mrgα-CLOSED SET IN MINIMAL STRUCTURE

N.Murugavalli[1], A.Pushpalatha[2]

Abstract : In this paper, we introduce mrgα-closed set and some of its basic properties.

Key words: mr- closed set, mrgα-closed set and mrα-open set

1. INTRODUCTION

Noiri[11] introduced concept of minimal structure on a nonempty set. Also they introduced the notion of m_x-open set and m_x-closed set and characterize those sets using m_x-cl and m_x-int operators respectively. Further they introduced m-continuous functions and studied some of its basic properties. T.Noiri[11] introduced the concept of mg-closed sets in minimal structures which is analogs to g-closed sets in topological space introduced by Levine. N[8]

N. Levine [8] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Balachandran, Sundaram and Maki [3], Bhattacharyya and Lahiri [4], Arockiarani [1], Dunham [6], Gnanambal [7], Malghan [9], Palaniappan and Rao [12], Park [13] Arya and Gupta [2] and Devi [5] have worked on generalized closed sets, their generalizations and related concepts in general topology.A.Vadivel and K.Vairamanickam [15] introduced rgα-closed. In this paper, we define and study the properties of minimal regular generalized α-closed sets (briefly m$rg\alpha$-closed).

2. PRELIMINARIES

Definition 2.1 [11] A subfamily m_x of the power set P(X) of a non empty set X is called as minimal structure on X if $\Phi \in m_x$ and $X \in m_x$. Minimal structure is denoted by (X, m_x) where X is a nonempty set with a minimal structure m_x on X.

Definition 2.2 [11] Let (X, m_x) be a minimal space with a minimal structure m_x on X for a subset A of X, the m-closure of A and the m-interior of A

$$\text{mint}(A) = \cup \{ U : U \subseteq A, U \in m_x \}$$

$$\text{mcl}(A) = \cap \{ F : A \subseteq F, X-F \in m_x. \}$$

Definition 2.3 [11] Let (X, m_x) be a space with a minimal structure m_x on X and $A \subset X$. A subset A of X is called m-semi open set if $A \subseteq$ mcl [mint (A)] the complement of m-semi open set is called m-semi closed set.

Definition2.4 [11] Let (X, m_x) be an m-space, A subset A of X is said to be mg-closed if

m_x -cl(A) ⊆ G whenever A ⊆ G and G is m_x -open. The complement of an mg-closed set is said to be mg-open set.

Definition 2.5 [11] A minimal structure m_x on a nonempty set X is said to have property B if the union of any family of subsets belong to m_x.

Lemma 2.6 [11] Let X be a nonempty set and mx a minimal structure on X satifying property B. For a subset A of X, the following properties hold:

(i) A∈ m_x if and only if m_x -int(A) = A
(ii) A is m_x -closed if and only if m_x-cl(A) = A
(iii) m_x -int(A)∈ m_x and m_x -cl(A) is m-closed

Definition 2.7. [10] A subset A of a Minimal space (X,mx) is called

a) a minimal preopen set (briefly mp-open) if

A ⊆ mx- int(mx-cl (A)) and a minimal preclosed set if mx –cl(mx- int (A)) ⊆ A.

b) a minimal semiopen set (briefly ms-open) if
A ⊆ m_x- cl(m_x- int (A)) and a minimal semiclosed set (briefly ms-closed)if m_x -int(m_x- cl (A)) ⊆ A.

c) a minimal α-open set (briefly mα-open) if
A ⊆ m_x- int(m_x- cl(m_x-int (A))) and a minimal α-closed set (briefly mα-closed) if m_x –cl(m_x- int(m_x- cl (A))) ⊆ A.

d) a minimal semi-preopen set (briefly msp-open) if
A⊆m_x-cl(m_x-int(m_x- cl(A))) and a minimal semi-preclosed set (briefly msp-closed) if m_x -int(m_x- cl(m_x- int (A))) ⊆ A.

e) a minimal regular open set (briefly mr-open) if
A = m_x -int(m_x- cl (A)) and a minimal regular closed set (briefly msp-closed) if A = m_{x-} cl(m_x- int (A)).

(f) a minimal regular semiopen if there is a minimal regular open U such U ⊆ A ⊆ m_x- cl(U). The family of all regular semiopen sets of X is denoted by RMSO(X).

(g) minimal generalized closed set (briefly, mg-closed) if
m_x -cl (A) ⊆ U whenever A ⊆ U and U is m_{x-} open in X.

Note 2.8 (i) The minimal closure of a minimal semiopen set is minimal regular closed.
(ii) Every minimal regular closed set is minimal semiopen set.

The intersection of all minimal semiclosed (resp-minimal semiopen) subsets of *X* containing *A* is called the minimal semi-closure of A and is denoted by mscl(A).

Also the intersection of all minimal preclosed (resp. minimal semi-preclosed and minimal α-closed) subsets of X containing A is called minimal pre-closure (resp. minimal semi-pre closure and minimal α-closure) of A and is denoted by mpcl(A) (resp.mspcl(A) and mα-cl (A)

3. MRGA-CLOSED SETS IN MINIMAL STRUCTURES

In this section, we introduce the notion minimal regular α -open set(mrα-closed set) and minimal regular generalized α - closed set (mrgα-closed set) and investigate some of their properties.

Definition 3.1 A subset A of a minimal space (X,m_x) is called minimal regular α-open set (mrα-open set) if there is a minimal regular open set U such that $U \subset A \subset$ mαcl(U).

The family of all minimal regular α - open sets of (X,m_x) is denoted by MRαO(X).

Definition 3.2 A subset A of a minimal space (X,m_x) is called minimal regular generalized α- closed set (mrgα-closed set) if mαcl(A) $\subset$ U whenever $A \subset U$ and U is a minimal regular α- open set in X. We denote the set of all mrgα-closed sets in (X,m_x) by MRGαcl(X).

Theorem 3.3. The union of two mrgα-closed subsets of X is also mrgα-closed subset of X.

Proof: Assume that A and B are mrgα-closed set in X. Let U be minimal regular α-open in X such that $(A \cup B) \subset U$.
Then $A \subset U$ and $B \subset U$. Since A and B are mrgα-closed,
mαcl(A) $\subset$ U and mαcl(B) $\subset$ U.
Hence mαcl(A ∪ B) = (mαcl(A)) ∪ (mαcl(B)) $\subset$ U. That is
mαcl (A ∪B) $\subset$ U. Therefore (A ∪ B) is mrgα-closed set in X.

Theorem 3.4. If a subset A of X is mrgα-closed set in X. Then mαcl(A)\A does not contain any nonempty minimal regular α-open set in X.

Proof: Suppose that A is mrgα-closed set in X. We prove the result by contradiction. Let U be a minimal regular α-open set such that mαcl(A) \ A $\supset$ U and $U \neq \Phi$. Now U $\subset$ mαcl(A) \A. Therefore $U \subset X \setminus A$ which implies $A \subset X \setminus U$. Since U is minimal regular α-open set, X\U is also minimal regular α-open in X. Since A is mrgα-closed set in X, by definition we have
mαcl(A) $\subset$ X\U. So U $\subset$ X\mαcl(A). Also U $\subset$ mαcl(A).
Therefore U $\subset$ (mαcl(A) ∩ X\ mαcl(A))) = Φ. This shows that,

U = Φ which is contradiction. Hence mαcl(A) \ A does not contains any non-empty minimal regular α-open set in X. The converse of the above theorem need not be true seen from following example.

Example 3.5 .If macl(A) \ A contains no non-empty mrgα-open subset in X, Then A need not be mrgα-closed set. Let X = {a, b, c, d, e} with minimal structure
m_x = {X, Φ, {a} , {d} , {e} , {a, d} , {a, e} , {d, e} , {a, d, e}} and
A = {a, b}.Then macl(A)\A = {a, b, c}\ {a, b} = {c} does not contain non-empty minimal regular α-open set in X, but A is not a mrgα-closed set in X.

Corollary 3.6. If a subset A of X is mrgα-closed set in X, then macl(A)\A does not contain any mr- open set in X, but not conversely.
Proof: Follows from Theorem 3.4 and the fact that every mr-open set is mrα-open.

Corollary 3.7 If a subset A of (X,m_x) is mrgα-closed set in X, then macl(A)\A does not contain any non-empty mr - closed set in X, but not conversely.
Proof: Follows from theorem 3.4. and the fact that every mr- open set is mrα-open.

Theorem 3.8 For an element x ∈ X, the set X\ {x} is mrgα-closed or mrα-open..

Proof: Suppose X\ {x} is not mrα-open set. Then X is the only mrα-open set containing X\ {x}. This implies macl(X\ {x}) ⊂ X. Hence X\ {x}is mrgα-closed set in X.

Theorem 3.9 If A is mrgα-closed subset of X such that A ⊂ B ⊂ macl(A).Then B is mrgα-closed set in X.

Proof: If A is mrgα-closed subset of (X,m_x) such that
A ⊂ B ⊂ macl(A). Let U be a mrα-open set of X such that B ⊂U. Then A ⊂ U. Since A is a mrgα-closed we have macl(A) ⊂ U. Now macl(B) ⊂ macl (macl(A)) = macl(A) ⊂ U. Therefore B is mrgα-closed set in X.

Remark 3.10. The converse of the theorem 3.9 need not be true in general.Consider the minimal space (X,m_x) , where
X = {a, b, c, d, e} with m-structure
m_x = {X, Φ, {a} , {d} , {e} , {a, d} , {a, e} , {d, e} , {a, d, e}}. Let
A = {b} and B = {b, c}. Then A and B are mrgα-closed sets in (X,m_x), but A ⊂ B is not subset in macl(A).

Theorem 3.11. Let A be a mrgα-closed in (X,m_x). Then A is mα-closed if and only if macl(A)\A is a mrα-open.

Proof: Suppose A is a mα-closed in X. Then macl(A) = A and so macl(A)\A = Φ, which is mrα-open in X. Conversely, suppose macl(A)\A is a mrα-open set in X. Since A is mrgα-closed, by theorem 3.4. macl(A)\A does not contain any nonempty mrα-open in X. Then macl(A)\A = Φ, hence A is mα-closed set in X.

Theorem 3.12 If A is mr-open and mrg-closed, then A is mrgα-closed set in (X,m_x).

Proof: Let U be any mrα-open set in X such that A ⊂ U. Since A is mr-open and mrg-closed, we have mαcl(A) ⊂ A. Then mαcl(A) ⊂ A ⊂ U. Hence A is mrgα-closed set in X.

Theorem 3.13 If a subset A of minimal space (X,m_x) is both mrα-open and mrgα-closed, then it is mα-closed.

Proof: Suppose a subset A of minimal space (X,m_x) is both mrα-open and mrgα-closed. Now A ⊂ A. Then mαcl(A) ⊂ A. Hence A is mα-closed.

Corollary 3.14. Let A be mrα-open and mrgα-closed subset in X. Suppose that F is mα-closed set in X. Then A ∩ F is an mrgα-closed set in X.

Proof: Let A be a mrα-open and mrgα-closed subset in X and F be m-closed. By theorem 3.13, A is mα-closed. So A ∩ F is a mα-closed and hence A ∩ F is mrgα-closed set in X.

Theorem 3.15. If A is both m-open and mg-closed set in (X,m_x), then it is mrgα-closed set in X.

Proof: Let A be an m-open and mg-closed set in X. Let A ⊂ U and let U be a mrα-open set in X. Now
A ⊂ A. By hypothesis mαcl(A) ⊂ A. That is mαcl(A) ⊂ U. Thus A is mrgα-closed in X.

Remark 3.16 If A is both m-open and mrgα-closed in X, then A need not be mg-closed, in general, as seen from the following example.

Example 3.17. Consider X = {a, b, c, d, e} with minimal structure
m_x = {X, Φ, {a} , {d} , {e} , {a, d} , {a, e} , {d, e} , {a, d, e}}. In this minimal space the subset {a, d, e} is an m-open and mrgα-closed set, but not mg-closed.

4. ACKNOWGMENT

The authors are grateful to the referees for accepted this paper.

5. REFERENCES

1. I.Arockiarani, Studies on generalizations of generalized closed sets and maps in topological spaces, Ph.D Thesis, Bharathiar University,Coimbatore, (1997).
2. S.P.Arya and R.Gupta, On strongly continuous mappings, Kyungpook Math., J. 14(1974), 131-143.
3. K. Balachandran, P.Sundaram and H.Maki, On generalized continuous maps in topological spaces, Mem. I ac Sci. Kochi Univ. Math., 12(1991), 5-13.
4. P.Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(1987), 376-382.
5. R.Devi, K.Balachandran and H.Maki, On generalized α-continuous maps, Far. East J. Math. Sci. Special Volume, part 1 (1997), 1-15.
6. W.Dunham, A new closure operator for non-T1 topologies, Kyungpook Math. J., 22(1982), 55-60.
7. Y. Gnanambal, On generalized pre-regular closed sets in topological spaces, Indian J. Pure Appl. Math., 28(1997), 351-360.

8. N.Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo,19(1970), 89-96.
9. S.R. Malghan, Generalized closed maps, J. Karnatak Univ. Sci., 27(1982),82-88.
10. N.Murugavalli and A.Pushpalatha , Some more sets in Minimal Structures – Submitted
11. T.Noiri,11th meetings on topological spaces Theory and its applications, Fukuoka University seminar House, 2006,1-9.
12. N.Palaniappan and K.C.Rao, Regular generalized closed sets, Kyungpook,Math. J., 3(1993), 211-219.
13. J.K. Park and J.H. Park, Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos, Solutions and Fractals 20(2004), 1103-1111.
14. V.Popa & T.Noiri,On M- continous functions,Anal.Univ."Dunurea de Jos" Galati,ser.Mat.Fiz..Mec.Teor.(2),18 (23) (2000),31-41.
15. A. Vadivel and K. Vairamanickam, rgα-closed sets and rgα-open sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 3, 2009, no. 37,1803-1819.

*****

----

[1]N.Murugavalli, Department of Mathematics, Sri Eshwar College of Engineering,Coimbatore-641 202, Tamil Nadu, India,
n.murugavalli@yahoo.com
[2]A.Pushpalatha, Department of Mathematics,
Government Arts College, Udumalpet-642 126, Tamil Nadu, India
pushpa_velu@yahoo.co.in