FIXED POINT THEOREMS FOR SELF-MAPS ON *G*-METRIC SPACES

Dr. C. Jaya Subba Reddy[1], V.Srinivas Chary[2], T. Mahesh Kumar[3], K.Hemavathi[4]

Abstract: *Metric spaces are playing an increasing role in mathematics and applied sciences. Over past two decades the development of fixed point theory in metric spaces has attracted considerable attention due to more applications in this area. Gahler [2] introduced the notion of 2-metric spaces as a generalization of usual metric spaces and Ha [3] proved that, the 2-metric need not be continuous and further there is no easy relation between the two settings. Dhage [1] introduced the concept of D-metric spaces and developed the topological structure in it. But unfortunately it was proven that the topological structure of D-metric space is incorrect and almost all results related to the topological structure of D-metric spaces.*

As a probable modification of D-metric spaces, Mustafa [4] introduced the concept of G-metric spaces as follows:

Definition: Let X be a nonempty set and let $G:X\times X\times X\to[0,\infty)$ be a function satisfying the following axioms:

(G1) $G(x,y,z)=0$ if $x=y=z$,

(G2) $0<G(x,x,y)$ for all $x,y\in X$ with $x\neq y$,

(G3) $G(x,x,y)\leq G(x,y,z)$ for all $x,y,z\in X$ with $y\neq z$,

(G4) $G(x,y,z)=G(z,x,y)=G(y,z,x)=\cdots$ (Symmetry in all three variables),

(G5) $G(x,y,z)\leq G(x,a,a)+G(a,y,z)$ for all $x,y,z,a\in X$ (rectangle inequality)

Then the function G is called a generalized metric or general metric on X and the pair (X,G) is called a *G-metric space.*

Example: Let (X,d) be a usual metric space, then (X,G_s) and (X,G_m) are *G*-metric spaces, where

$$G_s(x,y,z)=d(x,y)+d(y,z)+d(z,x)\ \forall\, x,y,z\in X$$

$$G_m(x,y,z)=\max\{d(x,y),d(y,z),d(z,x)\}\ \forall\, x,y,z\in X$$

Definition: Let (X,G) be a *G*-metric space and $\{x_n\}$ is a sequence of points of *X*, we say that, $\{x_n\}$ is *G*-converges to x if $\lim\limits_{n,m\to\infty} G(x,x_n,x_m)=0$;

That is, for any $\varepsilon>0$, there exists $N\in N$ such that, $G(x,x_n,x_m)<\varepsilon$ for all $n,m\geq N$

One can easily prove the following results:

1.1 Theorem: Let (X,G) be a *G*-metric space, then the following are equivalent:

$\{x_n\}$ is *G*-converges to x.

$G(x_n,x_n,x)\to 0$ as $n\to\infty$.

$G(x_n,x,x)\to 0$ as $n\to\infty$.

Proof: Proof is trivial and it is given in [8].

Definition: Let (X,G) be a *G*-metric space and $\{x_n\}$ is said to be *G*-Cauchy sequence if for a given $\varepsilon>0$, there exists $N\in N$ such that, $G(x_n,x_m,x_l)<\varepsilon$ for all $n,m,l\geq N$, that is, $\lim\limits_{n,m,l\to\infty} G(x_n,x_m,x_l)\to 0$ as $n,m,l\to\infty$.

1.2 Theorem: In a *G*-metric space (X,G), the following are equivalent:

The sequence $\{x_n\}$ is *G*-Cauchy.

For every $\varepsilon>0$, there exists $N\in N$ such that, $G(x_n,x_m,x_m)<\varepsilon$ for all $n,m\geq N$.

Proof: Proof is trivial and it is given in [9].

Definition: Let (X,G) and (X',G') be *G*-metric space and let $f:X\to X'$ be a function, then f is said to be *G-continuous at a point* $a\in X$ if for every given

$\varepsilon > 0$, there exists $\delta > 0$ such that $x, y \in X; G(a, x, y) < \in$ implies $G^1(f(a), f(x), f(y)) < \in$.

A function f is G – continuous on x if and only if it is

G – continuous at all $a \in X$.

1.3. Theorem: Let (X, G) and (X', G') be G-metric spaces, then a function $f : X \longrightarrow X'$ is *G-continuous at a point* $a \in X$ if and only if it is G-sequentially continuous at a; that is, when ever $\{x_n\}$ is G-converges to a implies $f(x_n)$ is G-converges to $f(a)$.

Proof: Proof is trivial and it is given in [9].

1.4. Theorem: Let (X, G) be a G-metric space, then the function $G(x, y, z)$ is jointly continuous in all the three variables.

Proof: Proof is trivial and it is given in [9].

To Prove The Main Theorems

2.1 Theorem: Let (X, G) be a complete G-metric space and let $T : X \longrightarrow X$ be a mapping which satisfies the following condition, for all $x, y, z \in X$,

$$G(Tx, Ty, Tz) \le k.\max\{G(x, y, z), G(x, Tx, Tx), G(y, Ty, Ty), G(z, Tz, Tz), G(x, Ty, Ty), G(y, Tz, Tz), G(z, Tx, Tx)\} \qquad (2.1.1)$$

Where $0 \le k < 1/2$. Then prove that T has unique fixed point (say u) and T is G-continuous at u .

Proof: Suppose that T satisfies the condition (2.1.1), let $x_0 \in X$ be an arbitrary point and define the sequence $\{x_n\}$ by $x_n = T^n x_0$, them by (2.1.1), we have

$$G(x_n, x_{n+1}, x_{n+1}) = k.\max\{G(x_{n-1}, x_n, x_n), G(x_{n-1}, x_{n+1}, x_{n+1}), G(x_n, x_{n+1}, x_{n+1})\}$$

so,

$$G(x_n, x_{n+1}, x_{n+1}) \le k.\max\{G(x_{n-1}, x_n, x_n), G(x_{n-1}, x_{n+1}, x_{n+1})\} \quad (2.1.2)$$

But, by (G5)

$G(x_{n-1}, x_{n+1}, x_{n+1}) \le G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})$ so, (2.1.2) now becomes

$$G(x_n, x_{n+1}, x_{n+1}) \le k.\max\{G(x_{n-1}, x_n, x_n), G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})\}$$

$$G(x_n, x_{n+1}, x_{n+1}) \le k\{G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})\}$$

This implies that,

$$G(x_n, x_{n+1}, x_{n+1}) \le \frac{k}{1-k}.G(x_{n-1}, x_n, x_n) \quad (2.1.3)$$

Let $q = k/_{1-k}$, then, $q < 1$ and by repeated application of (2.1.3), we have

$$G(x_n, x_{n+1}, x_{n+1}) \le q^n.G(x_{n-1}, x_n, x_n) \quad (2.1.4)$$

Then, for all $n, m \in N, n < m$, we have by repeated use of rectangle inequality, and (2.1.4) that,

$$\begin{aligned} G(x_n, x_m, x_m) &\le G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + G(x_{n+2}, x_{n+3}, x_{n+3}) \\ &+ \ldots + G(x_{m-1}, x_m, x_m) \\ &\le \left(q^n + q^{n+1} + q^{n+2} + \ldots + q^{m-1}\right) G(x_0, x_1, x_1) \\ &\le \frac{q^n}{1-q} G(x_0, x_1, x_1) \end{aligned}$$

Then, since $\lim_{n,m\to\infty} \frac{q^n}{1-q} G(x_0, x_1, x_1) = 0$, showing that, $\{x_n\}$ is a convergent sequence and hence a Cauchy sequence. There exists $u \in X$ such that $\{x_n\}$ converges to some point, since (X, G) is a complete *G*-metric space.

To show that, u is a fixed point of T. Assume that, $Tu \neq u$, then

$$G(x_n,Tu,Tu) \leq k.\max\left\{\begin{matrix} G(x_{n-1},u,u), G(x_{n-1},x_n,x_n), G(u,Tu,Tu) \\ G(x_{n-1},Tu,Tu), G(u,x_n,x_n) \end{matrix}\right\}$$

Letting $n \to \infty$ on both the sides and using the fact that, the function G is continuous on its variables, we have $G(u,Tu,Tu) \leq k.G(u,Tu,Tu)$, which is a contradiction since $0 \leq k < 1/2$. Hence $Tu = u$.

To prove that, T has unique fixed point, assume $v \neq u$, such that, $Tv = v$, then (2.1.1) implies that

$$G(u,v,v) \leq k.\max\left\{\begin{matrix} G(u,v,v), G(u,u,u), G(v,v,v), G(v,v,v), \\ G(u,v,v), G(v,v,v), G(v,u,u) \end{matrix}\right\}$$

$$G(u,v,v) \leq k.\max\{G(u,v,v), G(v,u,u)\}$$

$$G(u,v,v) \leq k.G(v,u,u) \qquad (2.1.5)$$

Similarly, we can prove that $G(v,u,u) \leq k.G(u,v,v)$ (2.1.6)

From (2.1.5) and (2.1.6) we have $G(u,v,v) \leq k^2.G(u,v,v)$, which is a contradiction since $0 \leq k < 1/2$. Therefore $u = v$.

To see that T is G-continuous at u, let $\{y_n\}$ is sequence in X be such that $\lim\limits_{n\to\infty} y_n = u$, then

$$G(Ty_n,Tu,Ty_n) \leq k.\max\left\{\begin{matrix} G(y_n,u,y_n), G(y_n,Ty_n,Ty_n), G(u,Tu,Tu), \\ G(y_n,Tu,Tu), G(u,Ty_n,Ty_n), \end{matrix}\right\}$$

$$G(Ty_n,u,Ty_n) \leq k.\max\{G(y_n,u,y_n), G(y_n,Ty_n,Ty_n), G(y_n,u,u)\}$$

$$G(Ty_n,u,Ty_n) \leq k.\max\left\{\begin{matrix} G(y_n,u,u) + G(u,Ty_n,Ty_n), \\ G(y_n,u,y_n) \end{matrix}\right\} \qquad 2.1.7)$$

(2.1.7) leads to the following cases:

(i) $G(Ty_n,u,Ty_n) \leq k.G(y_n,y_n,u)$,

(ii) $G(Ty_n,u,Ty_n) \le q.G(y_n,u,u)$

In each case take $n \longrightarrow \infty$ on both the sides, then we obtain

$\lim_{n\to\infty} G(Ty_n,u,Ty_n)=0,$ and hence $Ty_n \longrightarrow Tu = u$ as $n \longrightarrow \infty$, showing that, T is *G*-continuous at u .

2.1 Corollary: Let (X,G) be a complete *G*-metric space and let $T: X \longrightarrow X$ be a mapping which satisfies the following condition for some $m \in N$ and for all $x, y, z \in X$,

$$G\left(T^m x,T^m y,T^m z\right) \le k.\max\{G(x,y,z),G(x,T^m x,T^m x),G(y,T^m y,T^m y), G(z,T^m z,T^m z),G(x,T^m y,T^m y), G(y,T^m z,T^m z),G(z,T^m x,T^m x)\} \quad (2.1.8)$$

where $0 \le k < 1/2$. Then T has unique fixed point (say u) and T^m is *G*-continuous at u .

Proof: From theorem 2.1, we have T^m has unique fixed point (say u), that is, $T^m u = u$. But $Tu = T(T^m u) = T^{m+1}u = T^m(Tu)$, so Tu is another fixed point of T^m and by uniqueness $Tu = u$.

2.2 Theorem: Let (X,G) be a complete *G*-metric space and let $T: X \longrightarrow X$ be a mapping which satisfies the following condition, for all $x, y, z \in X$,

$$G\left(Tx,Ty,Tz\right) \le k.\max\begin{Bmatrix} G(x,Ty,Ty)+G(y,Tx,Tx), \\ G(y,Tz,Tz)+G(z,Ty,Ty), \\ G(z,Tx,Tx)+G(x,Tz,Tz) \end{Bmatrix} \quad (2.2.1)$$

where $0 \le k < 1/2$. Then prove that T has unique fixed point (say u) and T is *G*-continuous at u .

Proof: Suppose that T satisfies the condition (2.2.1), let $x_0 \in X$ be an arbitrary point and define the sequence $\{x_n\}$ by $x_n = T^n x_0$, them by (2.2.1), we have

$$G(x_n, x_{n+1}, x_{n+1}) = k.\max\begin{Bmatrix} G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_n, x_n, x_n), \\ G(x_n, x_{n+1}, x_{n+1}) + G(x_n, x_{n+1}, x_{n+1}), \\ G(x_n, x_n, x_n) + G(x_{n-1}, x_{n+1}, x_{n+1}) \end{Bmatrix},$$

so, $G(x_n, x_{n+1}, x_{n+1}) \leq k.\max\{G(x_{n-1}, x_{n+1}, x_{n+1}), 2G(x_n, x_{n+1}, x_{n+1})\}$ (2.2.2)

But, by (G5)

$$G(x_{n-1}, x_{n+1}, x_{n+1}) \leq G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})$$

so, (2.2.2) now becomes

$$G(x_n, x_{n+1}, x_{n+1}) \leq k.\{G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})\}$$

This implies that,

$$G(x_n, x_{n+1}, x_{n+1}) \leq \frac{k}{1-k}.G(x_{n-1}, x_n, x_n) \tag{2.2.3}$$

Let $q = k/_{1-k}$, then, $q < 1$ and by repeated application of (2.1.12), we have

$$G(x_n, x_{n+1}, x_{n+1}) \leq q^n.G(x_{n-1}, x_n, x_n) \tag{2.2.4}$$

Then, for all $n, m \in N, n < m$, we have by repeated use of rectangle inequality, and (2.2.4) that,

$$G(x_n, x_m, x_m) \leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + G(x_{n+2}, x_{n+3}, x_{n+3}) + \ldots + G(x_{m-1}, x_m, x_m)$$

$$\leq \left(q^n + q^{n+1} + q^{n+2} + \ldots + q^{m-1}\right) G(x_0, x_1, x_1)$$

$$\leq \frac{q^n}{1-q} G(x_0, x_1, x_1)$$

Then $\lim_{n,m\to\infty} G(x_n, x_m, x_m) = 0$, since $\lim_{n,m\to\infty} \frac{q^n}{1-q} G(x_0, x_1, x_1) = 0$, showing that, $\{x_n\}$ is a convergent sequence and hence a Cauchy sequence. There exists $u \in X$ such that $\{x_n\}$ converges to some point, since (X, G) is a complete G-metric space.

To show that, u is a fixed point of T. Assume that, $Tu \neq u$, then

$$G(x_n, Tu, Tu) \leq k.\max\left\{\begin{matrix} G(x_{n-1}, Tu, Tu) + G(u, x_n, x_n), \\ G(u, Tu, Tu) + G(u, Tu, Tu) \\ G(u, x_n, x_n) + G(x_{n-1}, Tu, Tu) \end{matrix}\right\},$$

That is,

$$G(x_n, Tu, Tu) \leq k.\max\{G(x_{n-1}, Tu, Tu) + G(u, x_n, x_n), 2G(u, Tu, Tu)\}$$

Letting $n \to \infty$ on both the sides and using the fact that, the function G is continuous on its variables, we have

$$G(u, Tu, Tu) \leq k.\max\{G(u, Tu, Tu), 2G(u, Tu, Tu)\}$$

$G(u, Tu, Tu) \leq 2k.G(u, Tu, Tu)$, which is a contradiction since $0 \leq k < 1/2$. Hence $Tu = u$.

To prove that, T has unique fixed point, assume $v \neq u$, such that, $Tv = v$, then (2.2.1) implies that

$$G(u, v, v) \leq k.\max\left\{\begin{matrix} G(u, v, v) + G(v, u, u), \\ G(v, v, v) + G(v, v, v), \\ G(v, u, u) + G(u, v, v) \end{matrix}\right\}$$

$$G(u, v, v) \leq k.\{G(u, v, v) + G(v, u, u)\}$$

$$G(u, v, v) \leq \frac{k}{1-k} G(v, u, u) \tag{2.2.5}$$

Similarly, we can prove that $G(v,u,u) \le \frac{k}{1-k} G(u,v,v)$ (2.2.6)

From (2.2.5) and (2.2.6) we have $G(u,v,v) \le \left(\frac{k}{1-k}\right)^2 G(u,v,v)$, which is a contradiction since $\frac{k}{1-k} < 1 \Rightarrow \left(\frac{k}{1-k}\right)^2 < 1$. Therefore $u = v$.

To see that T is G-continuous at u, let $\{y_n\}$ is sequence in X be such that $\lim_{n\to\infty} y_n = u$, then

$$G(Ty_n,Tu,Tu) \le k.\max\begin{Bmatrix} G(y_n,Tu,Tu)+G(u,Ty_n,Ty_n), \\ G(u,Tu,Tu)+G(u,Tu,Tu), \\ G(u,Ty_n,Ty_n)+G(y_n,Tu,Tu) \end{Bmatrix}$$

$$G(Ty_n,u,u) \le k.\{G(y_n,u,u)+G(u,Ty_n,Ty_n)\} \quad (2.2.6)$$

By (G5), we have $G(u,Ty_n,Ty_n) \le 2G(Ty_n,u,u)$

(2.2.2) now becomes

$$G(Ty_n,u,u) \le \frac{k}{1-2k}.G(y_n,u,u) \quad (2.2.7)$$

Taking $n \longrightarrow \infty$ on both the sides, then we obtain

$\lim_{n\to\infty} G(Ty_n,u,u) = 0,$ and hence $Ty_n \to Tu = u$ as $n \longrightarrow \infty$, showing that, T is G-continuous at u.

2.2 Corollary: Let (X,G) be a complete G-metric space and let $T: X \to X$ be a mapping which satisfies the following condition for some $m \in N$ and for all $x, y, z \in X$,

$$G\left(T^m x,T^m y,T^m z\right)\le k.\max\left\{\begin{array}{l}G(x,T^m y,T^m y)+G(y,T^m x,T^m x),\\ G(y,T^m z,T^m z)+G(z,T^m y,T^m y),\\ G(z,T^m x,T^m x)+G(x,T^m z,T^m z)\end{array}\right\} \quad (2.2.8)$$

Where $0\le k<1/2$. Then T has unique fixed point (say u) and T^m is G-continuous at u .

Proof: From theorem 2.2, we have T^m has unique fixed point (say u), that is, $T^m u=u$. But $Tu=T(T^m u)=T^{m+1}u=T^m(Tu)$, so Tu is another fixed point of T^m and by uniqueness $Tu=u$.

2.3 Theorem: Let (X,G) be a complete G-metric space and let $T:X\to X$ be a mapping which satisfies the following condition, for all $x,y,z\in X$,

$$G\left(Tx,Ty,Tz\right)\le k.\max\left\{\begin{array}{l}G(z,Tx,Tx)+G(y,Tx,Tx),\\ G(y,Tz,Tz)+G(x,Tz,Tz),\\ G(x,Ty,Ty)+G(z,Ty,Ty)\end{array}\right\} \quad (2.3.1)$$

Where $0\le k<1/3$. Then prove that T has unique fixed point (say u) and T is G-continuous at u .

Proof: Suppose that T satisfies the condition (2.3.1), let $x_0\in X$ be an arbitrary point and define the sequence $\{x_n\}$ by $x_n=T^n x_0$, them by (2.3.1), we have

$$G(x_n,x_{n+1},x_{n+1})=k.\max\left\{\begin{array}{l}G(x_n,x_n,x_n)+G(x_n,x_n,x_n),\\ G(x_n,x_{n+1},x_{n+1})+G(x_{n-1},x_{n+1},x_{n+1}),\\ G(x_{n-1},x_{n+1},x_{n+1})+G(x_n,x_{n+1},x_{n+1})\end{array}\right\}$$

So,

$$G(x_n,x_{n+1},x_{n+1})\le k\left\{G(x_{n-1},x_{n+1},x_{n+1})+G(x_n,x_{n+1},x_{n+1})\right\} \quad (2.3.2)$$

But, by (G5) $G(x_{n-1},x_{n+1},x_{n+1})\le G(x_{n-1},x_n,x_n)+G(x_n,x_{n+1},x_{n+1})$

so, (2.3.2) now becomes

$$G(x_n,x_{n+1},x_{n+1})\le k.\left\{G(x_{n-1},x_n,x_n)+2G(x_n,x_{n+1},x_{n+1})\right\}$$

This implies that,

$$G(x_n, x_{n+1}, x_{n+1}) \leq \frac{k}{1-2k}.G(x_{n-1}, x_n, x_n) \quad (2.3.3)$$

Let $q = k/_{1-2k}$, then, $q < 1$ and by repeated application of (2.3.3), we have

$$G(x_n, x_{n+1}, x_{n+1}) \leq q^n.G(x_{n-1}, x_n, x_n) \quad (2.3.4)$$

Then, for all $n, m \in N, n < m$, we have by repeated use of rectangle inequality, and (2.3.4) that,

$$G(x_n, x_m, x_m) \leq G(x_n, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+2}, x_{n+2}) + G(x_{n+2}, x_{n+3}, x_{n+3}) + \ldots + G(x_{m-1}, x_m, x_m)$$

$$\leq \left(q^n + q^{n+1} + q^{n+2} + \ldots + q^{m-1}\right) G(x_0, x_1, x_1)$$

$$\leq \frac{q^n}{1-q} G(x_0, x_1, x_1)$$

Then $\lim_{n,m\to\infty} G(x_n, x_m, x_m) = 0$, since $\lim_{n,m\to\infty} \frac{q^n}{1-q} G(x_0, x_1, x_1) = 0$, showing that, $\{x_n\}$ is a convergent sequence and hence a Cauchy sequence. There exists $u \in X$ such that $\{x_n\}$ converges to some point, since (X, G) is a complete G-metric space.

To show that, u is a fixed point of T. Assume that, $Tu \neq u$, then

$$G(x_n, Tu, Tu) \leq k.\max \begin{Bmatrix} G(u, x_n, x_n) + G(u, x_n, x_n), \\ G(u, Tu, Tu) + G(x_{n-1}, Tu, Tu) \\ G(x_{n-1}, Tu, Tu) + G(u, Tu, Tu) \end{Bmatrix},$$

That is,

$$G(x_n, Tu, Tu) \leq k.\max\{2G(u, x_n, x_n), G(u, Tu, Tu) + G(x_{n-1}, Tu, Tu)\}$$

Letting $n \longrightarrow \infty$ on both the sides and using the fact that, the function G is continuous on its variables, we have

$G(u,Tu,Tu) \leq 2k.G(u,Tu,Tu)$, which is a contradiction since $0 \leq k < 1/3$. Hence $Tu = u$.

To prove that, T has unique fixed point, assume $v \neq u$, such that, $Tv = v$, then (2.2.1) implies that

$$G(u,v,v) \leq k.\max\begin{Bmatrix} G(v,u,u)+G(v,u,u), \\ G(v,v,v)+G(u,v,v), \\ G(u,v,v)+G(v,v,v) \end{Bmatrix}$$

$$G(u,v,v) \leq k.\max\{2G(v,u,u), G(u,v,v)\}$$

$$G(u,v,v) \leq \frac{k}{1-2k} G(v,u,u) \qquad (2.3.5)$$

Similarly, we can prove that $G(v,u,u) \leq \frac{k}{1-2k} G(u,v,v)$ (2.3.6)

From (2.3.5) and (2.3.6) we have

$$G(u,v,v) \leq \left(\frac{k}{1-2k}\right)^2 G(u,v,v),$$

Which is a contradiction since $\frac{k}{1-k} < 1 \Rightarrow \left(\frac{k}{1-k}\right)^2 < 1$. Therefore $u = v$.

To see that T is G-continuous at u, let $\{y_n\}$ is sequence in X be such that $\lim_{n \to \infty} y_n = u$, then

$$G(Tu,Ty_n,Ty_n) \leq k.\max\begin{Bmatrix} G(y_n,Tu,Tu)+G(y_n,Tu,Tu), \\ G(y_n,Ty_n,Ty_n)+G(u,Ty_n,Ty_n), \\ G(u,Ty_n,Ty_n)+G(y_n,Ty_n,Ty_n) \end{Bmatrix}$$

$$G(u,Ty_n,Ty_n) \le k.\max\left\{\begin{matrix} G(y_n,Ty_n,Ty_n)+G(u,Ty_n,Ty_n), \\ 2G(y_n,u,u) \end{matrix}\right\} \quad (2.3.6)$$

By (G5), we have $G(y_n,Ty_n,Ty_n) \le G(y_n,u,u)+G(u,Ty_n,Ty_n)$

(2.3.6) now becomes

$$G(u,Ty_n,Ty_n) \le k.\max\left\{\begin{matrix} G(y_n,u,u)+2G(u,Ty_n,Ty_n), \\ 2G(y_n,u,u) \end{matrix}\right\} \quad (2.3.7)$$

Now (2.3.7) gives two cases:

$$G(u,Ty_n,Ty_n) \le \frac{k}{1-2k} G(y_n,u,u)$$

$$G(u,Ty_n,Ty_n) \le 2kG(y_n,u,u)$$

Taking $n \longrightarrow \infty$ in both of the above cases, we get

$$\underset{n\to\infty}{Lim} G(u,Ty_n,Ty_n)=0$$

hence $Ty_n \longrightarrow Tu = u$ as $n \longrightarrow \infty$, showing that, T is *G*-continuous at u .

2. REFERENCES

1. Dhage, B.C., "Generalized metric space and mapping with fixed points", Bulletin of the calcutta mathematical society, Vol. 84, pp. 329 – 336, 1992.
2. Gahler. S, "2 – metriche Raume undihre topologischestruktur," Mathematische Nachrichten. Vol. 26, no. 1-4, pp. 115 – 148, 1963.
3. Ha. R.S., Cho. Y.J. and White A, "Strictly Convex and Strictly 2 – convex 2 – normed spaces," Mathematica Japanica, vol. 33, no. 3, pp. 375 – 384, 1988.
4. Mustafa. Z and Sims. B, "Some remarks concerning D-metric spaces," in proceedings of the International conference on Fixed point theorey and Applications, pp. 189 – 198, Valencica, Spain, July 2003.
5. Mustafa. Z, A new structure for generalized metric spaces – with applications to fixed point theory, Ph.D. thesis, the University of New castle, Callaghan, Australia, 2005.

*****

----

[1]*Author 1 Assistant Professor, Department of Mathematics, S.V.University, Tirupathi*
cjsreddysvu@gmail.com
[2]*Author 2 Research Scholars, Department of Mathematics, S.V.University, Tirupathi.*
mahesh.reddy555@gmail.com
[3]*Author 3 Research Scholars, Department of Mathematics, S.V.University, Tirupathi*
hemavathiphd@gmail.com