

ARBITRARY SUPERSUBDIVISION OF HELMS, CENTIPEDES AND LADDER GRAPHS ARE GRACEFUL

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Abstract: In the present paper, we discuss the graceful labeling of arbitrary supersubdivision of all the edges of helms, centipedes and ladder graphs by a complete bipartite graphs $K_{2,m}$ for any m .

1. INTRODUCTION

Graceful labeling is assigned to both vertices and edges subject to certain conditions. The labeling of graphs is an area of significance now-a-days. A.Rosa[1] was the pioneer in introducing labeling techniques as α -valuation and Golomb[2] has given the name graceful for such a concept of labeling.

A complete survey on graceful labeling is given in Gallian[3]. G.Sethuraman and P.Selvaraju[4] have discussed about the gracefulness of arbitrary supersubdivision of paths and cycles. But while considering cycles the arbitrary supersubdivision of all edges of cycles by the bipartite graphs, except few, are not graceful. In our study we have proved that the arbitrary supersubdivision of the cycles C_n by $K_{2,n-1}$ is always graceful. K.M.Kathiresan and S.Amutha[5] have discussed the arbitrary supersubdivision of stars to be graceful.

In this paper, we have proved that the arbitrary supersubdivision of the edges of helms, centipedes and ladder graphs are graceful.

Definition 1: A graph $G = (V(G), E(G))$ with p vertices and q edges is said to admit *graceful labeling* if $f : V(G) \rightarrow \{0, 1, \dots, q\}$ such that distinct vertices receive distinct numbers and $\{|f(u) - f(v)| : uv \in E(G)\} = \{1, 2, 3, \dots, q\}$. A graph which is labeled gracefully is known as graceful graph.

Definition 2: The *n-centipede* (C_n) is tree with $2n$ vertices obtained by joining the bottoms of n copies of the path graph (P_n) laid in a row with edges.

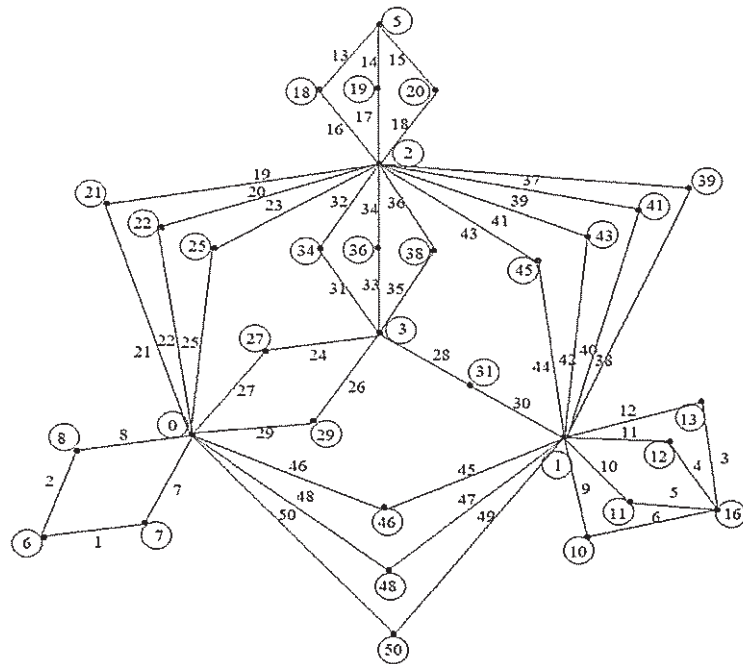
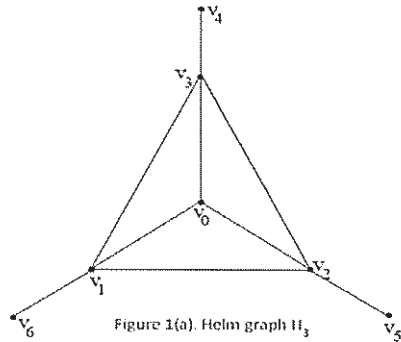
Definition 3: The *ladder graph* (L_n) is obtained as the Cartesian product of two path graphs P_{n+2} and P_1 considering the end vertices as pendent vertices. L_n consists of $2(n+2)$ vertices and $3n+2$ edges.

Theorem 1: Arbitrary supersubdivision of helm graph by $K_{2,m}$ for any m is graceful.

Proof: Supersubdivision of all edges of helm graph by a complete bipartite graph $K_{2,m}$ can be labeled gracefully where m may take any value.

This is illustrated as follows:

Illustration 1:



Theorem 2: Arbitrary supersubdivision of centipede graph C_n by $K_{2,m}$ for any m is graceful.

Proof : Supersubdivision of all the edges of the centipede graph by a complete bipartite graph $K_{2,m}$ can be labeled gracefully where n may take any value.

This is illustrated as follows:

Illustration 2:

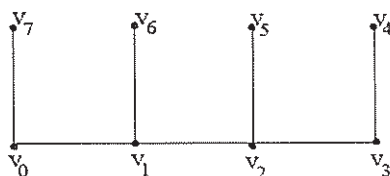


Figure 2(a). Centipede graph C_4 .

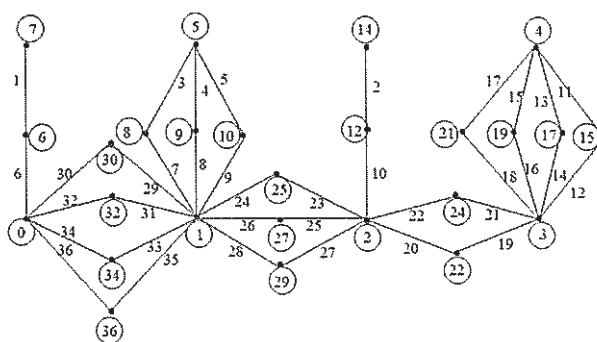


Figure 2(b). Supersubdivision of C_4 by $K_{2,1}, K_{2,2}, K_{2,3}, K_{2,4}$.

Theorem 3: Arbitrary supersubdivision of ladder graph L_n by $K_{2,m}$ for any m is graceful.

Proof : Supersubdivision of all the edges of the ladder graph by a complete bipartite graph $K_{2,m}$ can be labeled gracefully where m may take any value.

This is illustrated as follows.

Illustration 3:

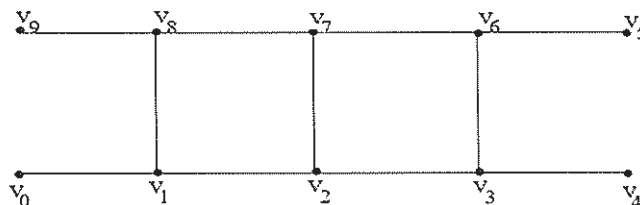


Figure 3(a). Ladder graph L_3

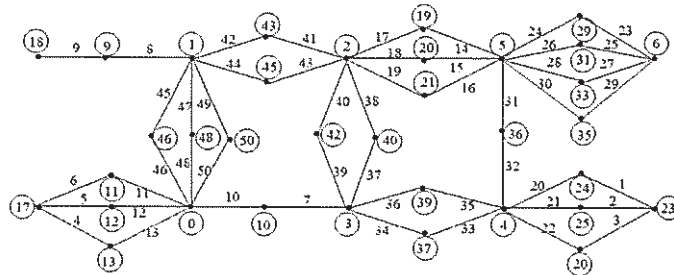


Figure 3(b). Supersubdivision of ladder graph L_3 by $K_{2,1}, K_{2,2}, K_{2,3}, K_{2,4}$.

2. CONCLUSION

The gracefulness of larger graphs obtained from helm, centipede and ladder graphs by taking the supersubdivision of all the edges by a complete bipartite graph $K_{2, m}$ for any m is proved.

3. REFERENCES

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