SOLVING NONLINEAR PROGRAMMING PROBLEMS IN A FUZZY ENVIRONMENT THROUGH A GENETIC ALGORITHM

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Abstract: In this paper,a fuzzy programming techniqiue through genetic algorithm for solving fuzzy nonlinear optimization problem is proposed,and obtain the numerical solution for fuzzy nonlinear programming using Gaussian membership function to the fuzzy numbers considering in its parametric form.Numerical examples are provided to show the effectiveness of the proposed method.

Keywords: Fuzzy numbers,Fuzzy nonlinear programming,Optimization problem,Fuzzy ranking,Genetic algorithms

1. INTRODUCTION

Over the past decades,solving fuzzy(linear and nonlinear)optimization problems one of the fundamental search subjects in the field of fuzzy sets and systems.the primary works of fuzzy mathematical programming have been presented by Zimmermann(1978,1983).Since then,many papers have appeared on fuzzy linear programming [5,10,11].When nonlinear programming is convex,we can obtain a global optimal solution by some convex programming techniques,e.g,the sequential quadratic programming.otherwise,i.e,when it is non-convex,because it is difficult to find a global optimal solution,we search an approximate optimal solution by some approximate solution methods such as genetic algorithms or simulated annealing.so in the fuzzy nonlinear case the situation is quite different,as there is a wide variety of specific and both practically and theoretically relevant nonlinear problems,each having a different solution method.Here a genetic algorithm procedure is used to find the solution of fuzzy nonlinear programming with defining the Gaussian membership function for fuzzy numbers.

2. PRELIMINARIES

Definition:A fuzzy number $\tilde{u}$ in parametric form is a pair($\underline{u}$,$\bar{u}$) of function $\underline{u}(r)$, $\bar{u}(r)$,$0 \le r \le 1$ which satisfies the following recquirements:

1) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,

2) $\bar{u}(r)$is a bounded monotonic decreasing left continuous function,

3) $\underline{u}(r) \le \bar{u}(r)$, $0 \le r \le 1$

A crisp number α is simply reperesent by $\underline{u}(r)=\bar{u}(r)= \alpha$, $0 \le r \le 1$.By appropriate definitions the fuzzy number space $\{\underline{u}(r),\bar{u}(r)\}$ become a convex cone,E^1 using the

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extention principle[15],the addition and scalar multiplication of fuzzy number are defined by

(u+v)(x)=sup min {u(r),v(s)},x=r+s,

(ku)(x)=u(x/k),k≠0,

For u,v ϵE^1,k ϵR.Equivalently,for arbitrary $\tilde{u}=(u_1,u_2),\tilde{v}=(v_1,v_2)$ and kϵR, the eddition and scalar multiplication may be defined as

$(u_1+v_1)(x)=u_1(x)+v_1(x)$,

$(u_2+v_2)(x)=u_2(x)+v_2(x)$, (1)

$(ku_1)(x)=ku_1(x),(ku_2)(x)=ku_2(x),k\geq 0$

$(ku_1)(x)=ku_2(x),(ku_2)(x)=ku_1(x),k\leq 0$

Fuzzy Number With Gaussian Membership function: Now the membership of Gaussian type for a fuzzy number is proposed as $\mu(x)=\exp\{-k(x-\alpha)^2\}$,which in parametric form is written as

$\underline{u}(r)=\alpha-\sqrt[2]{\log(r)/k},\bar{u}(r)=\alpha+\sqrt[2]{\log(r)/k}$ (2)

calculate the objective function and left side of the constraints using fuzzy addition and fuzzy scalar multiplication.

Ranking function: A simple method for ordering fuzzy numbers consists in the definition of a ranking function F,mapping each fuzzy number to the real number R,where a natural order exists.suppose S={$\mu_{A1}, \mu_{A2}, \mu_{A3},\cdots,\mu_{An}$ }is a set of n fuzzy numbers,and the ranking function F is a mapping from S to the real numbers R,i.e F:S→R.then for any distinct pair of fuzzy numbers μ_{Ai}, μ_{Aj} ϵS,the ranking function can be defined as

If $F(\mu_{Ai})<F(\mu_{Aj})$;then $\mu_{Ai}<\mu_{Aj}$, If $F(\mu_{Ai})=F(\mu_{Aj})$;then $\mu_{Ai}=\mu_{Aj}$

If $F(\mu_{Ai})>F(\mu_{Aj})$;then $\mu_{Ai}>\mu_{Aj}$

This implies for example,that if $F(\mu_{Ai})>F(\mu_{Aj})$,the fuzzy number μ_{Ai} is numerically greater than fuzzy number μ_{Aj}.the higher μ_{Ai} is,the larger $F(\mu_{Ai})$ is.A useful technique for ranking fuzzy numbers is **liou and wang** ranking function,defined by

$LW^{\alpha}(\tilde{u})=\alpha\int_0^1 R^{-1}(y)dy+(1-\alpha)\int_0^1 L^{-1}(y)dy$ (3)

Where L(.),R(.) are left and right shape functions of ũ,respectively and α ϵ[0,1] is the index of optimism which reperesents the degree of optimism of a decision maker.

3.Problem Formulation: Fuzzy optimization describes an optimization problem with fuzzy objective function and fuzzy constraints.the results obtained from classical methods of optimization involved deterministic variables exhibit various shortcoming.in practicular,the effects of the uncertainty attached to input information is often ignored altogether or only taken into account to the limited degree.the classical deterministic optimization problem according to

$$\text{Min(Max)} \quad Z=f(x,e)$$
$$\text{Subject to} \quad g_i(x,e)\leq r_i \;, i=1,..,n \qquad (4)$$
$$x \epsilon X$$

is considered under the aspect of uncertainty,and is extended.for the objective function f(x,e) the optimal solution X_{opt} from the set of design variables X(design space)is determined under compliance with the equality(inequality) constraints g_i(x,e),i=1,..,n.input parameters such as geometrical parameters,material parameters,external load parameters.reliability parameters and economic parameters are lumped together in the vector e.considering the uncertain(fuzzy)parameters to be deterministic variables.the deterministic optimization problem is extended to a fuzzy optimization problem

$$\text{Min(Max)} \quad \tilde{Z}=f(x,\tilde{e})$$
$$\text{Subject to} \quad g_i(x, \tilde{e}))\leq \tilde{r}_i, i=1,..,n \qquad (5)$$
$$x \epsilon X$$

here a genetic algorithm is used to find the solution of fuzzy optimization problem(3),we use genetic algorithm to solve this fuzzy problem with defining the membership functions for fuzzy numbers.

4.ALGORITHM FOR SOLVING FUZZY NONLINEAR PROGRAMMING THROUGH A GENETIC ALGORITHM

Step 0. Convert the coefficients of objective function and the constraints into fuzzy number using Gaussian membership function,and calculate the objective function and left and right side of the constraints usinge quations(1).Set the parameters, population size(popsize),mutation rate (p_m),selection rate(p_s),crossover rate(p_c),the maximum generation (max gen) and initialize number of generations gen=0.

Step 1. Select initial population at random.For using genetic algorithm,one must first code the decision variable 'x' into binary string with finite length.

Step 2.obtain the decoded values for the current population generated.if the current population are satisfy of all constraints,go to step 3.Otherwise choose the rest of the population randomly so that all the constraints are satisfied.

Step 3. Calculate objective value for each chromosome and compute the fitness value using the equation(2).

Step4. Selection:parents selection is made on the basis of fitness function, individuals with the best fitness values are chosen more often.The best fitness value of an individual the more likely that the individual will be selected for recombination.The slection of mating parents is done by roulette wheel selection.

Step 5. Impeliment crossover and mutation operator on selected members from the old generation.Crossover combines information from two parents that two children have a"resembelance"to each parent.Standard crossovers such as one-point and two-point,are used in GA model and bit-wize mutation operator performed here.

Step 6. If the termination condition is fulfilled,stop,otherwise.let t=t+1 and go to step 2.

5.ILLOSTRATIVE NUMERICAL EXAMPLE

Consider the following nonlinear problem[7]:

Min $f(x)=x_1x_2x_3$

Subject to $2ox_1x_2+6x_1x_3+6x_2x_3\leq 108$ (6)

$x_2+x_3\leq 3$

$x_1,x_2,x_3\geq 0$

The optimal solution is $x^*=(2.9952,1.0031,1.9968)$,$f(x^*)=5.9999$. Now,the fuzzy version of the problem is

Min $\widetilde{f(x)} = \tilde{1}x_1x_2x_3$

$\widetilde{20}x_1x_2+\tilde{6}x_1x_3+\tilde{1}x_2x_3\leq\widetilde{108}$ (7)

$\tilde{1}x_2+\tilde{1}x_3\leq\tilde{3}$

$x_1,x_2,x_3\geq 0$

Fuzzy coefficients of objective function and constraints are presented in **Table.1**

Table.1 Fuzzy number reperesentation

$\tilde{1}$	$\widetilde{20}$	$\tilde{6}$	$\widetilde{108}$	$\tilde{3}$
k=1, α=1	k=1, α=20	k=1, α=6	k=1,α=108	k=1, α=3

The later convert the coefficients of objective function and constraints into fuzzy numbers in parametric form,we calculate the objective function and constraints using equations(1), then apply genetic algorithm on new problem:

Max $\widetilde{f(x)}=(x_1x_2x_3-\{-\log(x)x_1^2x_2^2x_3^2\}^{1/2}, x_1x_2x_3+\{-\log(x)x_1^2x_2^2x_3^2\}^{1/2})$

S.to $(20x_1x_2+6x_1x_3+6x_2x_3-\{-\log(x)(x_1x_2+\sqrt{2}\ x_1x_3+\sqrt{2}\ x_2x_3)^2\}^{1/2},$
$20x_1x_2+6x_1x_3+6x_2x_3+\{-\log(x)(x_1x_2+\sqrt{2}\ x_1x_3+\sqrt{2}\ x_2x_3)^2\}^{1/2},)\leq(108-(-\log(x))^{1/2},108+(-\log(x))^{1/2})$ (8)
$(x_1+x_2-\{-\log(x)(x_2+x_3)^2\}^{1/2},\ x_1+x_2+\{-\log(x)(x_2+x_3)^2\}^{1/2})\leq(3-(-\log(x))^{1/2},$
$(3+(-\log(x))^{1/2})$
$x_1,x_2,x_3\geq 0$

The parameter values used in genetic algorithm for solving fuzzy nonlinear problem (8) were set as follows:p_s=0.40,p_c=0.40,p_m=0.03, Maxgen=100,Popsize=70. The method had 5 runs.The results obtained in 5 runs are shown in **table.2**, and The best results of $\widetilde{f(x)}$ obtainded in five runs are shown in **Fig.1**. This genetic algorithm program is run to obtain 100 different best values of fitness function (liu and wang function),which are shown in **Fig.2**

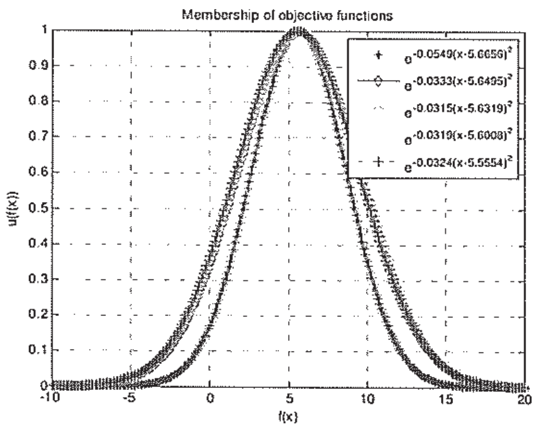

Fig.1 Optimal values obtained from solving problem(8).

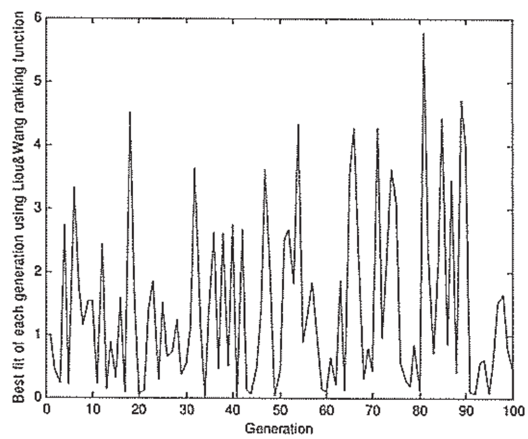

Fig.2 The curves for the best value obtained in 100 generations in a run for the fuzzy problem(8).

Table.2 The results obtained in 5 runs for the fuzzy problem(8).

x^*	$\hat{f}(\overline{x^*})$	Gen opt	Time (sec.)	$LW^{1/2}$ Left side	$LW^{1/2}$ hand side
[2.5211,1.1519, 1.6066]	k=0.0549α= 5.6656	40	80	[93.4874, 2.7585]	[108,3]
[3.2128,0.9059, 1.8826]	k=0.0333α= 5.6495	22	64	[104.7327 ,2.7885]	[108,3]
[3.0906,0.9852, 1.8496]	k=0.0315 α=5.6319	12	76	[106.1288 ,2.8348]	[108,3]
[3.2139,0.8094, 2.1530]	k=0.0319 α=5.6008	48	85	[103.9996 ,2.9624]	[108,3]
[3.3293,0.86071 .9387]	K=0.0324 α=5.5554	73	59	[106.0495 ,2.7994]	[108,3]

6. CONCLUSION: In this paper, we proposed a genetic algorithm approach to find a solution of fuzzy nonlinear programming problem.The proposed method is generalizd form solving fuzzy nonlinear programming problem with fuzzy coefficients,since it can solve fuzzy nonlinear programming with fuzzy number as coefficients and it can find both crisp and fuzzy solutions.Ilusstrative numerical examples were provided to demonstrate the feasibility and efficiency of the proposed method.

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