SOME CHAIN GRAPHS ARE GRACEFUL

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Abstract: In this paper, we have shown that the chain graphs obtained by considering the cluster of diamonds having a vertex or a diamond in common at their join are graceful. We have also considered the chain of cycles of different order and the chain of complete bipartite graphs of different order and proved them to be graceful.

1. INTRODUCTION

Rosa.A [1] in 1967, initiated the method of labeling by the name α-valuation and Golomb[2] gave the name graceful for such a concept of labeling. C.Delorme *et al*[3] have proved that any cycle with a chord is graceful. Christian Barrientons[4] has proved that the chain graph of blocks $B_1, B_2, B_3, \ldots, B_m$ accepts an α-labeling. In this present paper, we have proved that the chian of cluster of diamonds are graceful if they have either a vertex or a diamond in common at the join. In addition, we have focused our attention on the chain of cycles of different order and the chain of complete bipartite graphs of different order and proved that they are graceful if they have a common edge at the join.

Definition 1: A graph G = (V(G) , E(G)) with p vertices and q edges is said to admit *graceful labeling* if $f : V(G) \rightarrow \{0, 1, \ldots, q\}$ such that distinct vertices receive distinct numbers and $\{|f(u) - f(v)| : uv \in E(G)\} = \{1, 2, 3, \ldots, q\}$. A graph which is labeled gracefully is known as graceful graph.

To start with we consider a simple chain of diamonds which is nothing but the chain of cycle C_4 and the idea is developed for the chain of different cluster of diamonds.

Theorem 1: The link graph of diamonds is graceful.

Proof : The link graph which is a chain of diamonds (cycles C_4) is shown in figure-1.

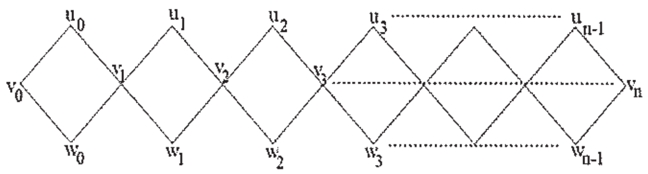

Figure 1.

The link graph is labeled gracefully by defining the function f as follows:

$f : V(G) \rightarrow \{0, 1, \ldots, q\}$

$$f(u_i) = q-2i, \qquad 0 \leq i \leq n-2.$$

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$f(v_i) = 2i, \quad 0 \le i \le n-1.$

$f(w_i) = (q-1)-2i, \quad 0 \le i \le n-2.$

Where q is the number of edges in the graph and n is the number of vertices in the middle of the graph.

Illustration 1:

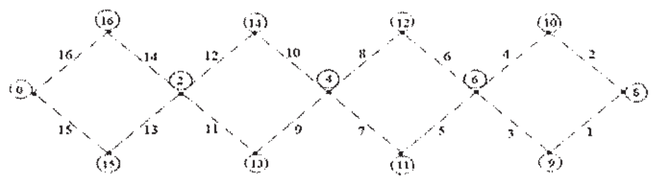

Figure 2.

Theorem 2: The chain of cluster-4 of diamonds is graceful.

Proof : The chain graph with the repetition of pattern is shown in figure-3.

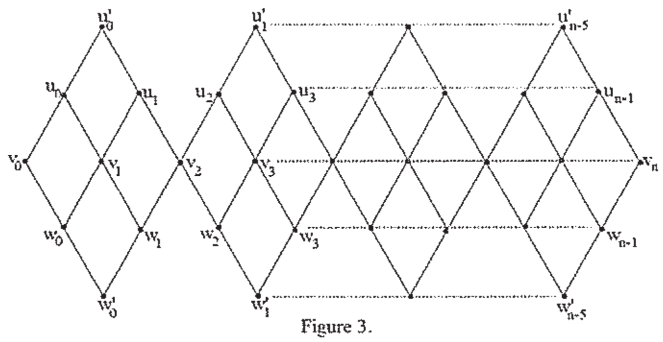

Figure 3.

This type of link graph is labeled gracefully by defining the function f as follows:

$f : V(G) \to \{0, 1, \ldots, q\}$ such that

$f(u_i') = 2+6i, \quad 0 \le i \le n-6.$

For the vertices ui, $0 \le i \le n-2$ the labeling is given by

$f(u_i) = (q-1)-3i,$ if i is odd.

$f(u_i) = q-3i,$ if i is even.

$f(v_i) = 3i, \quad 0 \le i \le n-1.$

For the vertices wi, $0 \le i \le n-2$ the labeling is given by

$f(w_i) = (q-2)-3i,$ if i is odd.

$f(w_i) = (q-1)-3i,$ if i is even.

$f(w_i') = 4+6i, \quad 0 \le i \le n-6.$

where q is the number of edges in the graph and n is the number of vertices in the middle of the graph.

Illustration 2:

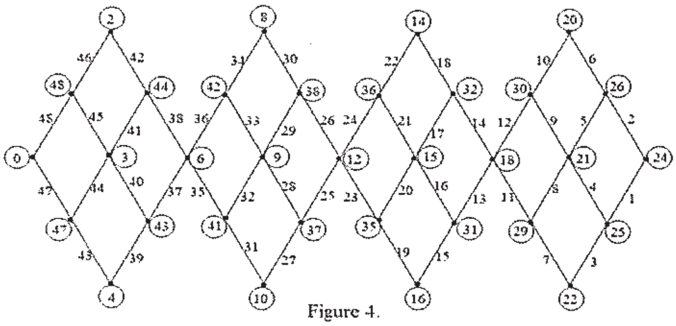

Figure 4.

Theorem 3: The chain of cluster-4 of diamonds having a common diamond at the join is graceful.

Proof : The chain graph of this type is shown in figure-5.

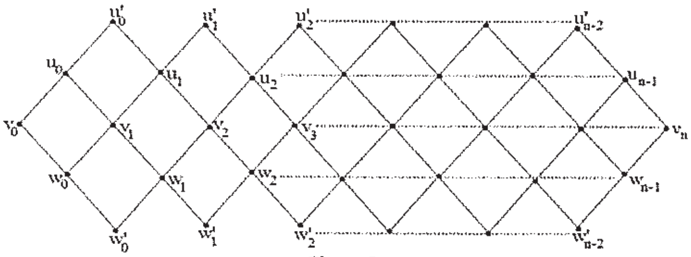

Figure 5.

This chain can be labeled gracefully by defining the function f as follows:

$f : V(G) \rightarrow \{0, 1, \ldots, q\}$ such that

$f(u_i') = 2+4i,$ $\quad 0 \le i \le n-3.$

$f(u_i) = q-4i,$ $\quad 0 \le i \le n-2.$

$f(v_0) = 0.$

$f(v_i) = 3+4(i-1),$ $\quad 1 \le i \le n-2.$

$f(v_{n-1})$ $\quad = 2+4(n-2).$

$f(w_i) = q-1-4i,$ $\quad 0 \le i \le n-2.$

$f(w_i') = 4+4i,$ $\quad 0 \le i \le n-3.$

where q is the number of edges in the graph and n is the number of vertices in the middle of the graph.

Illustration 3:

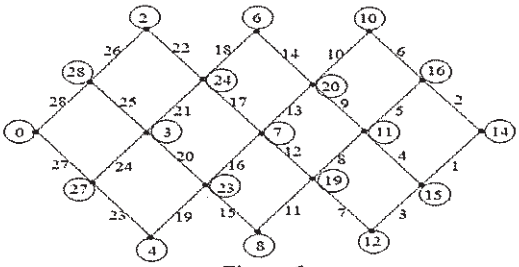

Figure 6.

Theorem 4: The chain of diamonds of different clusters is graceful.

Proof : The diamonds of different clusters can be joined at the end vertices in such a way that the resulting graph is a chain graph. This graph can be labeled gracefully. This is illustrated as follows.

Illustration 4:

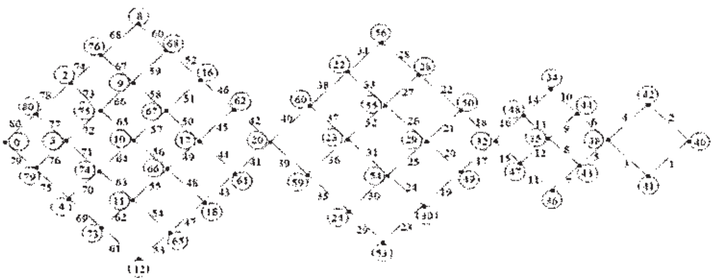

Figure 7.

Theorem 5: The chain joining the cycles of different order is graceful.

Proof : Consider the cycles of different order. These cycles are joined to each other in such a way that adjacent cycles share a common edge. This newly obtained graph is labeled gracefully and is shown in the following illustration.

Illustration 5:

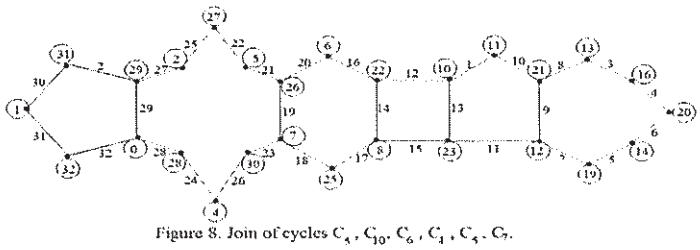

Figure 8. Join of cycles C_5, C_{10}, C_6, C_4, C_4, C_7.

Theorem 6: The chain joining the complete bipartite graphs $K_{2,m}$ for different m is graceful.

Proof : Consider complete bipartite graphs $K_{2,m}$ (here m takes different values) and join the complete bipartite graphs in such a way that they share a common edge at the join. This type of resultant graph can also be labeled gracefully. This is illustrated as follows.

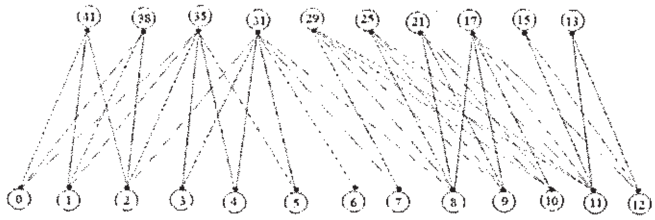

Figure 9. Join of complete bipartite graphs $K_{3,3}$, $K_{2,4}$, $K_{1,3}$, $K_{2,2}$, $K_{4,4}$, $K_{3,2}$.

Illustration6:

The vertex labeled 0 has the incident edges labeled as 41, 38 and 35; the vertex labeled 1 has the incident edges labeled as 40, 37 and 34; the vertex labeled 2 has the incident edges labeled as 39, 36, 33 and 29; the vertex labeled 3 has the incident edges labeled as 32 and 28; the vertex labeled 4 has the incident edges labeled as 31 and27; the vertex labeled 5 has the incident edges labeled as 30 and 26; the vertex labeled 6 has the incident edges labeled as 25; the vertex labeled 7 has the incident edges labeled as 24 and22; the vertex labeled 8 has the incident edges labeled as 23, 21, 17, 13 and 9; the vertex labeled 9 has the incident edges labeled as 20, 16,12 and 8; the vertex labeled 10 has the incident edges labeled as 19, 15, 11 and7; the vertex labeled 11 has the incident edges labeled as 18, 14, 10, 6, 4 and 2; the vertex labeled 12 has the incident edges labeled as 5, 3 and 1.

2. CONCLUSION

The gracefulness of different types of link graphs and the join of cycles of different order at the edge and the join of complete bipartite graph $K_{2,m}$ for different m at the edge is proved.

3. REFERENCES

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