FUZZY TRANSPORTATION PROBLEMS SOLVING BY SIMPLEX TYPE ALGORITHMS

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***Abstract:** Fuzzy transportation problems have an essential role in fuzzy modeling, which can formulate uncertainty in actual environment. The present paper have simple simplex type algorithm to minimize the fuzzy objective value of the fuzzy transportation problem with fuzzy render and fuzzy requirements parameters. L.R. fuzzy number, which was defined and used by Dubois and Prade with some useful and easy approximation arithmetic operators. Our main object of this paper is how can we solve a fuzzy transportation problem using simplex algorithms.*

Keywords:Fuzzy arithmetic, Fuzzy number, Fuzzy transportation problem, Fuzzy variables.

AMS Subject Classifications: 90C10, 90C70, 90C90.

1. INTRODUCTION

One of the most important applications of the simplex method is the transportation model [1]. The simplex method is an iterative algebraic procedure for solving linear programming problems [2]. In the late 1940s, George Dantzig and his contemporaries were faced with monumental problems that arose in the areas of military logistics, management, economics etc. In 1947 Dantzig devised the simplex method a way to reduce the number of calculations. This was the advent of linear programming [1]. The simplex algorithm was the forerunner of many computer programs that are used to solve complex optimization problems [3]. These applications are used extensively in a variety of situations.

Zadeh [4] and Bellman and Zadeh [5] acquainted the concept of fuzziness. Chanas et.al. [6] investigated transportation problems with fuzzy render and requirement quantities and solved them using the parametric programming technique in terms of the Bellman-Zadeh criterion. Chen [7] acquainted the concept of function principle, which is used to calculate the fuzzy transportation cost. The Graded Mean Integration The representation method, used to defuzzify the fuzzy transportation cost, which was acquainted by Chen [8]. The transportation model is a special case of the linear programming models, and obviously, it can be solved by the regular simplex method (big - M) or the dual simplex method. However, these algorithms require additional variables, which complicate the formulation, enlarge the tableaux and increase the number of iterations

2. FUZZY PRELIMINARIES

The concept of fuzzy numbers and arithmetic operation with these numbers were first introduced and investigated by Zimmerman [9]. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al

[10] which was the frame work of the fuzzy decision of Bellman and Zadeh [5]. Now we present some necessary definitions:

Definition 2.1: A fuzzy number $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in R\}$ is nonnegative if and only if $\mu_{\tilde{A}}(x) = 0$, for all $x < 0$. Then a triangular fuzzy number $\tilde{A} = (a, b, c)$ is nonnegative if $a - b \geq 0$.

Definition 2.2: Two triangular fuzzy numbers $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are said to be equal if and only if $a_1 = a_2$, $b_1 = b_2$, and $c_1 = c_2$.

Definition 2.3: A fuzzy number $\tilde{A} = (a, b, c)$ is called symmetric, if $a = b$.

Definition 2.4: A matrix A is called nonnegative and denoted by $A \geq 0$ if each element of A be a nonnegative number.

Definition 2.5: A fuzzy vector $b = (b_i)_{m\times 1}$ is called nonnegative and denoted by $b \geq 0$, if each element of b be a nonnegative fuzzy, that is $b_i \geq 0$.

3. LINEAR FUZZY REAL NUMBERS

Considering the real numbers R, one way to associate a fuzzy number with a fuzzy subset of real numbers is as a function $\mu: R \rightarrow [0, 1]$, where the value $\mu(x)$ is to represent a degree of belonging to the subset of R. The linear fuzzy real numbers as described by Neggers and Kim [5, 3] is a triple of real numbers (a, b, c) where $a \leq b \leq c$ of real numbers, such that:

$$\mu(x) = \begin{cases} 0, & \text{if } x \leq a \text{ or } x \geq c; \\ \dfrac{(x-a)}{(b-a)}, & \text{if } a < x < b; \\ \dfrac{(c-x)}{(c-b)}, & \text{if } b < x < c. \\ 1, & \text{if } x = b; \end{cases}$$

For a real number c, we let $\varepsilon(c) = \mu$ with associated triple (a, b, c). Then μ is a linear fuzzy real number with $\mu(c) = 1$ and $\mu(x) = 0$ otherwise. As a linear fuzzy real number we consider $\varepsilon(c) = \mu$ to represent the real number c itself. Thus by this interpretation we note that the set R of all real numbers is a subset of the set containing the linear fuzzy real numbers. The set of the linear fuzzy real numbers is a hybrid set showing intermediate properties, which are unique to the set and not those of either the real numbers or the "general" fuzzy numbers. Let LFR = $\{\mu: R \rightarrow [0, 1] \mid \mu$ is a linear fuzzy real number$\}$. Each μ has a set of descriptive parameters. The base is defined as the triple (a, b, c) that occurs in the definition of a linear fuzzy real number. Thus one may write an element of LFR as $\mu = \mu(c, c, c)$.

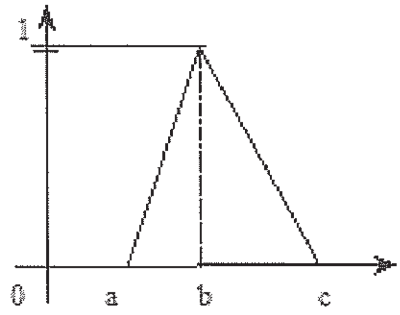

Fig. 1. Linear Fuzzy Real number $\mu(a, b, c)$

4. ARITHMETIC OPERATIONS ON TRIANGULAR FUZZY NUMBERS

Let μ_1(or $\tilde{A}$) = $\mu(a_1, b_1, c_1)$ and μ_2(or $\tilde{B}$) = $\mu(a_2, b_2, c_2)$ be two triangular fuzzy numbers, then arithmetic operations on them is defined as:

4.1: Addition: $\mu_1 + \mu_2$ (or $\tilde{A} \oplus \tilde{B}$) $= \mu(a_1 + a_2, b_1 + b_2, c_1 + c_2)$.

This operation is not the usual definition of addition of functions. Using this definition we have
$(a_1 + a_2) = \mu(a_1 + a_2, a_1 + a_2, a_1 + a_2) = (a_1) + (a_2)$.
It is also clear that $\mu + (0) = \mu$, for all $\mu \in$ LF R.

4.2: Subtraction: For subtraction, first we define $-\mu$ as $-\mu(a, b, c) = \mu(-c, -b, -a)$ and $\mu_1 - \mu_2$ (or $\tilde{A} - \tilde{B}$) $= \mu(a_1, b_1, c_1) + \mu(-c_2, -b_2, -a_2) = \mu(a_1 - c_2, b_1 - b_2, c_1 - a_2)$.

If $\mu_1 = \mu_2 = \mu(a, b, c)$ then $\mu_1 - \mu_2 = \mu(a - c, 0, c - a)$. This is not the same as (0).

4.3: Multiplication: Given the linear fuzzy real numbers $\mu_1 = \mu(a_1, b_1, c_1)$ and $\mu_2 = \mu(a_2, b_2, c_2)$, $\mu_1 . \mu_2$ is defined by $\mu_1 . \mu_2 = \mu(\min\{a_1 . a_2, a_1 . c_2, a_2 . c_1, c_1 . c_2\}, b_1 . b_2, \max\{a_1 . a_2, a_1 . c_2, a_2 . c_1, c_1 . c_2\})$.

Thus if $\mu_i = \mu(a_i, b_i, c_i)$ for i = 1, 2, 3, then $\mu_1 \cdot \mu_2 \cdot \mu_3 = \mu(\min\{a_1 a_2 a_3, \cdots, c_1 c_2 c_3\}, b_1 b_2 b_3, \max\{a_1 a_2 a_3, \cdots, c_1 c_2 c_3\})$.
Also, $\mu(a, b, c) \cdot \mu(1, 1, 1) = \mu(\min\{a, c\}, b, \max\{a, c\}) = \mu(a, b, c)$, i.e. $\mu \cdot \varepsilon(1) = \mu$ for all $\mu \in$ LFR.

4.4: Scalar multiplication: For any scalar λ, we have

$$\lambda \tilde{A} = \lambda(a, b, c) = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1), & \text{if } \lambda \geq 0 \\ (\lambda a_2, \lambda b_2, \lambda c_2), & \text{if } \lambda \leq 0 \end{cases}$$

4.5: Division: Given the linear fuzzy real numbers $\mu_1 = \mu(a_1, b_1, c_1)$ and $\mu_2 = \mu(a_2, b_2, c_2)$, $\frac{\mu_1}{\mu_2}$ is defined by $\frac{\mu_1}{\mu_2} = \mu_1 . \frac{1}{\mu_2}$ where $\frac{1}{\mu_2} = \mu(\min\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\}$, median $\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\}$, $\max\{\frac{1}{a_2}, \frac{1}{b_2}, \frac{1}{c_2}\})$,

Note that $\mu = \mu(a, b, c)$, if $0 < a \le b \le c$, then $\frac{1}{\mu} = \mu\{\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\}$,

4.6: Law of trichotomy: A linear fuzzy real number $\mu(a, b, c)$ is defined to be positive if $a > 0$, negative if $c < 0$, and zeroic if $a \le 0$ and $c \ge 0$. The following properties also hold:

- If μ is positive , then $-\mu$ is negative;
- If μ is negative, then $-\mu$ is positive;
- If μ is zeroic, then $-\mu$ is also zeroic;
- If μ_1 and μ_2 are positive, then so is $\mu_1 + \mu_2$;
- If μ_1 and μ_2 are negative, then so is $\mu_1 + \mu_2$;
- If μ_1 and μ_2 are zeroic, then so is $\mu_1 + \mu_2$;
- For any μ, $\mu - \mu$ is zeroic.

5. FUZZY TRANSPORTATION PROBLEM (FTP)

A Fuzzy Transportation Problem (FTP) is a linear programming problem of specific structure. As in the transportation problem, let $\tilde{a}_i$ represents the renders and $\tilde{e}_j$ represents the requirements respectively. We define the following quantities

$\tilde{a}_i$ = fuzzy renders at source i ,

$\tilde{e}_j$ = fuzzy requirements at term in ij ,

$\tilde{c}_{ij}$ = unit cost of transportation from source i to term in ij ,

$\tilde{x}_{ij}$ = number of units transported from source i to term in ij.

The objective is to nd how much material should be transported from source i to term in ij.

Minimize $z = \sum_{i=1}^{m} \tilde{c}_{ij} \sum_{j=1}^{n} \tilde{x}_{ij}$

Subject to the constraints $\sum_{j=1}^{n} \tilde{x}_{ij} \le \tilde{a}_i$, for i = 1, 2, 3, ... M

$\sum_{i=1}^{m} \tilde{x}_{ij} \leq \tilde{e}_j$, for j = 1, 2, 3, ... N

$\tilde{x}_{ij} \geq 0$, for i = 1, 2, 3, ... m, j = 1, 2, 3, ...n.

The matrix form of transportation problem is

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

Subject to the constraints

$Ax \leq b$

$x \geq 0$

where $b = (a_1, a_2, ...a_m , e_1, e_2, ...e_n)$ and A is coefficient matrix of transportation problem. However, in the real-world decision problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be of a probabilistic type. In this situation, the decision maker can model the inexactness by means of fuzzy parameters [2].

The matrix form of fuzzy transportation problem is

$$\text{Min } \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

Subject to the constraints $Ax \leq \tilde{b}$ $x \geq 0$

Where $\tilde{b} = (\tilde{a}_1 , \tilde{a}_2 ,..... \tilde{a}_m , \tilde{e}_1 , \tilde{e}_2 ,.... \tilde{e}_m)$ and A is the coefficient matrix of FTP. It can be demonstrated that, matrix A is a unimodular one. It means that, determinant of each square sub matrix of A is ± or zero.

Using coefficient matrix characteristic, we use a simple simplex algorithm (two phase) for the transportation problem. In the first phase, a feasible or an infeasible basic solution is obtained. In the second phase we led to feasibility and turns into a basic optimal solution. Here two phases utilize the row-column matrix as Gauss-Jordan. Also the algorithm defines the redundant constraint.

6. NUMERICAL INSTANCES

Example 6.1: A company has two factories (Availabilities) A_1, A_2 and two retail stores (Demands) D_1, D_1. The production quantities/month A_1 = (100, 199, 254) and A_2 = (100, 101, 146) tons respectively.

The demands/month D_1 = (100, 150, 200) and D_1 = (100, 150, 200) tons respectively.

The transportation cost/ton $\tilde{c}_{ij}$, i = 1, 2, ; j = 1, 2 is

$\tilde{c}_{11}$ = (10, 14, 24), $\tilde{c}_{12}$ = (17, 26, 29), and $\tilde{c}_{21}$ = (3, 5, 7), $\tilde{c}_{22}$ = (25, 34, 49).

Row-Column matrix

Availabilities			
Demands	(10, 14, 24)	(17, 26, 29)	A_1 = (100, 199, 254)
	(3, 5, 7)	(25, 34, 49)	A_2 = (100, 101, 146)
	D_1 = (100, 150, 200)	D_1 = (100, 150, 200)	

Step 1: Row-Column Reduction

Availabilities			
Demands	(-14, 0, 14)	(-26, 0, 26)	A_1 = (100, 199, 254)
	(-4, 0, 4)	(-1, 17, 53)	A_2 = (100, 101, 146)
	D_1 = (100, 150, 200)	D_1 = (100, 150, 200)	

Step 2: We use the following algorithms:

$(-14, 0, 14)\ \tilde{x}_{11} + (-26, 0, 26)\ \tilde{x}_{12} = (100, 199, 254)$

$(-14, 0, 14)\ \tilde{x}_{21} + (-1, 17, 53)\ \tilde{x}_{22} = (100, 101, 146)$

$(-14, 0, 14)\ \tilde{x}_{11} + (-4, 0, 4)\ \tilde{x}_{21} = (100, 150, 200)$

$(-26, 0, 26)\ \tilde{x}_{12} + (-1, 17, 53)\ \tilde{x}_{22} = (100, 150, 200)$

Step 3: Simplex Tableau

	Var	$\tilde{x}_{11}$	$\tilde{x}_{12}$	$\tilde{x}_{21}$	$\tilde{x}_{22}$	RHS
A1	?	(1, 1, 1)	(1, 1, 1)	°	°	(100, 199, 254)
A2	?	°	°	(1, 1, 1)	(1, 1, 1)	(100, 101, 146)
D1	?	(1, 1, 1)	°	(1, 1, 1)	°	(100, 150, 200)
D2	?	°	(1, 1, 1)	°	(1, 1, 1)	(100, 150, 200)
		(-14, 0, 14)	(-26, 0, 26)	(-4, 0, 4)	(-1, 17, 53)	
A1	?	(1, 1, 1)	(1, 1, 1)	°	°	(100, 199, 254)
A2	?	°	°	(1, 1, 1)	(1, 1, 1)	(100, 101, 146)
D2	?	°	(1, 1, 1)	°	(1, 1, 1)	(100, 150, 200)
		(-14, 0, 14)	(-26, 0, 26)	(-4, 0, 4)	(-1, 17, 53)	
A1	$\tilde{x}_{11}$	°	(1, 1, 1)	°	°	(100, 199, 254)
A2	$\tilde{x}_{21}$	°	°	°	(1, 1, 1)	(100, 101, 146)
D2	?	°	(1, 1, 1)	°	(1, 1, 1)	(100, 150, 200)
		°	(-26, 0, 26)	°	(-1, 17, 53)	
A1	$\tilde{x}_{11}$	°	°	°	(-1, -1, -1)	(-126, 49, 180)
A2	$\tilde{x}_{21}$	°	°	°	(1, 1, 1)	(50, 99, 154)
D2	$\tilde{x}_{12}$	°	°	°	(1, 1, 1)	(74,150, 226)
		°	°	°	(-1, 17, 53)	

Right hand side values are non-negative, therefore from[ ] simplex tableau is now optimum.

Step 4:

(10, 14, 24) $\tilde{x}_{11}$ = (-100, 49, 154)	(17, 26, 29) $\tilde{x}_{12}$ = (100, 150, 200)	(100, 199, 254)
(3, 5, 7) $\tilde{x}_{21}$ = (100, 101, 146)	(25, 34, 49) $\tilde{x}_{22}$ = 0	(100, 101, 146)
(100, 150, 200)	(100, 150, 200)	

Step 4: Cost- The optimum fuzzy transportation cost is (-400, 5091, 10518). Defuzzified fuzzy transportation cost is 5087.

Example 6.2: A company has two Big stores B_1, B_2 and three retail shops with Demands D_1 = (400, 448, 508), D_2 = (300, 351, 396). The supply in the shops are A_1 = (300, 399, 504), A_2 = (250, 301, 346) and A_3 = (300, 399, 504). The transportation cost/ton

$\tilde{c}_{ij}$, i = 1, 2, 3 ; j = 1, 2 is

$\tilde{c}_{11}$ = (1, 3, 5), $\tilde{c}_{12}$ = (3, 5, 13)

$\tilde{c}_{21}$ = (92, 3, 10), $\tilde{c}_{22}$ = (3, 4, 1)

and $\tilde{c}_{31}$ = (5, 6, 13), $\tilde{c}_{32}$ = (1, 3, 5)

This matrix needs to be balanced with a dummy destination, B_3 = { $\tilde{c}_{13}$ = (0, 0, 0), $\tilde{c}_{23}$ = (0, 0, 0), $\tilde{c}_{33}$ = (0, 0, 0)}. To balance Demand with Availabilities we have D_1 = (100, 202, 292)

	B_1	B_2	B_3	Availability
	(4, 7, 8) $\tilde{x}_{11}$	(6, 8, 10) $\tilde{x}_{12}$	(0, 0, 0) $\tilde{x}_{13}$	A_1 = (250, 301, 346)
	(6, 9, 14) $\tilde{x}_{21}$	(5, 8, 14) $\tilde{x}_{22}$	(0, 0, 0) $\tilde{x}_{23}$	A_2 = (300, 399, 504)
	(7, 11, 18) $\tilde{x}_{31}$	(6, 8, 10) $\tilde{x}_{32}$	(0, 0, 0) $\tilde{x}_{33}$	A_3 = (250, 301, 346)
Demand	(300, 351, 396)	(400, 448, 508)	(100, 202, 292)	

We use the same method to solve it. The optimum fuzzy transportation cost = (2252, 6141, 11884). Defuzzified fuzzy transportation cost is 6244.

7. CONCLUSIONS

In this paper we presented a simplex type algorithm to solve the fuzzy transportation problem in phase first and then use fuzzy penalty transportation method. This particular method manually is effective and efficient without using any artificial variables.

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