|$)+1 is divisible by 2^n, not by 2^{n+1} and each $H^j(G^i(z_0)), 1\le j\le n-2$ is in (b). By Theorem 3 we have $H^{n-1}(G^i(z_0))$ is in (a).

case:3 $G^i(z_0)$ is in (c) and ceiling ($|G^i(z_0)|$) $= 3k+1$ for an odd integer $k\equiv -1 \bmod 4$. Then by Lemma 2, $k-1$ is divisible by 2. Hence the iteration of F gives

$$F\ (G^i(z_0)) = H(\frac{G^i(z_0)-1}{3}) \tag{10}$$

case:4 $G^i(z_0)$ is in (c) and ceiling ($|G^i(z_0)|$) $= 3k+1$ for an odd integer $k \equiv 1$ mod 4. Then by Lemma 1, $k-1$ is divisible by 4. Hence the iteration of F (twice) yield

$$F^2(G^i(z_0)) = G(\frac{G^i(z_0)-1}{3}) \tag{11}$$

case:5 $G^i(z_0)$ is in (c) and ceiling $(|G^i(z_0)|)$ is not equal to 1 mod 3. Note that ceiling $(|G^i(z_0)|)$ is equal to either $3k$ or $3k+2$ for an even integer k. Then we have $F^n(G^i(z_0)) = \frac{G^i(z_0)}{2^n}$ provided ceiling $(|\frac{G^i(z_0)}{2^{n-1}}|)$ is in (c) and not equal to 1 mod 3.

We remark that in case 3, case 4 and case 5, by iteration of F we mean the iteration of the function $F(z_0) = \frac{z_0}{2}$ when ceiling $(|z_0|)$ is even as in (2).

Theorem:5 *Let* z_0 *be in (b) and* $H(z_0)$ *is given by (17). The successive iterations of* $H(z_0)$ *join with the orbit A.*

Theorem:6 *If* z_0 *is in (c) then the successive iterations of* $F(z_0)$ *where* $F(z_0)$ *is given by (2), join with the orbit A.*

Properties of the functions G(z), H(z) and F(z).

1. If z_0 is in (a) then $G(z_0) < z_0$ as the derivative of $G(z_0)$ with respect to z_0 is less than 1.

2. If z_0 is in (b) then $H(z_0) > z_0$ as the derivative of $H(z_0)$ with respect to z_0 is greater than 1.

3. If z_0 is in (c) then $F(z_0) < z_0$ as the derivative of $F(z_0)$ with respect to z_0 is less than 1.

Theorem:7 *If* z_0 *be in (a) such that ceiling* $(|z_0|) = 1$ *then the successive iterations of* z_0 *under G stays in the orbit A itself.*

ALGORITHM

We introduce a function.

$$L(z) = \begin{cases} G(z) & \text{for } z \in (a) \\ H(z) & \text{for } z \in (b) \\ \dfrac{z-1}{3} & \text{for } z \in (c) \text{ with ceiling}(|z|) = 3k+1, \\ & k \text{ is odd integer.} \\ F(z) & \text{for } z \in (c) \text{ with ceiling}(|z|) \neq 3k+1, \\ & k \text{ is odd integer.} \end{cases}$$

where $G(z), H(z)$ and $F(z)$ are given by (5), (17) and (2) respectively.

To apply L(z) successively, denote

$L^j(z_0) = L(L^{j-1}(z_0)), j \geq 1$ and $L^\circ(z_0) = z_0$. Then by theorems 4, 5 and 6 the successive iterations of L yields, for j ≥ 1.

$$L^j(z_0) = \begin{cases} G(L^{j-1}(z_0)) & \text{if} \quad L^{j-1}(z_0) \in (a) \to (\alpha) \\ H(L^{j-1}(z_0)) & \text{if} \quad L^{j-1}(z_0) \in (b) \to (\beta) \\ \dfrac{L^{j-1}(z_0) - 1}{3} & \text{if} \quad L^{j-1}(z_0) \in (c) \text{andceiling} \\ & \quad (|L^{j-1}(z_0)|) = 3k+1 \\ & \quad \text{foranoddintegerk.} \to (\gamma) \\ F(L^{j-1}(z_0)) & \text{if} \quad L^{j-1}(z_0) \in (c) \text{andceiling} \\ & \quad (|L^{j-1}(z_0)|) \neq 3k+1 \\ & \quad \text{foranoddintegerk.} \to (\delta) \end{cases}$$

By Theorem 4, (α),(β) and (δ) follow from case 1, case 2 and case 5 respectively. In (γ), if ceiling ($|L^{j-1}(z_0)|$) is even and divisible by just 2 then by case 3. L^{j+1} = H and if ceiling ($|L^{j-1}(z_0)|$) is even and divisible by at least 4 then by case 4 L^{j+1} = G.

To justify the algorithm we prove that

$$\{L^j(z_0) : j \geq 0\} \subset \{F^j(z_0) : j \geq 0\}$$

case:1 Let $\omega_0 \in \{L^j(z_0)\}$ for some j > 1 and ceiling ($|\omega_0|$) ≡ 0mod3 or 1mod3 or 2mod3 with respect to G ≡ F^3.

- By Theorem 4 there exists z_0 with ceiling ($|z_0|$) ≡ 1mod4 such that $\omega_0 = (F^3)^i(z_0)$ for some $i > 1$.
- When ceiling ($|z_0|$) ≡ 0mod3 or 1mod3 or 2mod3 and also ≡ 1 mod 4 by Theorem 5 there exists z_1 with ceiling ($|z_1|$) ≡ -1 mod 4 such that $z_0 = (F^2)^{i_1}(z_1)$ for some i_1 which implies $\omega_0 = (F^3)^i(F^2)^{i_1}(z_1)$.
- When ceiling ($|z_0|$) ≡ 1 mod 4 by Theorem 4 there exists z_3 with ceiling ($|z_3|$) ≡ 1 mod 4 such that $z_2 = (F^3)^{i_2}(z_3)$ for some i_2 with ceiling

$(|z_2|) \equiv 1 \bmod 3$ (even) which gives $z_0 = \frac{z_2 - 1}{3}$ and $\omega_0 = F^2(z_2)$. Hence $\omega_0 = (F^2)(F^3)^{i_2}(z_3)$.

- When ceiling $(|z_0|) \equiv 1 \bmod 4$ by Theorem 5 there exists z_5 with ceiling $(|z_5|) \equiv -1 \bmod 4$ such that $z_4 = (F^2)^{i_3}(z_5)$ for some i_3 with ceiling $(|z_4|) \equiv 1 \bmod 3$ (even) which gives $z_0 = \frac{z_4 - 1}{3}$ and $\omega_0 = F^2(z_4)$. Hence $\omega_0 = (F^2)(F^2)^{i_3}(z_5); (F^2)^{i_3+2}(z_5)$.

- When ceiling $(|z_0|) \equiv 1 \bmod 4$ by Theorem 6 there exists z_6 with ceiling $(|z_6|) \equiv 1 \bmod 3$ (even) such that $z_0 = \frac{z_6 - 1}{3}$ and hence $\omega_0 = F^2(z_6)$.

- When ceiling $(|z_0|) \equiv 1 \bmod 4$ by Theorem 6 there exists z_7 with ceiling $(|z_7|) \not\equiv 1 \bmod 3$ (even) such that $z_0 = F(z_7)$ which implies $\omega_0 = (F^3)^i(F(z_7))$.

- When ceiling $(|z_0|) \equiv 1 \bmod 4$ by Theorem 6 there exists z_9 with ceiling $(|z_9|) \not\equiv 1 \bmod 3$ (even) such that $z_8 = (F)^{i_4}(z_9)$ for some i_4 where ceiling $(|z_8|) \equiv 1 \bmod 3$ (even) which gives $z_0 = \frac{z_8 - 1}{3}$ and $\omega_0 = F^2(z_8)$. Hence $\omega_0 = F^2(F)^{i_4}(z_9)$.

case:2 Let $\omega_{0'} \in \{L^j(z_{0'})\}$ for some $j > 1$ and ceiling $(|\omega_{0'}|) \equiv 0 \bmod 3$ or $1 \bmod 3$ or $2 \bmod 3$ with respect to H $\cong F^2$.

- By Theorem 5 there exists $z_{0'}$ with ceiling $(|z_{0'}|) \equiv -1 \bmod 4$ such that $\omega_{0'} = (F^2)^j(z_{0'})$ for some $j > 1$.

- when ceiling $(|z_{0'}|) \equiv -1 \bmod 4$ by Theorem 4 there exists $z_{1'}$ with ceiling $(|z_{1'}|) \equiv 1 \bmod 4$ such that $z_{0'} = (F^3)^{j_1}(z_{1'})$ for some j_1 which implies $\omega_{0'} = (F^2)^j(F^3)^j_1(z_{1'})$

- When ceiling $(|z_{0'}|) \equiv -1 \bmod 4$ by Theorem 4 there exists $z_{3'}$ with ceiling $(|z_{3'}|) \equiv 1 \bmod 4$ such that $z_{2'} = (F^3)^{j_2}(z_{3'})$ for some j_2 with ceiling

($|z_{2'}| \equiv 1 \bmod 3$ (even) which gives $z_{0'} = \frac{z_{2'} - 1}{3}$ and $\omega_{0'} = F(z_{2'})$. Hence $\omega_{0'} = F(F^3)^{j_2}(z_{3'})$

- When ceiling ($|z_{0'}|$) $\equiv$ -1 mod 4 by Theorem 5 there exists $z_{5'}$ with ceiling ($|z_{5'}|$) $\equiv$ -1 mod 4 such that $z_{4'} = (F^2)^{j_3}(z_{5'})$ for some j_3, with ceiling ($|z_{4'}|$) $\equiv$ 1 mod 3 (even) which gives $z_{0'} = \frac{z_{4'} - 1}{3}$ and $\omega_{0'} = F(z_{4'})$ whch implies $\omega_{0'} = F(F^2)^{j_3}(z_{4'})$.

- When ceiling ($|z_{0'}|$) $\equiv$ -1 mod 4 by Theorem 6 there exists $z_{6'}$ with ceiling ($|z_{6'}|$) $\equiv$ 1 mod 3 (even) such that $z_{0'} = \frac{z_{6'} - 1}{3}$ and hence $\omega_{0'} = F(z_{6'})$.

- When ceiling ($|z_{0'}|$) $\equiv$ -1 mod 4 by Theorem 6 there exists $z_{7'}$ with ceiling ($|z_{7'}|$) $\not\equiv$ 1 mod 3 (even) such that $F(z_{7'}) = z_{0'}$ which gives $\omega_{0'} = (F^2)^j(F(z_{7'}))$.

- When ceiling ($|z_{0'}|$) $\equiv$ -1 mod 4 by Theorem 6 there exists $z_{9'}$ with ceiling ($|z_{9'}|$) $\not\equiv$ 1 mod 3 (even) such that $z_{8'} = (F)^{j_4}(z_{7'})$ for some j_4 with ceiling ($|z_{8'}|$) $\equiv$ 1 mod 3 (even) which gives $z_{0'} = \frac{z_{8'} - 1}{3}$ and $\omega_{0'} = F(z_{8'})$ Hence $\omega_{0'} = F(F)^{j_4}(z_{8'})$.

case:3 Let $\omega_{0''} \in \{L^j(z_{0''})\}$ for some $j > 1$ and ceiling ($|\omega_{0''}|$) $\equiv$ 0 mod 3 or 1 mod 3 or 2 mod 3 with respect to F.
By Theorem 6 there exists $z_{0''}$ with ceiling($|z_{0''}|$) $\equiv$ 1 mod 3 (even) such that $\omega_{0''} = F(z_{0''})$ if $\frac{z_{0''} - 1}{3} \equiv -1 \bmod 4$, otherwise $\omega_{0''} = F^2(z_{0''})$.

case:4 Let $\omega_{0'''} \in \{L^j(z_{0'''})\}$ for some $j > 1$ and ceiling($|\omega_{0'''}|$) $\equiv$ 0 mod 3 or 1 mod 3 or 2 mod 3 with respect to F.

- By Theorem 6 there exists $z_{0'''}$ with ceiling ($|z_{0'''}| \not\equiv$ 1 mod 3(even) such that $\omega_{0'''} = (F)^k(z_{0'''})$ for some k.

- By Theorem 4 there exists $z_{1'''}$ with ceiling ($|z_{1'''}| \equiv$ 1 mod 4 such that $z_{0'''} = (F^3)^{k_1}(z_{1'''})$ for some k_1 which implies $\omega_{0'''} = (F)^k(F^3)^{k_1}(z_{1'''})$.

- By Theorem 5 there exists $z_{2'''}$ with ceiling $(|z_{2'''}| \equiv -1 \bmod 4$ such that $z_{0'''} = (F^2)^{k_2}(z_{2'''})$ for some k_2 which implies $\omega_{0'''} = (F^k)(F^2)^{k_1}(z_{2'''})$.

Let $z_0 \in$ (a). We have the orbit A of z_0 under L given by (α). On account of property 1 and 5 this orbit A forms a decreasing sequence of complex numbers $\{z_i\}$ with ceiling $(|z_i|) \equiv$ 0mod3 or 1mod3 or 2mod3. When ceiling $(|z_i|) \equiv 1$ mod 4 the orbit A stays completely in it. (case 1 of Theorem 4). When ceiling $(|z_i|) \equiv -1$ mod 4, the orbit A moves to orbit B (case 2 of Theorem 4). On account of property 2 and (17) the orbit B under L given by (β) forms an increasing sequence of complex numbers $\{z_{i'}\}$ with ceiling $(|z_{i'}|) \equiv$ 0mod3 or 1mod3 or 2mod3. But by Theorem 5, we observe that this orbit B joins with the orbit A. That is when ceiling $(|z_{i'}| \equiv 1$ mod 4 (odd) (case 1 of Theorem 5) or $\equiv 1$ mod 3 (even) (case 2, 3 of Theorem 5) or $\not\equiv 1$ mod 3 (even) (case 4 of Theorem 5). We have the above observation. If ceiling $(|z_i|) \equiv 1$ mod 3 (even) or $\not\equiv 1$ mod 3 (even) this orbit A moves to orbit C. This orbit C under L given by (γ) and (δ) forms a decreasing sequence of complex numbers $\{z_{i''}\}$. But by theorem 6, the orbit C joins with orbit A directly or through orbit B. That is when ceiling $(|z_i|) \equiv 1$ mod 3 (even) say 3k+1 for an odd integer k $\equiv 1$ mod 4 or -1 mod 4 (case 1, 2 of Theorem 6) this orbit C joins with orbit A directly or moves to orbit B and hence (case 2, 3 of Theorem 5) it returns to orbit A respectively. When ceiling $(|z_i|) \not\equiv 1$ mod 3 (even), (case3 of Theorem 6) we observe that this orbit C joins with orbit A. That is when ceiling$(|z_{i''}|)$ for some j $\equiv 1$ mod 4 or -1 mod 4 this orbit C joins with orbit A directly or through the orbit B respectively. Finally, if ceiling $(|z_i|) \equiv 1$ mod 3, (even) and is divisible by 4, by an application of L given by (γ) may yield $z_{j'}$ with ceiling $(|z_{j'}|) = 1$ for some j.

Let $z_0 \in$ (b). We have orbit B of z_0 under L given by (β). On account of property 2 and (17), this orbit B forms an increasing sequence of complex numbers $\{z_i\}$ such that ceiling$(|z_i|) \equiv$ 0mod3 or 1mod3 or 2mod3. But by Theorem 5 we observe that the orbit B joins with the orbit A and we have to apply L given by (α) to the orbit A.

Let $z_0 \in$ (c). We have orbit C of z_0 under L given by (γ) and (δ). On account of property 3 and by (2) and (γ) the orbit C forms a decreasing sequence of complex numbers $\{z_i\}$ such that ceiling $(|z_i|) \equiv 1$ mod 3 (even) or $\not\equiv 1$ mod 3 (even). But by theorem 6 this orbit C joins with the orbit A directly or through the orbit B correspondingly. Again we apply L given by (α) to the orbit A. Thus the transition from orbit A to other orbits B or C and vice versa taken place many time.

If ceiling ($| z_i |$) $\equiv$ 1 mod 3 (even) and is divisible by 4 ,an application of L given by (γ) to z_i may yield some z_j with ceiling ($| z_j |$) = 1 or we may have to return orbit A and repeat iterating. But by Theorems 4, 5 and 6, on returning each time to orbit A, we get a sequence of complex numbers z_m with ceiling ($| z_m |$) $\equiv$ 1 mod 4. When ceiling ($| z_m |$) = 4 the iteration of L given by (γ) yields z_j such that ceiling ($| z_j |$) = 1. In otherwords, by Theorems 4, 5 and 6, there exists a positive number m > 0 such that $L^m(z_0) = \frac{L^{m-1}(z_0) - 1}{3}$ where ceiling ($| L^m(z_0) |$) = 1 and hence ceiling ($| L^{m-1}(z_0) |$) = 4. Moreover by Theorem 5, $L^m(z_0) \neq H(L^{m-1}(z_0))$ as ceiling ($| L^{m-1}(z_0) |$) is even. Also by Theorem 4, $L^m(z_0) \neq G(L^{m-1}(z_0))$ as ceiling($| L^{m-1}(z_0) |$) > 1. Hence by Theorem 7 the succesive iterations of z_0 eventually reaches 1+0i.

Conjecture :For each z_0 , applying successive iterations of F eventually reaches 1.

(For practical purpose one can consider 1+0i ;$x + iy$ in C as $0.9 \leq x \leq 1$ and y is in the neighbourhood of zero).

The Theorem follows from Theorems 4, 5, 6 and 7 with the aid of algorithm.

Numerical Illustration The data was generated using iframe in netbeans 5.5 for java development environment and stored as text file.

1. Let $z_0 = -3 - 5i$, ceiling ($| z_0 |$) = 6 $\equiv$ 0 mod 3 $\Rightarrow z_0 \in$ (c). By Theorem 6, we have $z_1 = L(z_0) = F(z_0) = -1.5 - 2.5i$ and ceiling($| z_1 |$) $= 3 \equiv$ 0 mod 3 also -1 mod 4. $\Rightarrow z_1 \in (b)$. By Theorem5, we have $z_2 = L^2(z_0) = L(z_1) = H(z_1) = -1.75 - 3.75i$,ceiling ($| z_2 |$) = 5 $\equiv$ 2 mod 3,also 1 mod 4 $\Rightarrow z_2 \in (a)$. By theorem 4,we have $z_3 = L^3(z_0) = L(z_2) = G(z_2) = -1.0625 - 2.8125$, ceiling ($| z_3 |$) = 4 $\equiv$ 1 mod 3, even. $\Rightarrow z_3 \in (c)$. By Theorem 6, we have $z_4 = \frac{z_3 - 1}{3} = -0.6875 - 0.9375i$. As $\frac{\text{ceiling} | z_3 | - 1}{3} = 1 \in (a)$ we have $z_5 = L^5(z_0) = L(z_4) = L(\frac{z_3 - 1}{3}) = G(\frac{z_3 - 1}{3}) = -0.265625 - 0.703125i$ ceiling ($| z_5 |$) $= 1 \Rightarrow z_5 \in (a)$. Now by Theorem 7, we have $z_{50} = L^{50}(z_0) = 0.9999774 - 1.2584924E - 6i$ with ceiling ($| z_{50} |$) $= 1$.

By repeated iteration we get $z_{99} = L^{99}(z_0) = 0.9999999 - 9.502818E-13 \approx 1+0i$.

2. Let $z_0 = 88.0 + 99.0i$, ceiling$(|z_0|) = 133 \equiv$ 1 mod 3 also 1 mod 4 $\Rightarrow z_0 \in$ (a). By Theorem 4, we have $z_1 = L(z_0) = G(z_0) = 66.25 + 74.25i$, ceiling $(|z_1|) = 100 \equiv$ 1 mod 3,even. $\Rightarrow z_1 \in (c)$ By Theorem 6, we have $z_2 = \frac{z_1 - 1}{3} = 21.75 + 24.75i$. As $\frac{\text{ceiling}\,|z_1| - 1}{3} = 33 \equiv$ 1 mod 4 in (a) we have $z_3 = L^3(z_0) = L(z_2) = L(\frac{z_1-1}{3}) = G(\frac{z_1-1}{3}) = 16.5625 + 18.5625i$ ceiling $(|z_3|) = 25 \Rightarrow z_3 \in (a)$.

By Theorem 4, we have $z_4 = L^4(z_0) = L(z_3) = G(z_3) = 12.671875 + 13.921875\,i$ ceiling $(|z_4|) = 19 \equiv$ 2 mod 3,also -1 mod 4 $\Rightarrow z_4 \in (b)$ .By Theorem 5, we have $z_5 = L^5(z_0) = L(z_4) = H(z_4) = 19.507812 + 20.882812$,ceiling $(|z_5|) = 29 \equiv$ 2 mod 3,also 1 mod 4 $\Rightarrow z_5 \in (a)$. By repeated process, we get $z_{14} = L^{14}(z_0) = 2.8737297 + 2.4777946$,ceiling$(|z_{14}|) = 4 \Rightarrow z_{14} \in$ (c).

By Theorem 6,we have $z_15 = \frac{z_14 - 1}{3} = 0.624574666 + 0.8259315333i$. As $\frac{\text{ceiling}\,|z_14| - 1}{3} = 1 \equiv$ 1 mod 4 in (a) we have $z_16 = L^16(z_0) = L(z_15) = L(\frac{z_14-1}{3}) = G(\frac{z_14-1}{3})$ $= 0.718430999 + 0.619448649i$ ceiling $(|z_16|) = 1 \Rightarrow z_16 \in (a)$. Now by theorem 7, we get

$$z_{49} = L^{49}(z_0) = 0.9999933 + 1.4766314\,E-5i$$
$$\approx 1+0i$$

2. CONCLUSION

The complex function is defined by considering the other two variants, instead of ceiling $(|z|)$, namely floor $(|z|)$ = greatest integer $\leq |z|$ and round $(|z|)$ = integer nearest to $|z|$, rounding up incase of ambiguity and one can verify the $3x$ +1 conjecture for the complex function with respect to floor $(|z|)$ and round $(|z|)$

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