AN INTRODUCTION TO ROUGH SOFT SET

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***Abstract:** Soft set theory and rough set theory can be used as generic mathematical tool for dealing with uncertainty. In this study a connection between soft set and rough set has been made and a new model rough soft set is introduced to deal with vagueness and impreciseness. Basic properties of soft approximation space are presented with some examples.*

Keywords: Soft Sets , Rough Sets , Rough Soft Sets , Soft approximation Space .

1. INTRODUCTION

The introduction of probability theory, fuzzy set theory ([10]) , Rough set theory ([7]) foster a progressive trend to deal with mathematics as the advance mathematical tool. But difficulties crop up in the process of solving the complicacies through the application of these theories in economics, engineering and environmental sciences. However, the introduction of the concept of soft set by Molodtsov ([5]) in 1999 has been presented as a new mathematical tool to deal with uncertainties to a considerable extend. Advancing research on soft set theory and its application in various sphere is moving forward as an extension of the scope of the soft theory. Researcher have introduced several operators of set theory along with the notion of soft groups including barbaric structure of soft sets and the notion of soft topological spaces. In this regard, Molodtsov ([5]) has demonstrated in his brilliant paper about the potential application of the soft set theory in different fields addressing the smoothness of functions, game theory, operation research, perron integration, probability theory measurement theory. This paper aims at introducing the new concept rough soft set, the hybrid model combining rough set with soft set which can be se from any practical purposes through height on the basic concept of fuzzy set, rough set and soft set.

Defining fuzzy set , soft set and rough set in the beginning of the stepping stone in the advancement of new theories , this paper expanses the scope and application of fuzzy set theory introducing by L.A.Zadeh (1965) to put the fuzzy set theory in to a more general form for establishing the classical notion of a set and to face fuzziness with sufficient clarity for judgement . Zadeh's introduction of fuzzy set theory empowers the method of comprehension the conceptual frame work to handle the problematic issues in the areas of pattern classification and information processing.

Definition 1.1 Consider the classical set A of the universe U ($A \subseteq U$). A fuzzy set **B** is defined by a set of ordered pairs

$B = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$, where $\mu_A(x)$ is a function called membership function, $\mu_A(x)$ specifies the grade or degree to which any element

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x in A belongs to the fuzzy set **B**. This definition associates with each element x ϵ A areal number $\mu_A(x)$ in the interval [0,1] which is assign to x. Larger values of $\mu_A(x)$ indicates higher degrees of membership.

2. SOFT SET

In this section we find some basic definitions for soft set. Throughout this article , U denotes an initial Universe set and E is a set of parameters ; the power set of U be denoted by P(U).

Definition 2.1 A pair (F, A) is called a soft set over U, where $A \subseteq E$ and F is a mapping given by $F : A \rightarrow P(U)$. That is, a soft set over U is a parametarized family of subsets of the universe U. For $x \in A$, F(x) may be considered as set of x-approximate elements of the soft set (F,A).

Definition 2.2 Let (F, A) and (G,B) soft sets, $A, B \subseteq E$ over a common universe U. Then (G,B) is called a subset of (F,A) denoted by $(G,B) \subseteq (F,A)$ iff

(i) $B \subseteq A$

(ii) $G(x) \subseteq F(x)$ for all $x \in B$

In this case (F,A) is said to be a soft super set of (G,B).

Definition 2.3 Two soft sets (F,A) and (G,B) , $A, B \subseteq E$ over the common universe U are said to be soft equal if $(F,A)\tilde{\subset}(G,B)$ and $(G,B)\tilde{\subset}(F,A)$. This is denoted by $(F,A)\tilde{=}(G,B)$.

Definition 2.4 Let (F,A) and (G,B) be two soft sets over common universe U. The union of (F,A) and (G,B) is defined as the soft set (H,C) satisfying the following condition

$C = AUB$

For all $x \in C$

$$H(x) = \begin{cases} F(x), if, x \in A - B \\ G(x), if, x \in B - A \\ F(x) \cup G(x), if, x \in A \cap B \end{cases}$$

We use the notion $(H,C)=(F,A)\tilde{\cup}(G,B)$

Definition 2.5 Let (F,A) and (G,B) be two soft sets over a common universe U. The intersection of (F,A) and (G,B), denoted by

$(F,A)\tilde{\cap}(G,B)$ be defined as the soft set (H,C) iff $C=(A\cap B)$ For all $x\in C, H(x)=F(x)\cap G(x)$.

Definition 2.6 Let U he an initial universe, E be the set of parameters and $A\subseteq E$.

(a) (F,A) is called relative null soft set (with respect to the parameter set A,) denoted by ϕ_A, if $F(a)=\phi$ for all $a\in A$.

The relative soft set with respect to E is called the null soft set over U and is denoted by ϕ_E, if $F(x)=\phi$ for all $a\in E$.

(b) (G,A) is called a relative whole soft set with respect to the parameter set A, denoted by U_A if G(e)=U for all $e\in A$.

The relative whole soft set with respect to the set of parameter e is called absolute soft set over u and denoted by U_E if G(x)=U for all $x\in E$.

The soft set over U with empty parameter set is called empty soft set over U and is denoted by ϕ_ϕ. We note here that ϕ_ϕ and ϕ_A are different soft sets over U and $\phi_\phi \subseteq \phi_A \subseteq (F,A) \subseteq U_A \subseteq U_E$ for all soft set (F, A) over U.

Definition 2.7 Let $E=\{e_1,e_2,.......e_n\}$ be a set of parameter. The not set of E denoted by –E, is defined by $-E=\{-e_1,-e_2,......-e_n\}$ where $-e_k = not\ e_k$, $1\leq k\leq n$.

The complement of a soft set (F, A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c=(F^c,-A)$ where $F^c:-A\rightarrow P(U)$ is a mapping given by $F^c(x)=U-F(-x)$ for all $x\in -A$.

Clearly $(U_A)^c=\phi_{-A}$. and $(U_E)^c=\phi_{-E}$

Definition 2.8 Let (F,A) and (G, B) be two soft sets over the same universe U and $A,B \subseteq E$. The difference of the soft sets be denoted by $(F,A)\tilde{-}(G,B)=(H,C)$, where $C=A-B$ and for all $x \in C$, $H(x)=F(x)-G(x)$. We note here that $(F,A)^c \neq U_A \tilde{-}(F,A)$,

$(F,A)\tilde{\cup}(F,A)^c \neq U_A$ and

$(F,A)\tilde{\cap}(F,A)^c = \phi_\phi$ as $A \cap -A = \phi$.

3. EXAMPLES

This example is quoted directly from D.Molodtsov ([5])

Example 3.1 Let the universe $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$ be six houses under consideration and let $E=\{x_1,x_2,x_3,x_4,x_5\}$ be set of decision parameters. The parameter x_1 stands for expensive, parameter x_2 stands for beautiful ; x_3 stands for wooden ; x_4 for cheap and x_5 for green surroundings. Consider the mapping $F:E \rightarrow P(U)$ given by $F(x_1)$ means house expensive and its functional value is the set $\{h \in U : h$ is an expensive house$\}$.Suppose it is given that $F(x_1)=$ expensive house = $\{h_2, h_4\}$; $F(x_2)=\{h_1,h_3\}$, $F(x_3)=\phi$, $F(x_4)=\{h_1,h_3,h_5\}$, $F(x_5)=\{h_1\}$. Then the soft set (F, E) is consisting of the following collection of approximation:

(F , E) = {(expensive houses, $\{h_2,h_4\}$), (beautiful houses, $\{h_1,h_3\}$), (wooden houses, ϕ), (cheap houses, $\{ h_1, h_3, h_5 \}$),(in green surrounding,$\{h_1\}$)}

$$=\{(x_1,\{h_2,h_4\}),(x_2,\{h_1,h_3\}),(x_3,\phi),$$
$$(x_4,\{h_1,h_3,h_5\}),(x_5,\{h_1\})\}$$

Each approximation has two parts ; predicate and an approximate value set.

Example3.2 Let $U=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8\}$ be the initial universe and let $E = \{e_1,e_2,e_3,e_4,e_5,e_6\}$be set of parameters and let $F:E \rightarrow P(U)$ be a mapping given by F $(e_1) = \{h_1,h_3,h_5\}$, F$(e_2)=\phi$, F $(e_4) = \{h_7, h_8\}$ F$(e_5) =\{h_8\}$, F $(e_6) = \{h_4$ $h_6, h_8\}$

Let A= $\{e_1, e_2, e_3\}$, B= $\{e_1, e_4, e_5, e_6\}$, Then the soft set (F,A) = {(e_1, {h_1, h_3, h_5}), (e_2, ϕ), (e_3, {h_3, h_4, h_5, h_6})}; and the soft set (F, B) = { (e_1, {h_1, h_3, h_5}), (e_4, {h_7, h_8}) , (e_5, {h_8}), (e_6, {h_4, h_6, h_8}).

Then the soft set $(F,A)^c = (F^c,-A)$ = {(-e_1, {h_2, h_4, h_6, h_7, h_8}), (-e_2, U), (-e_3, {h_1, h_2, h_7, h_8)} be the complement of the soft set (F,A). Also,

(F, A) $\tilde{\cap}$ (F, B) = (F, C) = {(e_1{h_1, h_3, h_5} and

(F, A) $\tilde{\cup}$ (F, B) = (F, D) = (F, {e_1, e_2, e_3, e_4, e_5, e_6}) = {(e_1, F(e_1)) , (e_2 , F(e_2)), (e_3 , F (e_3)) , (e_4 , F(e_4)), (e_5, F(e_5)), (e_6 , F(e_6))

And (F, A) $\simeq$ (F , B) = (F, K) = (F, {e_2, e_3}) = {(e_2, ϕ) , (e_3, {h_3, h_4, h_5, h_6})}

Example 3.3 Let U = {h_1, h_2, h_3, h_4, h_5, h_6, h_7} be the initial universe of seven house under consideration, and let E = {x_1, x_2, x_3, x_4, x_5, x_6} = {Red , Blue, White, Black, Yellow, Green} be the colour of houses as a set of parameters and $F: A \rightarrow P(U)$ and $G: B \rightarrow P(U)$ be the mapping given by , for A, B $\subseteq$ E and A= {x_1, x_2, x_3, x_5} , B = {x_1, x_2, x_4, x_6}, it has F(x_1) = {h_1, h_6} ,

F(x_2) = {h_1, h_2, h_3} , F(x_3) = ϕ , F(x_5) = {h_1, h_4, h_7} and as the x – approximate elements differ from person to person , we have G(x_1) = {h_2, h_3} ,G(x_2) = {h_1, h_2, h_6}, G(x_4) = { h_2, h_4, h_6} ,G(x_6) = {h_1, h_2, h_6, h_7} ; thus (F, A) and (G, B) be two soft sets of U .

A, B $\subseteq$ E , then (F, A) $\tilde{\cap}$ (G, B)
= (H, C)= {(x_1, ϕ) , (x_2 , {h_1, h_2})}
(F, A) $\tilde{\cup}$ (G, B) = (H, D) =
{(x_1, {h_1, h_2, h_3, h_6}), (x_2 , { h_1, h_2, h_3, h_6}), (x_3, ϕ) , (x_5 , {h_1, h_4, h_7}) ,(x_4, {h_2, h_4, h_6}) , (x_6, {h_1, h_2, h_6, h_7}) } .
$(F,A)^c = (F^c,-A)$ = {(not x_1, {h_2, h_3, h_4, h_5, h_7}), (not blue , {h_4, h_5, h_6, h_7}), (not white, U), (not yellow , {h_2, h_3, h_5, h_6})}
(F , A) $\simeq$ (G,B) = (H, M)= {white, ϕ), (yellow, {h_1, h_4, h_7}}

Example 3.4 let U = {a, b, c, d, e, f } be the initial universe and E= {x_1, x_2, x_3, x_4} be the parameter set.
Let the mapping $F: E \rightarrow P(U)$ be given by F(x_1) = {a, b} , F(x_2) = {c, f} , F(x_3) = {a, c, f} and F(x_4) = {a, b, c } .
Here the elements c, d $\in$ U are not approximated any of the parameter of soft set (F, E).
Let A = { x_1, x_2, x_4 } and B={ x_1, x_2, x_4 } $\subseteq$ E be two subsets of parameters then (F, A) and (F, B) be two soft sets.

Let $G: E \rightarrow P(U)$ be the another mapping given by $G(x_1)$ = {a, b} , $G(x_2)$ = {d}, $G(x_3)$ = {b, c} , and $G(x_4)$ = {c, d, e, f}
We note here that S= (F, A) be a soft set over U where $A \subseteq E$ and $F: E \rightarrow P(U)$ be a mapping. Then P= (U, S) is a soft approximation space.

Let P(S) be the set of all soft sets over U. Then (U, P(S)) be called soft knowledge base.

4.ROUGH SET

Here we recall some fundamental facts and definitions of rough sets introduced by Z. Pawlak ([7]) . The rough set philosophy is founded on the assumption that with every object of interest, we associates some information (data, knowledge).

According to Pawlak ([8]), knowledge is based on the ability to classify the objects , and by objects we mean anything.

Definition 4.1 A knowledge representation system (or an information system) is a pair (U, A) where U is a non empty, finite universe of discourse and A is a set of attributes; each attribute $a \in A$, is a function $a : U \rightarrow V_a$ is the set of values of attributes a.

Let R be an equivalence relation (or knowledge) defined on the non empty finite universe U. The pair (U, R) is called an approximation space. R is also called indiscernibility relation defined on U. If (U, A) is an information system and $B \subseteq A$, then the indiscernibility relation R= IND (B) can be defined as

$(x,y) \in IND(B) \Leftrightarrow a(x)= a(y)$, for all $a \in B$; $x,y \in U$ and a(x) denotes the value of attribute a for the object x.

For the indiscernibility relation R (an equivalence relation R) we define two approximations , for any $X \subseteq U$,

$\underline{R}(X) = \{y \in U : [y]_R \subseteq X\}$ and

$\overline{R}(X) = \{y \in U : [y]_R \cap X \neq \phi\}$

are called R- lower and R- upper approximation of X , respectively .
The set $X \subseteq U$ is called rough (inexact)with respect to R if $\underline{R}(X) \neq \overline{R}(X)$, otherwise the set X is called crisp(exact) set. Also the sets $POS_R(X) = \underline{R}(X)$, $NEG_R(X) = U - \overline{R}(X)$

And $BN_R(X) = \overline{R}(X) - \underline{R}(X)$

are called as the R- positive , the R- negative and the R- boundary region of X , respectively . so that X is called rough with respect to R if $BN_R(X) \neq \phi$.

The elements which are certainly belonging to X under R is known as R -positive region of X, the elements which are possibly belonging to X but not with certainly is called R- boundary (or R- border line) region of X and the elements which certainly do not belong to X under R is called R- negative region of X.

5.ROUGH SOFT SETS

Here we introduce rough soft approximations. Let U be the initial universe, the compliment of Y in U is denoted by $Y^c = \sim Y = U \sim Y$. Let E be the set of all parameters. $A \subseteq E$.

Definition 5.1 let S = (F,A) be a soft set over U. Then the triplet P = (U, F, A) is called soft approximation space and we define two approximation, for $X \subseteq U$, by

$$\underline{apr}_F(X) = \{u \in U : \exists x \in A, [u \in F(x) \subseteq X]\}$$

$$= \bigcup_{x \in A}\{F(x) : F(x) \subseteq X\}$$

$$\overline{apr}_F(X) = M \ , forX \subseteq M$$

$$\overline{apr}_F(X) = M \cup N, forX \not\subset M$$

Where

$$M = \{u \in U : \exists x \in A, [u \in F(x), F(X) \cap X \neq \phi\} = \bigcup_{x \in A}\{F(x) : F(x) \cap X \neq \phi\}$$

and $N = \bigcap_{x}\{F^c(x) : x \in (-A)\}$

are called the soft F-lower approximation and the soft F- upper approximation of X, respectively. That, $\underline{apr}_F(X)$ and $\overline{apr}_F(X)$ are referred to rough soft approximations of X with respect to the parameterized mapping F_A , where

$F_A = F \rightarrow P(A)$ be the given mapping. The set $X \subseteq U$ is called F-rough soft set if $\underline{apr}_F(X) \neq \overline{apr}_F(X)$ otherwise X is called F-soft definable set.

We denote the notation F_A to just indicate the parameter set A and mapping F. That is, F_A and F have the name mapping turn A to P(U). In this note we use the notation F instead of F_A everywhere.

The sets

$$POS_F(X) = \underline{apr}_F(X),$$

$$NEG_F(X) = \bigcup_{x\in A}\{F(X) : F(X) \cap X = \phi\} = U \sim M, for X \subseteq M$$

And $$BN_F(X) = \overline{apr}_F(X) - \underline{apr}_F(X)$$

are called F –soft positive region, F-soft negative region and F-soft boundary region of X, respectively.

We note here that in ([2]), Feng Feng et al have given the definition of soft rough set, but actually it would be rough soft set. They defined as follows:

Let S = (F, A) be a soft set over U. Then the pair P=(S,U) is called a soft approximation space. Based on the soft approximation space P, we define the following two operations

$$\underline{apr}_p(X) = \{u \in U : \exists a \in A, [u \in F(a) \subseteq X]\}$$ and

$$\overline{apr}_p(X) = \{u \in U : \exists a \in A, [u \in F(a), (a) \cap X \neq \phi]\}$$

assigning to every subset X $\subseteq$ U, two sets $\underline{apr}_p(X)$ and $\overline{apr}_p(X)$ are called the soft P-lower approximation and the soft P-upper approximation of X.

The set X $\subseteq$ U is called soft P-rough set if $\underline{apr}_p(X) \neq \overline{apr}_p(X)$, otherwise X is called soft P-definable.

In an example of Feng Feng et al ([2]) it is known that a subset X $\subseteq$ U can not be approximated by $\overline{apr}_p$ (X), that is, $\overline{apr}_p$ (X) $\not\supseteq$ X and also there exits one $\phi \neq$ X $\subseteq$ U for which $\overline{apr}_p(X) = \phi$ and $\underline{apr}_p(X) = \phi$. So it is not a good definition to define roughness in soft set theory.

In this note we find the properties by using the definition 5.1 for F-rough soft set.

Proposition 5.1 let S= (F, A) be a soft set over U and A $\subseteq$ E be a set of parameters, and P=(U,F,A) be the corresponding soft approximation space. The F-rough approximation satisfy the following properties

a. $\underline{apr}_F(\phi) = \phi, \overline{apr}_F(\phi) = \phi$

b. $\underline{apr}_F(U) = U, \overline{apr}_F(U) = U$

c. $\underline{apr}_F(X) \subseteq X \subseteq \overline{apr}_F(X), \forall X \subseteq U$

d. $\underline{apr}_F(X \cap Y) = \overline{apr}_F(X) \cap \underline{apr}_F(Y)$

e. $\underline{apr}_F(X \cup Y) \supseteq \underline{apr}_F(X) \cup \underline{apr}_F(Y)$, $\forall X, Y \subseteq U$

f. $X \subseteq Y \Rightarrow \underline{apr}_F(X) \subseteq \underline{apr}_F(Y)$

g. $X \subseteq Y \Rightarrow \overline{apr}_F(X) \subseteq \overline{apr}_F(Y)$

h. $\overline{apr}_F(X \cup Y) = \overline{apr}_F(X) \cup \overline{apr}_F(Y)$

i. $\overline{apr}_F(X \cap Y) \subseteq \overline{apr}_F \cap \overline{apr}_F(Y)$

Example 5.1 In the example 3.2 suppose that $X = \{h_2, h_6, h_7, h_8\} \subseteq U$ and $B = \{e_1, e_4, e_5, e_6\} \subseteq E$ be the set of parameters, (F, B) be given soft set , then $\overline{apr}_F(X) = \{h_7, h_8\}$ and

$$\overline{apr}_F(X) = M \cup N = \{h_4, h_6, h_7, h_8\} \cup \{h_2\} = \{h_2, h_4, h_6, h_7, h_8\}$$

Thus X is rough soft set with respect to the approximation operator apr_F.

7. CONCLUSION

A new concept rough soft set is born out of the challenging situations we face for the imprecise and indefinite nature of information in the event of knowledge explosion in the branches of social sciences, pure sciences and applied sciences . The amalgamation of rough set and soft set gives the Heybridge form of rough soft set. The paper validates the new concept with illustrious examples for application in the required space. A verification of the properties of the rough set strengthen the new concept. To sum up, knowledge through rough soft set would empower us to tackle the problems of vague and impreciseness in information sciences.

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