mrgα-CLOSED AND mrgα-OPEN MAPS IN MINIMAL STRUCTURES

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Abstract: *In this paper, we introduce mrgα-closed map , mrgα-open map and some of its basic properties.*

Key words: *$M_{1/2}$ –space, m-space, mrgα-closed map and mrgα-open map*

1. INTRODUCTION

V.Popa and T.Noiri[9] introduced concept of minimal structure on a nonempty set. Also they introduced the notion of m_x-open set and m_x-closed set and characterize those sets using m_x-cl and m_x-int operators respectively. Further they introduced m-continuous functions and studied some of its basic properties. T.Noiri[8] introduced the concept of mg-closed sets in minimal structures which is analogs to g-closed sets in topological space introduced by Levine. N[3]

Generalized closed mappings were introduced and studied by Malghan[5]. Regular closed maps, gpr-closed maps and rg-closed maps have been introduced and studied by Long[4], Gnanambal[2] and Arockiarani[1] respectively. In this paper, a new class of maps called minimal regular generalized α-closed (briefly,mrgα-closed) maps, mrgα - closed maps have been introduced and studied their relations with various generalized closed maps. We prove that the composition of two mrgα-closed maps need not be mrgα-closed map. We also obtain some properties of mrgα-closed maps.

2. PRELIMINARIES

Definition 2.1:[9] A subfamily m_x of the power set P(X) of a non empty set X is called as minimal structure on X if $\emptyset \in m_x$ and $X \in m_x$. Minimal space is denoted by (X, m_x) where X is a nonempty set with a minimal structure m_x on X.

Definition 2.2: [9] Let (X, m_x) be a minimal space with a minimal structure m_x on X for a subset A of X, the closure of A and the interior of A are defined as follows

$$\text{mint}(A) = \bigcup\{U : U \subseteq A, U \in m_x\}$$
$$\text{mcl}(A) = \bigcap\{F : A \subseteq F, X-F \in m_x.\}$$

Definition 2.3:[8] Let (X, m_x) be an m-space, A subset A of X is said to be mg-closed if m_x -cl(A) G whenever $A \subseteq G$ and G is m_x -open. The complement of an mg-closed set is said to be mg-open set.

Definition 2.4: [9] A minimal structure m_x on a nonempty set X is said to have property B if the union of any family of subsets belong to m_x.

Lemma 2.1: [9] Let X be a nonempty set and m_x a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:

(i) $A \in m_x$ if and only if m_x -int(A) = A
(ii) A is m_x -closed if and only if m_x-cl(A) = A
(iii) m_x -int(A)$\in m_x$ and m_x -cl(A) is m_x -closed.

Definition 2.5:[7] A subset A of a minimal space (X,m_x) is called minimal regular generalized α- closed set (mrgα-closed set) if mαcl(A)$\subset$U whenever A$\subset$U and U is a minimal regular α- open set in X. We denote the set of all mrgα-closed sets in (X,m_x) by MRGαcl(X).

Definition 2.6: [6] A subset A of a Minimal space (X,m_x) is called

2.6.1) a minimal preopen set (briefly mp-open) if A $\subseteq$ m_x-int(m_x- cl(A)) and a minimal preclosed set if m_x–cl(m_x- int (A)) $\subseteq$ A.

2.6.2) a minimal semiopen set (briefly ms-open) if

A $\subseteq$ m_x-cl(m_x-int(A)) and a minimal semiclosed set (briefly ms-closed) if m_x -int(m_x- cl (A)) $\subseteq$ A.

2.6.3) a minimal α-open set (briefly mα-open) if A $\subseteq$ m_x- int(m_x- cl(m_x-int (A))) and a minimal α-closed set (briefly mα-closed) if m_x –cl(m_x- int(m_x- cl (A))) $\subseteq$ A.

2.6.4) a minimal semi-preopen set (briefly msp-open) if

A $\subseteq$ m_x- cl(m_x- int(m_x- cl(A))) and a minimal semi-preclosed set briefly msp-closed) if m_x -int(m_x- cl(m_x- int (A))) $\subseteq$ A.

2.6.5) a minimal regular open set (briefly mr-open) if

A = m_x-int(m_x- cl(A)) and a minimal regular closed set (briefly mr-closed) if A = m_{x}- cl(m_x- int (A)).

2.6.6) a minimal regular semiopen (briefly mrs-open) if there is a minimal regular open U such U $\subseteq$ A $\subseteq$ m_x- cl(U). The family of all regular semiopen sets of X is denoted by RMSO(X).

2.6.7) minimal generalized closed set (briefly, mg-closed) if m_x -cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is m_x- open in X.

Note 2.7: (i) The minimal closure of a minimal semiopen set is minimal regular closed.

(ii) Every minimal regular closed set is minimal semiopen set.

The intersection of all minimal semiclosed (resp-minimal semiopen) subsets of X containing A is called the minimal semi-closure of A and is denoted by mscl(A). Also the intersection of all minimal preclosed (resp. minimal semi-preclosed and minimal α-closed) subsets of X containing A is called minimal pre-closure (resp. minimal semi-pre closure and minimal α-closure) of A and is denoted by mpcl(A) (resp.mspcl(A) and mαcl (A)

Definition 2.7: [9] A map f : $(X, m_x) \to (Y, m_y)$ is called

mg-continuous if $f^{-1}(V)$ is mg-closed set of (X, m_x) for every m-closed set V of (Y, m_y),

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We introduce the following definitions

Definition 3.1: A map f : $(X, m_x) \to (Y, m_y)$ is called

(i) mrg-closed if f(F) is mrg-closed in (Y, m_y) for every m-closed set F of (X, m_x),
(ii) minimal regular closed (mr-closed) if f(F) is m-closed in (Y, m_y) for every mr- closed set F of (X, m_x)
(iii) mw-open if f(U) is mw-open in (Y, m_y) for every m-open set U of (X, m_x).

Definition 3.2: A map f: $(X, m_x) \to (Y, m_y)$ is said to be minimal regular generalized α- closed set (mrgα-closed) if the image of every m-closed set in (X, m_x) is mrgα-closed in (Y, m_y).

Definition 3.3: A minimal space (X, m_x) is called $M_{1/2}$ – space if every mg-closed set in X is m-closed in X.

Definition 3.4: A map f : $(X, m_x) \to (Y, m_y)$ is called

(i) mrgα-continuous if the inverse image of every m-closed set in (Y, m_y) is mrgα-closed set in (X, m_x).
(ii) mrgα-irresolute map if the inverse image of every mrgα-closed set in (Y, m_y) is mrgα-closed in (X, m_x).
(iii) strongly mrgα-continuous if the inverse image of every mrgα-open set in (Y, m_y) is m-open in (X, m_x)

Theorem 3.4: Every m-closed map is mrgα-closed map, but not conversely.

Proof: The proof follows from the definitions and fact that every m-closed set is mrgα-closed .

The converse of the above theorem need not be true, as seen from the following example.

Example 3.5: Consider X = Y = {a,b,c} with minimal structures m_x = {X, ϕ, {a} , {a,b}} and m_y = {Y, ϕ {a}.Let
f: (X, m_x) → (Y, m_y) be the identity map. Then this function is mrgα-closed but not m- closed, as the image of m-closed set {c} in X is {c} which is not m-closed in Y.

Theorem 3.6: Every mrgα-closed map is mrg-closed map but not conversely.

Proof: The proof follows from the definitions and fact that every mrgα-closed set is mrg-closed.
The converse of the above theorem need not be true, as seen from the following example.

Example3.7: Consider X = {a,b,c}, Y = {a,b,c,d} with minimal structures m_x = {X, ϕ, {a},{c},{a,c}} and m_y = {Y, ϕ, {a},{b}, {a,b}}.Let the map f: (X, m_x) → (Y, m_y) be defined by f(a) = a, f(b) = c and f(c) = d. Then this function is mrg-closed but not mrgα-closed, as the image of the m-closed set {a,b} in X is {a,c} which is not mrgα-closed in Y.

Theorem 3.8: A map f: (X, m_x) → (Y, m_y) is mrgα-closed if and only if for each subset S of (Y, m_y) and each m-open set U containing $f^{-1}(S) \subset U$, there is a mrgα-open set V of (Y, m_y) such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof: Suppose f is mrgα-closed. Let $S \subset Y$ and U be an m-open set of (X, m_x) such that $f^{-1}(S) \subset U$. Now X − U is m-closed set in (X, m_x). Since f is mrgα-closed, f(X − U) is mrgα-closed set in (Y, σ). Then V = Y − f(X − U) is a mrgα-open set in (Y, m_y). Note that $f^{-1}(S) \subset U$ implies $S \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X - U)) \subset X - (X - U) = U$. That is $f^{-1}(V) \subset U$.For the converse, let F be a m-closed set of (X, m_x). Then $f^{-1}(f(F)^c) \subset F^c$ and F^c is an m-open in (X, m_x). By hypothesis, there exists a mrgα-open set V in (Y, σ) such that $f(F)^c \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f(((f-1(V))^c) \subset V^c$ which implies $f(V) \subset V^c$. Since V^c is mrgα-closed, f(F) is mrgα-closed. That is f(F) is mrgα-closed in (Y, m_y) and therefore f is mrgα-closed.

Remark 3.9 : The composition of two mrgα-closed maps need not be mrgα-closed map in general and this is shown by the following example.

Example 3.10 : Let X = Y = Z = {a, b, c}, m_x = P(X), m_y = {Y, ϕ, {c} , {a, b}} and m_z = {Z, ϕ, {a} , {b} , {a, b}}. Define f: (X, m_x) → (Y, m_y) by f(a) = a, f(b) = b and f(c) = c and
g : (Y, m_y) → (Z, m_z) be the identity map. Then f and g are mrgα-closed maps, but their composition $g \circ f$: (X, m_x)→ (Z, m_z) is not mrgα-closed map, because F ={a} is m-closed in (X, m_x) but $g \circ f(F) = g \circ f(\{a\}) = g(f(\{a\})) = g(\{a\}) = \{a\}$ which is not mrgα-closed in (Z, m_z).

Theorem 3.11: If f : (X, m_x) → (Y, m_y) is m-closed map and

$g : (Y, m_y) \to (Z, m_z)$ is mrgα-closed map, then the composition $g \circ f : (X, m_x) \to (Z, m_z)$ is mrgα-closed map.

Proof: Let F be any m-closed set in (X, m_x). Since f is m-closed map, f(F) is m-closed set in (Y, m_y). Since g is mrgα-closed map, g(f(F)) is mrgα-closed set in (Z, m_z). That is $g \circ f(F) = g(f(F))$ is mrgα-closed and hence $g \circ f$ is mrgα-closed map.

Remark 3.12: If $f : (X, m_x) \to (Y, m_y)$ is mrgα-closed map and $g : (Y, m_y) \to (Z, m_z)$ is m-closed map, then the composition need not be mrgα-closed map as seen from the following example.

Example 3.13: Consider X = Y = Z = {a, b, c}, m_x = {X, ϕ, {a} , {b} ,{a, b}}, m_y = {Y, ϕ, {a} , {b, c}} and m_z = {Z, ϕ, {b} , {c} , {b, c}} Let $f : (X, m_x) \to (Y, m_y)$ be the identity map and
$g : (Y, m_y) \to (Z, m_z)$ is defined by g(a) = g(b) = a and g(c) = b. Then f is mrgα-closed map and g is a m-closed map. But their composition $g \circ f : (X, m_x) \to (Z, m_z)$ is not mrgα-closed map, since for the m-closed set {c} in (X, m_x), but $g \circ f(\{c\}) = g(f(\{c\})) = g(\{c\}) = \{b\}$ which is not mrgα-closed in (Z, m_z).

Theorem 3.14: Let (X, m_x), (Y, m_y) and (Z, m_z) be minimal spaces where "every mrgα-closed subset is m-closed". Then the composition $g \circ f : (X, m_x) \to (Z, m_z)$ of the mrgα-closed maps
$f : (X, m_x) \to (Y, m_y)$ and $g : (Y, m_y) \to (Z, m_z)$ is mrgα-closed.

Proof: Let A be a m-closed set of (X, m_x). Since f is mrgα-closed, f(A) is mrgα-closed in (Y, m_y). Then by hypothesis, f(A) is m-closed. Since g is mrgα-closed, g(f(A)) is mrgα-closed in (Z, m_z) and $g(f(A)) = g \circ f(A)$. Therefore $g \circ f$ is mrgα-closed.

Theorem 3.15.:If $f : (X, m_x) \to (Y, m_y)$ is mg-closed,
$g : (Y, m_y) \to (Z, m_z)$ be mrgα-closed and (Y, m_y) is $M_{1/2}$-space then their composition $g \circ f : (X, m_x) \to (Z, m_z)$ is mrgα-closed map.

Proof: Let A be a m-closed set of (X, m_x). Since f is mg-closed, f(A) is mg-closed in (Y, m_y). Since (Y, m_y) is $M_{1/2}$-space, f(A) is m-closed in (Y, m_y). Since g is mrgα-closed, g(f(A)) is mrgα-closed in (Z, m_z) and $g(f(A)) = g \circ f(A)$. Therefore $g \circ f$ is mrgα-closed.

Theorem 3.16: Let $f : (X, m_x) \to (Y, m_y)$ and
$g : (Y, m_y) \to (Z, m_z)$ be two mappings such that their composition $g \circ f : (X, m_x) \to (Z, m_z)$ be mrgα-closed mapping. Then the following statements are true.
(i) If f is m-continuous and surjective, then g is mrgα-closed.
(ii) If g is mrgα-irresolute and injective, then f is mrgα-closed.
(iii) If f is mg-continuous, surjective and (X, m_x) is a $M_{1/2}$-space, then g is mrgα-closed.
(iv) If g is strongly mrgα-continuous and injective, then f is mrgα-closed.

Proof: (i) Let A be a m-closed set of (Y, m_y). Since f is m-continuous, $f^{-1}(A)$ is m-closed in (X, m_x) and since g ∘ f is mrgα-closed, (g ∘ f)(f^{-1} (A)) is mrgα-closed in (Z, m_z). That is g(A) is mrgα-closed in (Z, m_z), since f is surjective. therefore g is mrgα-closed.

(ii) Let B be a m-closed set of (X, m_x). Since g ∘ f is mrgα- closed,
g ∘ f(B) is mrgα-closed in (Z, m_z). Since g is mrgα-irresolute, g^{-1} (g ∘ f(B)) is mrgα-closed set in (Y, m_y). That is f(B) is mrgα-closed in (Y, m_y), since f is injective. Therefore f is mrgα-closed.

(iii) Let C be a m-closed set of (Y, m_y). Since f is mg-continuous, f^{-1} (C) is mg-closed set in (X, m_x). Since (X, m_x) is a $M_{1/2}$-space, $f^{-1}(C)$ is m-closed set in (X, m_x). Since g ∘ f is mrgα-closed,
(g ∘ f)(f^{-1} (C)) is mrgα-closed in (Z, m_z). That is g(C) is mrgα-closed in (Z, m_z), since f is surjective. Therefore g is mrgα-closed.

(iv) Let D be a m-closed set of (X, m_x). Since g ∘ f is mrgα-closed, (g ∘ f)(D) is mrgα-closed in (Z, m_z). Since g is strongly mrgα-continuous, g^{-1}((g ∘f)(D) is m-closed set in (Y, m_y). That is f(D) is m-closed set in (Y, m_y), since g is injective, Therefore f is m-closed.

Theorem 3.17: If f : (X, m_x) → (Y, m_y) is an m-open, m-continuous, mrgα-closed surjection and mcl(F) = F for every mrgα-closed set in (Y, m_y), where X is m-regular,then Y is m-regular.

Proof: Let U be an m-open set in Y and p ∈ U. Since f is surjection, there exists a point x ∈ X such that f(x) = p. Since X is m-regular and f is m-continuous,there is an m-open set V in X such that x ∈ V ⊂ mcl(V) ⊂ $f^{-1}(U)$. Here p ∈ f(V) ⊂ f(mcl (V)) ⊂ U → (i). Since f is mrgα-closed, f(mcl(V)) is mrgα-closed set contained in the m-open set U. By hypothesis, mcl(f(mcl(V))) = f(mcl(V)) and mcl(f(V)) = mcl(f(mcl(V))) → (ii). From (i) and (ii), we have p ∈ f(V) ⊂ mcl(f(V)) ⊂ U and f(V) is m-open, since f is m-open. Hence Y is m-regular.

Definition 3.18: A map f : (X, m_x) → (Y, m_y) is called a mrgα-open map if the image f(A) is mrgα-open in (Y, m_y) for each m-open set A in (X, m_x).

From the definitions we have the following results.

Theorem 3.19:(i)Every m-open map is mrgα-open but not conversely.
(ii) Every mw-open map is mrgα-open but not conversely.
(iii) Every mrgα-open map is mrg-open but not conversely.
(iv) Every mrgα-open map is mrwg-open but not conversely.
(v) Every mrgα-open map is mgpr-open but not conversely.

Theorem 3.20: For any bijection map f : (X, m_x) → (Y, m_y), the following statements are equivalent:
(i) f^{-1} : (Y, m_y) → (X, m_x) is mrgα-continuous.
(ii) f is mrgα-open map and (iii) f is mrgα-closed map.

Proof:(i) $\Rightarrow$ (ii) Let U be an m-open set of (X, m_x). By assumption, $(f^{-1})^{-1}(U) = f(U)$ is mrgα-open in (Y, m_y) and so f is mrgα-open.
(ii) $\Rightarrow$ (iii) Let F be a m-closed set of (X, m_x). Then F^c is m-open set in (X, m_x). By assumption, $f(F^c)$ is mrgα-open in (Y, m_y). That is $f(F^c) = f(F)^c$ is mrgα-open in (Y, m_y) and therefore f(F) is mrgα-closed in (Y, m_y). Hence f is mrgα-closed.
(iii) $\Rightarrow$ (i) Let F be a m-closed set of (X, m_x). By assumption, f(F) is mrgα-closed in (Y, m_y). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is m-continuous.

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