SOME CONSIDRATION ON WAVE PROPAGATION IN MICROSTRETCH GENERALIZED THERMO ELASTIC PLATES

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1. INTRODUCTION

The propagation of waves in micropolar generalized thermoelastic plate subjected to stress free isothermal and thermally insulated conditions is investigated in the context of conventional coupled thermoelasticity (CT), Lord-Shulman (LS), and Green-Lindsay (GL) theories of thermoelasticity. The secular equations for the micropolar generalized thermoelastic plate in closed form and isolated mathematical conditions for symmetric and skew-symmetric wave mode propagation in completely separate terms are derived. The secular equations for micropolar coupled thermoelastic, micropolar elastic, thermoelastic and elastic plates have been deduced as particular cases from the derived secular equations. At short wave length limits, the secular equations for symmetric and skew symmetric waves in stresses free, thermally insulated and isothermal, micropolar, thermoelastic plate reduce to Rayleigh surface waves frequency equation. The amplitudes of displacements, micro-rotation and temperature change have also been obtained. Finally, in order to illustrate the analytical development, the numerical solution is carried out for Magnesium composite material. The dispersion curves and amplitudes of displacements, microrotation and temperature change for symmetric and skew symmetric wave modes, are computed numerically and presented graphically. The theory and numerical computations are found to be in close agreement.

2. FORMULATION OF THE PROBLEM

We consider an infinite homogeneous isotropic micropolar generalized thermoelastic plate of thickness 2d initially undisturbed and at uniform temperature T_0. We take origin of the coordinate system (x, y, z) on the middle surface of the plate. The x-y plane is chosen to coincide with the middle surface of the plate and z-axis normal to it along the thickness of the plate. The surfaces $z = \pm d$ are assumed to be stress free, thermally insulated or isothermal boundaries of the plate.

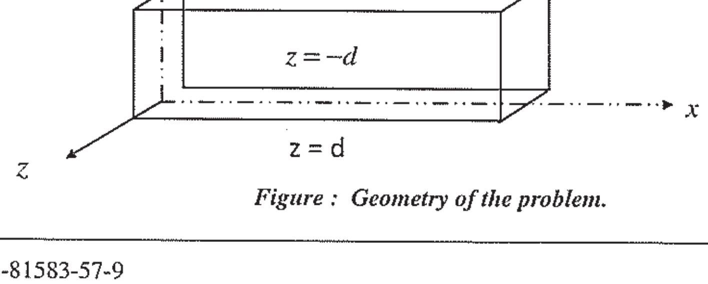

Figure : Geometry of the problem.

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The basic governing equations of micropolar generalized thermoelasticity Kumar and Singh[1998] in the absence of body forces and heat sources are

$$(\lambda+2\mu+K)\nabla(\nabla.\vec{u})-(\mu+K)\nabla\times(\nabla\times \ \vec{u})+K\nabla\times\phi$$
$$-\nu(1+t_1\delta_{2k}\frac{\partial}{\partial t})\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad , \tag{1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+K\nabla\times\vec{u}-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{2}$$

$$K^*\nabla^2T=\rho C^*(\frac{\partial T}{\partial t}+t_0\frac{\partial^2 T}{\partial t^2})+\nu T_0(\frac{\partial}{\partial t}+\delta_{1k}t_0\frac{\partial^2}{\partial t^2})\nabla.\vec{u} \ , \tag{3}$$

The constitutive relations are given by

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu(1+t_1\delta_{2k}\frac{\partial}{\partial t})T\delta_{ij} \tag{4}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{j,j}+\lambda\phi_{j,i}, \qquad \text{i, j, r} = 1, 2, 3 \tag{5}$$

where $\vec{u}=(u,v,w)$ is the displacement vector, T(x, y, z, t) is the temperature change, $\vec{\phi}$ is micro-rotation vector, λ,μ,γ,K are material constants, ρ is the density, K^* is thermal conductivity, $\nu=(3\lambda+2\mu+K)\alpha_T$, α_T is the coefficient of linear thermal expansion, C_e is the specific heat at constant strain, σ_{ij} and m_{ij} are respectively the component force stress tensor and couple stress tensor, and δ_{ij} is Kronecker's delta. Here k = 1 for Lord Shulman (LS) theory and k = 2 for Green-Lindsay (GL) theory. The comma notation denotes spatial derivatives. The thermal relaxation time t_0 and t_1 satisfy the inequalities

$$t_0\geq t_1\geq 0 \tag{6}$$

for GL theory only. However, it has been proved by Strunin [2001] recently that the inequalities (6) are not mandatory for t_0 and t_1 to follow

For two dimensional problems, we take

$\vec{u}=(u,0,w)$,

$\vec{\phi}=(0,\phi,0)$,

so that equation (1) to (3) in non-dimensional form can be rewritten as

$$\left(1-\delta^2\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x \partial z}\right)+\delta^2\nabla^2 u-p\frac{\partial \phi}{\partial z}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}=\frac{\partial^2 u}{\partial t^2}\,, \tag{7}$$

$$\left(1-\delta^2\right)\left(\frac{\partial^2 u}{\partial x \partial z}+\frac{\partial^2 w}{\partial z^2}\right)+\delta^2\nabla^2 w+p\frac{\partial \phi}{\partial x}-\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}=\frac{\partial^2 w}{\partial t^2}\,, \tag{8}$$

$$\nabla^2\phi+\delta^*\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\delta^*\phi=\frac{1}{\delta_1^2}\frac{\partial^2\phi}{\partial t^2}\,, \tag{9}$$

$$\nabla^2 T-\left(\frac{\partial T}{\partial t}+t_0\frac{\partial^2 T}{\partial t^2}\right)=\in_T\left(\frac{\partial}{\partial t}+\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x}+t_0\frac{\partial w}{\partial z}\right). \tag{10}$$

Here we have defined the quantities

$$\begin{aligned}
&x'=\frac{\omega^* x}{c_1},\ z'=\frac{\omega^* z}{c_1},\ t'=\omega^* t,\ t_1'=\omega^* t_1,\\
&u'=\frac{\rho\omega^* c_1}{\nu T_0}u,\ w'=\frac{\rho\omega^* c_1}{\nu T_0}w,\ T'=T/T_0,\ \delta^2=c_2^2/c_1^2,\\
&\phi'=\frac{\rho c_1^2}{\nu T_0}\phi,\ \sigma_{ij}'=\frac{\sigma_{ij}}{\nu T_0},\ m_{ij}'=\frac{\omega^*}{c_1\nu T_0}m_{ij},\\
&\omega^*=\frac{\rho C_e c_1^2}{K^*},\ c_1^2=\frac{\lambda+2\mu+K}{\rho},\ c_2^2=\frac{\mu+K}{\rho},\\
&\in_T=\frac{T_0\nu^2}{\rho C_e(\lambda+2\mu+K)},\ \delta_1^2=\frac{c_4^2}{c_1^2},\ c_3^2=\frac{K^*}{\rho C_e\omega^*},\\
&c_4^2=\frac{\gamma}{\rho J},\ c_7^2=\frac{K}{\rho J},\ p=\frac{K}{\rho c_1^2},\ \delta^*=\frac{Kc_1^2}{\gamma\omega^*},\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.
\end{aligned} \tag{11}$$

where primes have been suppressed for convenience.

Boundary Conditions

We consider the following mechanical and thermal boundary conditions at the plate surfaces z = $\pm d$

Mechanical Conditions

The plate surfaces $z = \pm d$ are assumed to be free of stresses. Hence we have

$$\sigma_{zz} = \sigma_{zx} = m_{yz} = 0\,. \tag{12}$$

Thermal Conditions

The thermal boundary conditions at z = $\pm d$ are given by

$$T,_z + hT = 0 \tag{13}$$

where h is the surface heat transfer coefficient. Here $h \to 0$ corresponds to thermal insulated boundaries and $h \to \infty$ refers to isothermal one.

3. SOLUTION OF THE PROBLEM

In order to solve the problem, we introduce the potential functions q and ψ through the relations

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}\,, \qquad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}\,. \tag{14}$$

Inserting (14) in (7) - (10) we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) q = \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t}\right) T\,, \tag{15}$$

$$\left(\nabla^2 \psi - \frac{1}{\delta^2}\ddot{\psi}\right) = \frac{p}{\delta^2}\phi\,, \tag{16}$$

$$\nabla^2 \phi - 2\delta^* \phi - \frac{1}{\delta_1^2}\ddot{\phi} + \delta^* \nabla^2 \psi = 0\,, \tag{17}$$

$$\nabla^2 T - \left(\dot{T} + t_0 \ddot{T}\right) = \in_T \nabla^2 \left(\dot{q} + t_0 \delta_{1k} \ddot{q}\right) \tag{18}$$

Upon using equations (14) in the constitutive relations (15) and (18), we obtain

$$\sigma_{zz} = \nabla^2 q - 2\left(\delta^2 - \frac{p}{2}\right)\left(q,_{xx} + \psi,_{xz}\right) - \left(T + t_1 \delta_{2k} \dot{T}\right),$$

$$\sigma_{xz} = \delta^2\nabla^2\psi + 2\left(\delta^2 - \frac{p}{2}\right)(q,_{xz} - \psi,_{xx}) - p\phi_2 \ ,$$

$$m_{yz} = \gamma\frac{\partial\phi}{\partial z} \ . \tag{19}$$

We take the solution of the form

$$(q,\psi,T,\phi_2) = (f(z), g(z), h(z), w(z))e^{i\xi(x-ct)} \tag{20}$$

where $c = \dfrac{\omega}{\xi}$, is the non-dimensional phase velocity, ω and ξ are respectively the non dimensional circular frequency and wave number. Upon equation and then solving the resulting system of equations, the expressions for q, ψ, T, ϕ are obtained as

$$q = (A\cos m_1 z + B\sin m_1 z + C\cos m_2 z + D\sin m_2 z)e^{i\xi(x-ct)} \ , \tag{21}$$

$$\psi = (A'\cos m_3 z + B'\sin m_3 z + C'\cos m_4 z + D'\sin m_4 z)e^{i\xi(x-ct)} \ , \tag{22}$$

$$T = i\tau_1^{-1}\omega^{-1}\begin{bmatrix}(\alpha^2 - m_1^2)(A\cos m_1 z + B\sin m_1 z) \\ (\alpha^2 - m_2^2)(C\cos m_4 z + D\sin m_4 z)\end{bmatrix}e^{i\xi(x-ct)} \ , \tag{23}$$

$$\phi = \frac{\delta^2}{p}\begin{bmatrix}(\beta^2 - m_3^2)(A'\cos m_3 z + B'\sin m_3 z) \\ +(\beta^2 - m_4^2)(C'\cos m_4 z + D'\sin m_4 z)\end{bmatrix}e^{i\xi(x-ct)} \ . \tag{24}$$

The displacements u_1 and u_3 are obtained from equation (14), (21) and (22) as

$$u_1 = \begin{Bmatrix} i\xi[A\cos m_1 z + B\sin m_1 z + C\cos m_2 z + D\sin m_2 z] \\ +[-A'm_3\sin m_3 z + B'm_3\cos m_3 z - m_4 C'\sin m_4 z + D'm_4\cos m_4 z] \end{Bmatrix} e^{i\xi(x-ct)} \ , \tag{25}$$

$$u_3 = \begin{Bmatrix} [-m_1 A\sin m_1 z + m_1 B\cos m_1 z - m_2 C\sin m_2 z + m_2 D\cos m_2 z] \\ -i\xi[A'\cos m_3 z + B'\sin m_3 z + C'\cos m_4 z + D'\sin m_4 z)] \end{Bmatrix} e^{i\xi(x-ct)} \ , \tag{26}$$

where

$$m_1^2 = \xi^2\left(a_1^2c^2 - 1\right), \quad m_3^2 = \xi^2\left(a_3^2c^2 - 1\right),$$
$$m_2^2 = \xi^2\left(a_2^2c^2 - 1\right), \quad m_4^2 = \xi^2\left(a_4^2c^2 - 1\right), \tag{27}$$

$$a_{1,2}^2 = \frac{1}{2}\left\{ \begin{aligned} &(1 + \tau_0 - \iota\omega \in_T \tau_0'\tau_1) \\ &\qquad \pm\left[(1 - \tau_0 - \iota\omega \in_T \tau_0'\tau_1)^2 - 4\iota\omega \in_T \tau_0'\tau_1\tau_0\right]^{\frac{1}{2}} \end{aligned} \right\}, \tag{28}$$

$$a_{3,4}^2 = \frac{1}{2}\left\{ \begin{aligned} &\left[\frac{1}{\delta^2} + \frac{1}{\delta_1^2} + \frac{\delta^*}{\delta^2\omega^2}(p - 2\delta^2)\right] \\ &\pm\left[\left(\frac{1}{\delta^2} - \frac{1}{\delta_1^2} + \frac{\delta^*}{\delta^2\omega^2}(p - 2\delta^2)\right)^2 + \frac{4\delta^*\left(p - 2(\delta^2 - \delta_1^2)\right)}{\delta^2\delta_1^2\omega^2}\right]^{\frac{1}{2}} \end{aligned} \right\}, \tag{29}$$

$$\alpha^2 = \xi^2\left(c^2 - 1\right), \; \beta^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right), \tag{30}$$

$$\tau_0 = t_0 + i\omega^{-1}, \; \tau_0' = t_0\delta_{1k} + i\omega^{-1}, \; \tau_1 = t_1\delta_{2k} + i\omega^{-1}.$$

4. DERIVATION OF SECULAR EQUATIONS

Invoking the boundary conditions at the surface $z = \pm d$ of the plate and using the equations we obtain the system of eight simultaneous linear equations as below

$$P[Ac_1 + Bs_1 + Cc_2 + Ds_2] + Q[(A's_3 - B'c_3)m_3 + (C's_4 - D'c_4)m_4] = 0\,,$$
$$P[Ac_1 - Bs_1 + Cc_2 - Ds_2] + Q[(-A's_3 - B'c_3)m_3 + (-C's_4 - D'c_4)m_4] =$$
$$Q[(-As_1 + Bc_1)m_1 + (-Cs_2 + Dc_2)m_2] + P[A'c_3 + B's_3 + C'c_4 + D's_4] =$$
$$Q[(As_1 + Bc_1)m_1 + (Cs_2 + Dc_2)m_2] + P[A'c_3 - B's_3 + C'c_4 - D's_4] = 0\,,$$

$$f_3[A'c_3 + B's_3] + f_4[C'c_4 + D's_4] = 0,$$

$$f_3[A'c_3 - B's_3] + f_4[C'c_4 - D's_4] = 0\,,$$

$$g_1\left[(hc_1 - m_1 s_1)A + (hs_1 + m_1 c_1)B\right] + g_2\left[(hc_2 - m_2 s_2)A' + (hs_2 + m_2 c_2)B'\right] = 0 ,$$

$$g_1\left[(hc_1 + m_1 s_1)A + (m_1 c_1 - hs_1)B\right] + g_2\left[(hc_2 + m_2 s_2)A' + (m_2 c_2 - hs_2)B'\right] = 0 ,$$

where

$$P = \beta^2 - \xi^2 + \frac{p\xi^2}{\delta^2},\quad \mathrm{Q} = -2\mathrm{i}\xi\left(1 - \frac{\mathrm{p}}{2\delta^2}\right),\quad f_i = \beta^2 - m_i^2,$$

$$g_i = \alpha^2 - m_i^2,\quad c_i = \cos m_i d, \mathrm{s}_i = \sin m_i d \qquad i = 1,2,3,4.$$

The system of equations defined above have non-trivial solution if the determinant of the coefficient of amplitudes $(A, B, C, D, A', B', C', D')^T$ vanishes This after applying lengthy reductions and manipulations leads to the secular equations for the plate with stress free isothermal and thermally insulated boundaries, we obtain

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_2(m_1^2 - \alpha^2)}{m_1(m_2^2 - \alpha^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} - \frac{m_4(m_3^2 - \beta^2)}{m_3(m_4^2 - \beta^2)}\left[\frac{T_1}{T_4}\right]^{\pm 1} + \frac{m_2 m_4(m_1^2 - \alpha^2)(m_3^2 - \beta^2)}{m_1 m_3(m_2^2 - \alpha^2)(m_4^2 - \beta^2)}\left[\frac{T_2}{T_4}\right]^{\pm 1}$$

$$= \frac{\left(\beta^2 - \xi^2 + \frac{p\xi^2}{\delta^2}\right)^2 (m_1^2 - m_2^2)(m_4^2 - m_3^2)}{4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2 m_1 m_3 (m_2^2 - \alpha^2)(m_4^2 - \beta^2)}, \tag{31}$$

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_1(m_1^2 - \alpha^2)}{m_2(m_2^2 - \alpha^2)}\left[\frac{T_2}{T_3}\right]^{\pm 1} + \frac{4\xi^2 m_1 m_4\left(1 - \frac{p}{2\delta^2}\right)^2 (m_2^2 - m_1^2)(m_3^2 - \beta^2)}{\left(\beta^2 - \xi^2 + \frac{p\xi^2}{\delta^2}\right)^2 (m_2^2 - \alpha^2)(m_4^2 - m_3^2)}\left[\frac{T_4}{T_3}\right]^{\pm 1}$$

$$= \frac{4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2 m_1 m_3 (m_2^2 - m_1^2)(m_4^2 - \beta^2)}{\left(\beta^2 - \xi^2 + \frac{p\xi^2}{\delta^2}\right)^2 (m_2^2 - \alpha^2)(m_4^2 - m_3^2)}, \tag{32}$$

where

$$\alpha^2 = \xi^2(c^2-1),\ \beta^2 = \xi^2\left(\frac{c^2}{\delta^2}-1\right), T_i = \tan m_i d,\ i = 1,2,3.4.$$

Here the superscript +1 corresponds to skew-symmetric and –1 refers to symmetric modes of wave propagation. The equations (30 and (31) are the most general dispersion relations involving wave number and phase velocity of various modes of propagation in a micropolar, generalized thermoelastic plates under the considered situation. These equations can be recognized as modified Rayleigh-Lamb equations which respectively govern the symmetric and skew symmetric modes of wave propagation in force stress and couple stress free, thermally insulated or isothermal, micropolar, thermoelastic plates. We refer to such waves as micropolar thermoelastic plate waves rather than Lamb waves whose properties were derived by Lamb in 1917 for isotropic solids in elastokinetics.

5. PARTICULAR CASE

Micropolar Coupled Thermoelastic Plate

In case of coupled theory of thermoelasticity (CT) the thermal relaxation times vanish i.e. $t_0 = 0 = t_1$ so that $\tau_0 = \tau_0' = \tau_1 = i\omega^{-1}$. Consequently, from equation (31) we have

$$a_{1,2}^2 = \frac{1}{2}\left\{1 + i\omega^{-1}(1+\in_T) \pm \left[1 - i\omega^{-1}(1-\in_T)^2 - 4\in_T\ \omega^{-2}\right]\right\}^{\frac{1}{2}}. \tag{33}$$

Micropolar Elastic Plate

In case of uncoupled thermoelasticity (UCT), thermomechanical coupling vanishes ($\in_T = 0$), which leads to $a_1^2 = 1$ and $a_2^2 = \tau_0$ so that

$$m_1^2 = \alpha^2, \quad m_2^2 = \xi^2(\tau_0 c^2 - 1).$$

Consequently, the secular equation defined above reduce to

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{4\xi^2\alpha m_4\left(1-\frac{p}{2\delta^2}\right)^2(m_3^2-\beta^2)}{\left(\beta^2-\xi^2+\frac{p\xi^2}{\delta^2}\right)^2(m_4^2-m_3^2)}\left[\frac{T_4}{T_3}\right]^{\pm 1} = \frac{4\xi^2\alpha m_3\left(1-\frac{p}{2\delta^2}\right)^2(m_4^2-\beta^2)}{\left(\beta^2-\xi^2+\frac{p\xi^2}{\delta^2}\right)^2(m_4^2-m_3^2)}, \tag{34}$$

Thermoelastic Plate

In the absence of micro polarity (K = 0 =p), we have

$$m_3^2 = \beta^2, m_4^2 = \xi^2\left(\frac{c^2}{\delta_1^2} - 1\right).$$

and consequently, the secular equations respectively reduce to

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_2\left(m_1^2 - \alpha^2\right)}{m_1\left(m_2^2 - \alpha^2\right)}\left[\frac{T_2}{T_3}\right]^{\pm 1} = \frac{\left(\beta^2 - \xi^2\right)^2\left(m_1^2 - m_2^2\right)}{4\xi^2 m_1 \beta\left(m_2^2 - \alpha^2\right)} \tag{35}$$

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \frac{m_1\left(m_1^2 - \alpha^2\right)}{m_2\left(m_2^2 - \alpha^2\right)}\left[\frac{T_2}{T_3}\right]^{\pm 1} = \frac{4\xi^2 m_1 \beta\left(m_2^2 - m_1^2\right)}{\left(\beta^2 - \xi^2\right)^2\left(m_2^2 - \alpha^2\right)}. \tag{36}$$

The equations (35) and (36 are the same as obtained and discussed by Sharma et al. [2000] and Sharma [2001a].

6. REGIONS OF SECULAR EQUATIONS

Here depending on whether $c < \delta, 1, \frac{1}{|a_i|}, i = 1,2,3,4$ we may have $\alpha, \beta, m_i\ (i = 1,2,3,4)$ being purely imaginary, complex or real. Then for example the frequency equation is correspondingly altered as follows:

Region I: This region is characterized by $c < \delta, 1, \frac{1}{|a_i|}, i = 1,2,3,4$. In this case $m_i = 1,2,3,4$, α and β, so we replace α, β, m_i, i = 1,2,3,4 with $\iota\alpha', \iota\beta'$ and $\iota\alpha_i$, i = 1,2,3,4 respectively. The secular equation becomes

$$\left[\frac{\tanh(\alpha_1 d)}{\tanh(\alpha_3 d)}\right]^{\pm 1} - \frac{\alpha_1\left(\alpha_1^2 - \alpha'^2\right)}{\alpha_2\left(\alpha_2^2 - \alpha'^2\right)}\left[\frac{\tanh(\alpha_2 d)}{\tanh(\alpha_3 d)}\right]^{\pm 1} - \frac{4\xi^2\alpha_1\alpha_4\left(1 - \frac{p}{2\delta^2}\right)^2\left(\alpha_2^2 - \alpha_1^2\right)\left(\alpha_3^2 - \beta'^2\right)}{\left(\frac{\xi^2 p}{\delta^2} - \xi^2 - \beta'^2\right)^2\left(\alpha_2^2 - \alpha'^2\right)\left(\alpha_4^2 - \alpha_3^2\right)}\left[\frac{\tanh(\alpha_4 d)}{\tanh(\alpha_3 d)}\right]^{\pm 1}$$

$$= -\frac{4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2\alpha_1\alpha_3\left(\alpha_2^2 - \alpha_1^2\right)\left(\alpha_4^2 - \beta'^2\right)}{\left(\frac{p\xi^2}{\delta^2} - \beta'^2 - \xi^2\right)^2\left(\alpha_2^2 - \alpha'^2\right)\left(\alpha_4^2 - \alpha_3^2\right)}.$$

Region II: This region is characterized by $\delta < c < 1$. In this case we have

$m_i = i\alpha_i, i = 1,2 \quad \alpha = i\alpha'$ and the frequency equation becomes

$$\left[\frac{\tanh(\alpha_1 d)}{\tan(m_3 d)}\right]^{\pm 1} - \frac{\alpha_1(\alpha_1^2 - \alpha'^2)}{\alpha_2(\alpha_2^2 - \alpha'^2)}\left[\frac{\tanh(\alpha_2 d)}{\tan(m_3 d)}\right]^{\pm 1} + \frac{4\xi^2\alpha_1 m_4\left(1 - \frac{p}{2\delta^2}\right)^2(\alpha_2^2 - \alpha_1^2)(m_3^2 - \beta^2)}{\left(\frac{\xi^2 p}{\delta^2} - \xi^2 + \beta^2\right)^2(\alpha_2^2 - \alpha'^2)(m_4^2 - m_3^2)}\left[\frac{\tan(m_4 d)}{\tan(m_3 d)}\right]^{\pm 1}$$

$$= \frac{4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2 \alpha_1 m_3 (\alpha_2^2 - \alpha_1^2)(m_4^2 - \beta^2)}{\left(\frac{p\xi^2}{\delta^2} + \beta^2 - \xi^2\right)^2 (\alpha_2^2 - \alpha'^2)(m_4^2 - m_3^2)}.$$

Region III:

This region is characterized by c > 1 and the frequency equation governs the motion in this case.

Waves of Short Wave Length

Some information on the asymptotic behaviour is obtainable by putting $\xi \to \infty$. If we take $\xi > \frac{\omega}{\delta}$, it follows that $c > \delta, 1$ then the roots of $\alpha, \beta, m_i, i = 1,2,3,4$ lie in the region-1 and the secular equations respectively, reduce to

$$4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2(\alpha_1\alpha_2 + \alpha'^2)(\alpha_3\alpha_4 + \beta'^2)$$
$$= \left(\beta'^2 + \xi^2 - \frac{\xi^2 p}{\delta^2}\right)^2(\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4),$$

$$4\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2 \alpha_1\alpha_2(\alpha_1 + \alpha_2)(\alpha_3\alpha_4 + \beta'^2)$$
$$= \left(\beta'^2 + \xi^2 - \frac{\xi^2 p}{\delta^2}\right)^2(\alpha_3 + \alpha_4)(\alpha_1^2 + \alpha_2\alpha_1 + \alpha_1^2 - \alpha'^2).$$

The equations are respectively, the Rayleigh surface wave equations for a force stress and couple stress free thermally insulated and isothermal, micropolar, generalized thermoelastic half space.

Upon Ignoring the micro polarity effect (p = 0 = K) in the equations , we get

$$4\xi^2\left(\alpha_1\alpha_2+\alpha'^2\right)\beta'=\left(\beta'^2+\xi^2\right)^2\left(\alpha_1+\alpha_2\right), \qquad (375.6.3)$$

$$4\xi^2\alpha_1\alpha_2\left(\alpha_1+\alpha_2\right)\beta'=\left(\beta'^2+\xi^2\right)^2\left(\alpha_1^2+\alpha_2\alpha_1+\alpha_1^2-\alpha'^2\right), \qquad (385.6.4)$$

for isothermal and thermally insulated stress free/thermoelastic plates. The equations (37) and (38 are merely Rayleigh surface wave equations. The Rayleigh results enter here since, for such wavelengths, the finite thickness plate appears as a semi – infinite medium and hence vibrational energy is transmitted mainly along the surface of the plate.

Lame Modes

A special case of exact solutions called Lame modes but evidently first identified by Lamb in 1917 can be obtained by considering the special case $\beta=\xi\left(1-\frac{p}{\delta^2}\right)$, the roots for this case are in region II and the frequency equations reduces to

$$\tan m_3 d=0$$

Anti symmetric modes:

$$\Rightarrow m_3=\frac{n\pi}{2d}, \text{n}=0,2,4......$$

$$\tan m_3 d=\infty$$

Symmetric modes:

$$\Rightarrow m_3=\frac{n\pi}{2d}, \text{n}=1,3,5...$$

Here, the frequency is given by

$$\omega=\frac{\sqrt{4\beta^2 d^2+n^2\pi^2\left(1-\frac{p}{\delta^2}\right)^2}}{2da_3\left(1-\frac{p}{\delta^2}\right)}.$$

However in the absence of micro polarity effect (p = 0), $\beta=m_3$ and consequently the frequency is given by

$$\omega = \frac{n\pi\delta}{\sqrt{2}d}$$

which agrees with Sharma et al. [2000]. As expected these modes do-not depend upon the thermo mechanical coupling and thermal relaxation time. However, it is obvious that these modes depend upon the micropolar parameter K (or p) and thickness of the plate.

Thin Plate Results

Let us consider the case when the transverse wavelength with respect to thickness is quite large, so that $\xi d << 1$. The regions I and II yield the results of interest in this case. In region I, the symmetric case has no roots. For skew symmetric case, on retaining the first two terms in the expression of hyperbolic tangents, the secular equation reduces to

$$\left(\beta'^2 + \xi^2 - \frac{p^2\xi^2}{\delta^2}\right)^2 - \frac{4\xi^2 p\beta'^2}{2\delta^2}\left(p^3 - 2p + 2\right)$$
$$= \frac{4}{3}\xi^2\left(1 - \frac{p}{2\delta^2}\right)^2 d^2\left[m_3'^2 m_4'^2 - \beta'^2\left(m_4'^2 + m_3'^2\right)\right]$$
$$+ \frac{1}{3}\alpha^2 d^2\left(\beta'^2 + \xi^2 - \frac{p^2\xi^2}{\delta^2}\right)^2.$$

If, we Ignore the micro polarity effect, i.e. (K = p = 0), so that $m_3' = \beta'$. Then equation) reduces to $-\left(\beta'^2 - \xi^2\right)^2 = \frac{4}{3}\xi^2 d^2 \beta'^4 + \frac{1}{3}\alpha^2 d^2\left(\beta'^2 + \xi^2\right)^2$.

In region II, anti-symmetric case has no roots. The secular equation for symmetric case provides us

$$\alpha_2^2 + \alpha_1^2 - \alpha'^2 = -\frac{4\xi^2\alpha_1^2\alpha_2^2\left(m_3'm_4' - \beta'^2\right)\left(1 - \frac{p}{2\delta^2}\right)^2}{\left(\beta'^2 + \xi^2 - \frac{\xi^2 p}{\delta^2}\right)^2 (m_4 + m_3)m_3}.$$

If we Ignore the micro polarity effect (i. e. p = 0) the equation reduces to

$$\alpha_2^2 + \alpha_1^2 - \alpha'^2 = -\frac{4\xi^2 \alpha_1^2 \alpha_2^2}{\left(\beta'^2 + \xi^2\right)^2},$$

In general, here the wave modes depend upon the micropolar parameters.

7. NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating the theoretical results obtained in the proceeding sections and comparing these in the context of various theories of thermoelasticity we now present some numerical results. The material chosen for this purpose is Magnesium, crystal like material the physical data for which is given by Sharma and Kumar[1998]

$$\rho = 1.74 gm/cm^3, \quad \lambda = 9.4\times10^{11} dyne/cm^2, \quad \mu = 4.0\times10^{11} dyne/cm^2,$$

$$K = 1.0\times10^{11} dyne/cm^2, \ \gamma = 0.779\times10^{-4} dyne, \ \mathrm{j} = 0.2\times10^{-15} cm^2, \ T_0 = 23^o C,$$

$$\alpha_0 = 0.779\times10^{-4} dyne, \ C^* = 0.23 cal/gm^o C, \ K^* = 0.6\times10^{-2} cal/cm \sec^o C,$$
$$t_0 = 6.13\times10^{-13} \sec, t_1 = 8.765\times10^{-13} \sec, \ \varepsilon = 0.073, \ \varepsilon_1 = 0.069, \ d = 1cm.$$

The non-dimensional phase velocity of symmetric and skew-symmetric modes of wave propagation have been computed for various values of non-dimensional wave number from dispersion relations for stress free, thermally insulated and isothermal, micropolar, thermoelastic plates, respectively. The corresponding numerically computed values of phase velocity have been obtained. Graphs are still to be plotted.

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